RD Sharma Solutions for Class 9 Maths Chapter 16 Circles

Question 1: Fill in the blanks:

(i) All points lying inside/outside a circle are called ______ points/_______ points.

(ii) Circles having the same centre and different radii are called _____ circles.

(iii) A point whose distance from the center of a circle is greater than its radius lies in _________ of the circle.

(iv) A continuous piece of a circle is _______ of the circle.

(v) The longest chord of a circle is a ____________ of the circle.

(vi) An arc is a __________ when its ends are the ends of a diameter.

(vii) Segment of a circle is a region between an arc and _______ of the circle.

(viii) A circle divides the plane, on which it lies, in _________ parts.

Solution:

(i) Interior/Exterior

(ii) Concentric

(iii) The Exterior

(iv) Arc

(v) Diameter

(vi) Semi-circle

(vii) Center

(viii) Three

Question 2: Write the truth value (T/F) of the following with suitable reasons:

(i) A circle is a plane figure.

(ii) Line segment joining the center to any point on the circle is a radius of the circle,

(iii) If a circle is divided into three equal arcs each is a major arc.

(iv) A circle has only finite number of equal chords.

(v) A chord of a circle, which is twice as long as its radius is the diameter of the circle.

(vi) Sector is the region between the chord and its corresponding arc.

(vii) The degree measure of an arc is the complement of the central angle containing the arc.

(viii) The degree measure of a semi-circle is 180^0.

Solution:

(i) T

(ii) T

(iii) T

(iv) F

(v) T

(vi) T

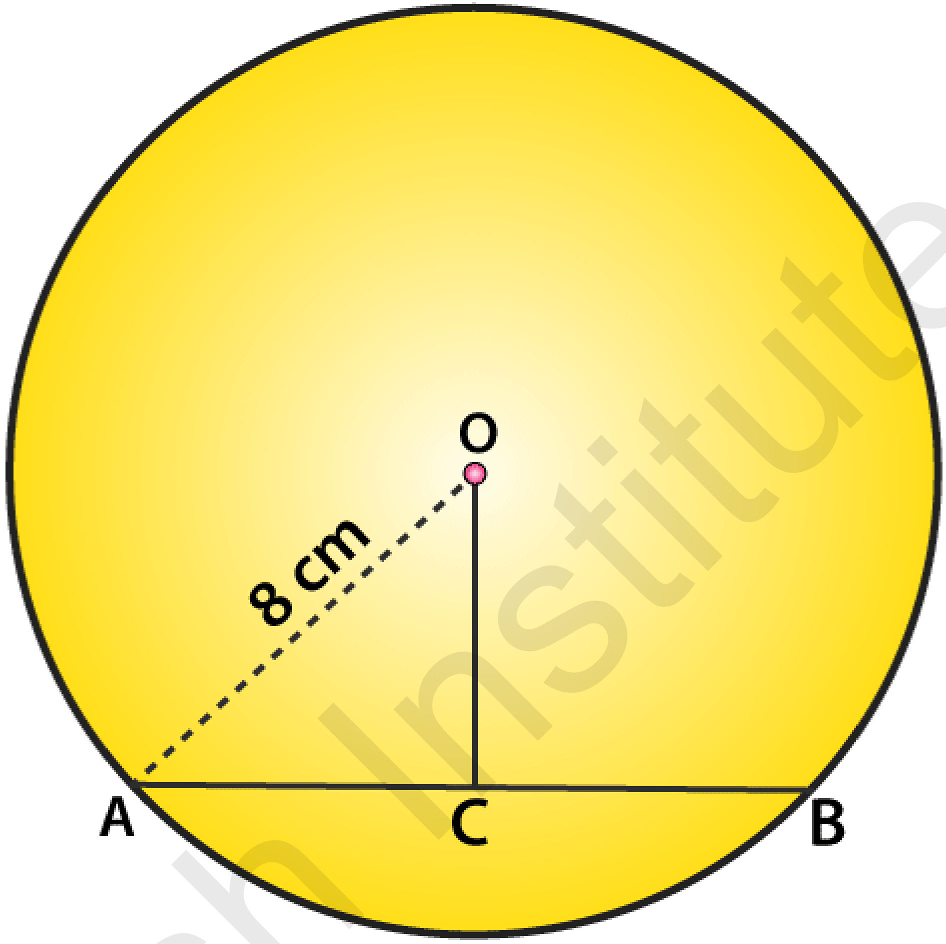
(vii) F

(viii) T

Exercise 16.2 Page No: 16.24

Question 1: The radius of a circle is 8 cm and the length of one of its chords is 12 cm. Find the distance of the chord from the centre.

Solution:



Radius of circle (OA) = 8 cm (Given)

Chord (AB) = 12cm (Given)

Draw a perpendicular OC on AB.

We know, perpendicular from centre to chord bisects the chord

Which implies, AC = BC = 12/2 = 6 cm

In right ΔOCA:

Using Pythagoras theorem,

$OA^2 = AC^2 + OC^2$

$64 = 36 + OC^2$

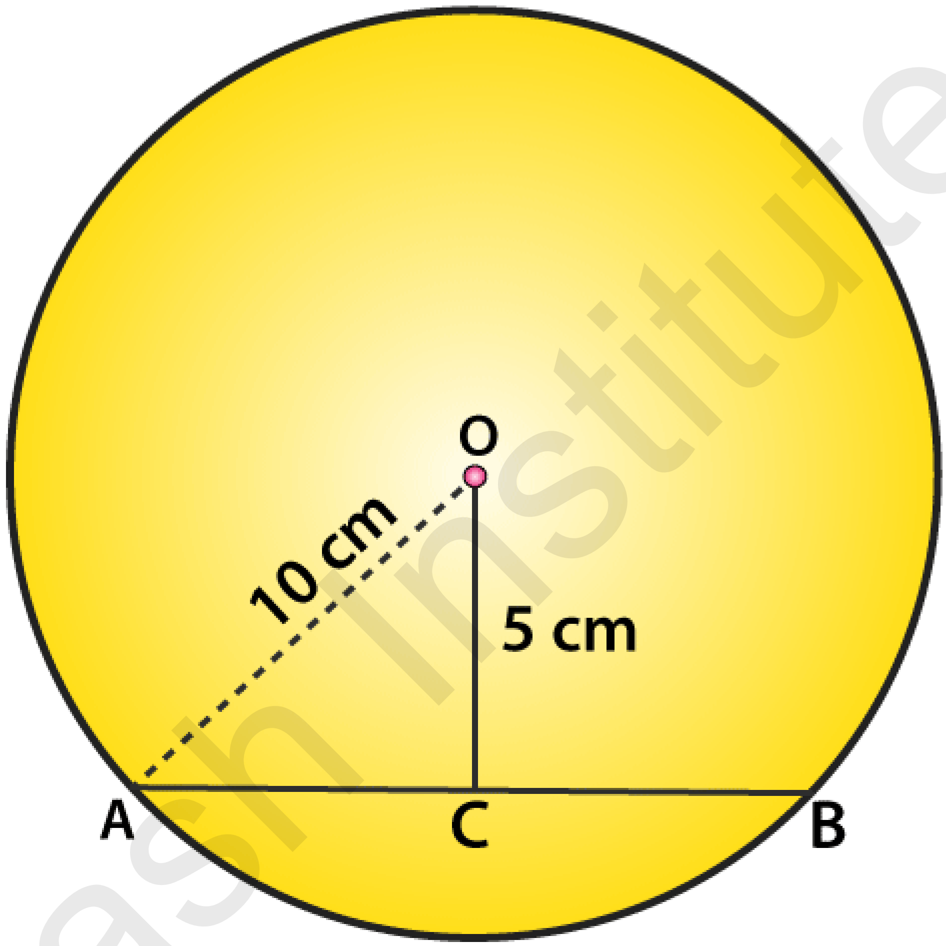
$OC^2 = 64 – 36 = 28$

or $OC = \sqrt{28} = 5.291$ (approx.)

The distance of the chord from the centre is 5.291 cm.

Question 2: Find the length of a chord which is at a distance of 5 cm from the centre of a circle of radius 10 cm.

Solution:



Distance of the chord from the centre = OC = 5 cm (Given)

Radius of the circle = OA = 10 cm (Given)

In ΔOCA:

Using Pythagoras theorem,

$OA^2 = AC^2 + OC^2$

$100 = AC^2 + 25$

$AC^2 = 100 – 25 = 75$

$AC = \sqrt{75} = 8.66$

As, perpendicular from the centre to chord bisects the chord.

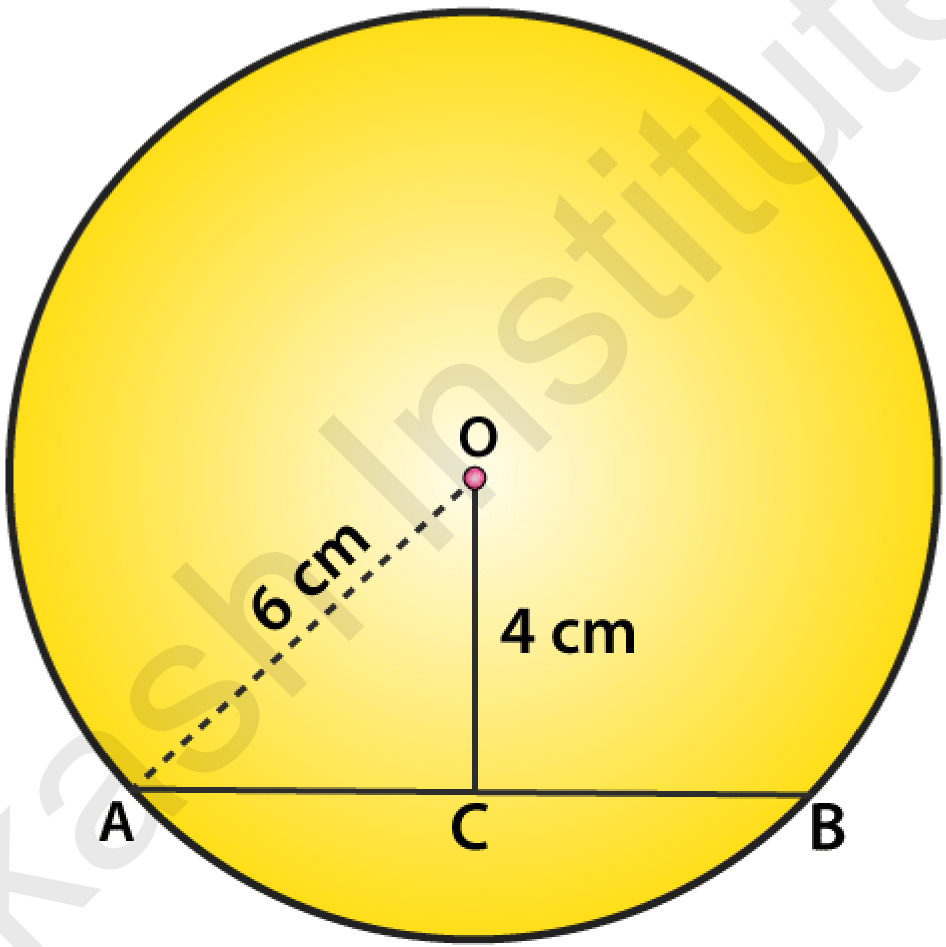
Therefore, AC = BC = 8.66 cm

=> AB = AC + BC = 8.66 + 8.66 = 17.32

Answer: AB = 17.32 cm

Question 3: Find the length of a chord which is at a distance of 4 cm from the centre of a circle of radius 6 cm.

Solution:



Distance of the chord from the centre = OC = 4 cm (Given)

Radius of the circle = OA = 6 cm (Given)

In ΔOCA:

Using Pythagoras theorem,

$OA^2 = AC^2 + OC^2$

$36 = AC^2 + 16$

$AC^2 = 36 – 16 = 20$

$AC = \sqrt{20} = 4.47$

Or AC = 4.47cm

As, perpendicular from the centre to chord bisects the chord.

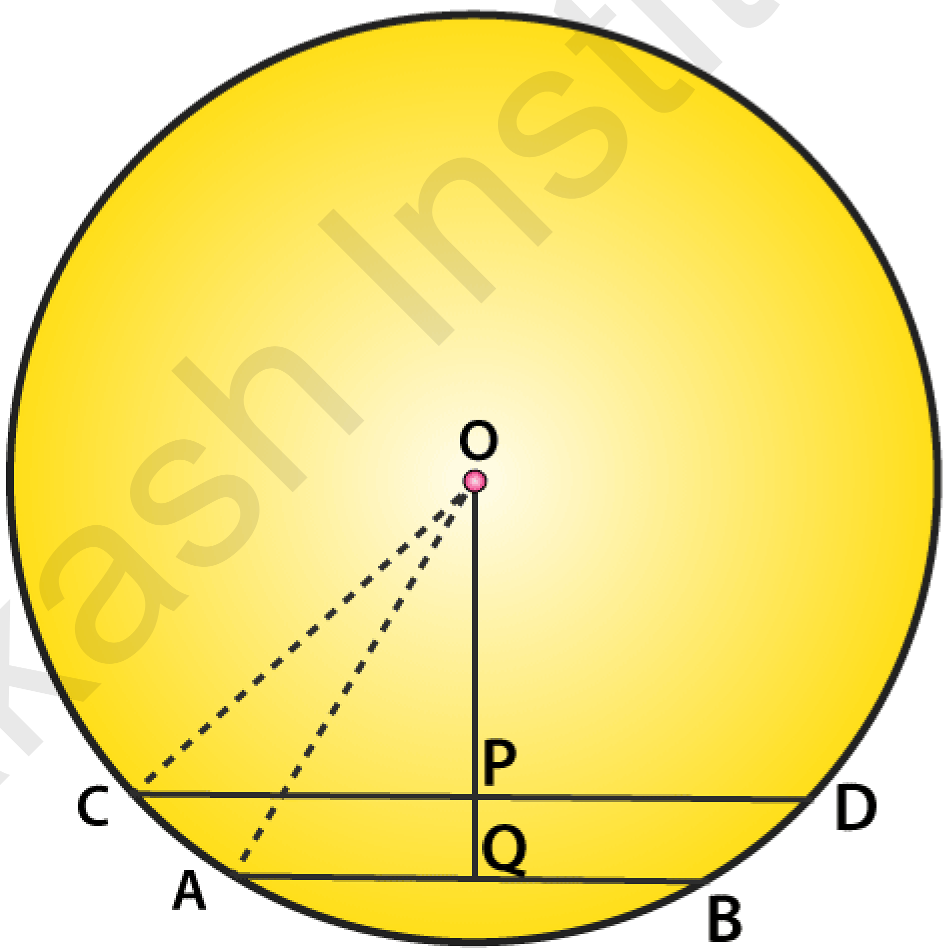
Therefore, AC = BC = 4.47 cm

=> AB = AC + BC = 4.47 + 4.47 = 8.94

Answer: AB = 8.94 cm

Question 4: Two chords AB, CD of lengths 5 cm, 11 cm respectively of a circle are parallel. If the distance between AB and CD is 3 cm, find the radius of the circle.

Solution:



Given: AB = 5 cm, CD = 11 cm, PQ = 3 cm

Draw perpendiculars OP on CD and OQ on AB

Let OP = x cm and OC = OA = r cm

We know, perpendicular from centre to chord bisects it.

Since $OP \perp CD$, we have

CP = PD = 11/2 cm

And $OQ \perp AB$

AQ = BQ = 5/2 cm

In ΔOCP:

By Pythagoras theorem,

$OC^2 = OP^2 + CP^2$

$r^2 = x^2 + (11/2)^2$ …..(1)

In ΔOQA:

By Pythagoras theorem,

$OA^2=OQ^2+AQ^2$

$r^2= (x+3)^2 + (5/2)^2$ …..(2)

From equations (1) and (2), we get

$(x+3)^2 + (5/2)^2 = x^2 + (11/2)^2$

Solve above equation and find the value of x.

$x^2 + 6x + 9 + 25/4 = x^2 + 121/4$

(using identity, $(a+b)^2 = a^2 + b^2 + 2ab$)

6x = 121/4 – 25/4 − 9

6x = 15

or x = 15/6 = 5/2

Substitute the value of x in equation (1), and find the length of radius,

$r^2 = (5/2)^2 + (11/2)^2$

= 25/4 + 121/4

= 146/4

or r = √146/4 cm

Question 5: Give a method to find the centre of a given circle.

Solution:

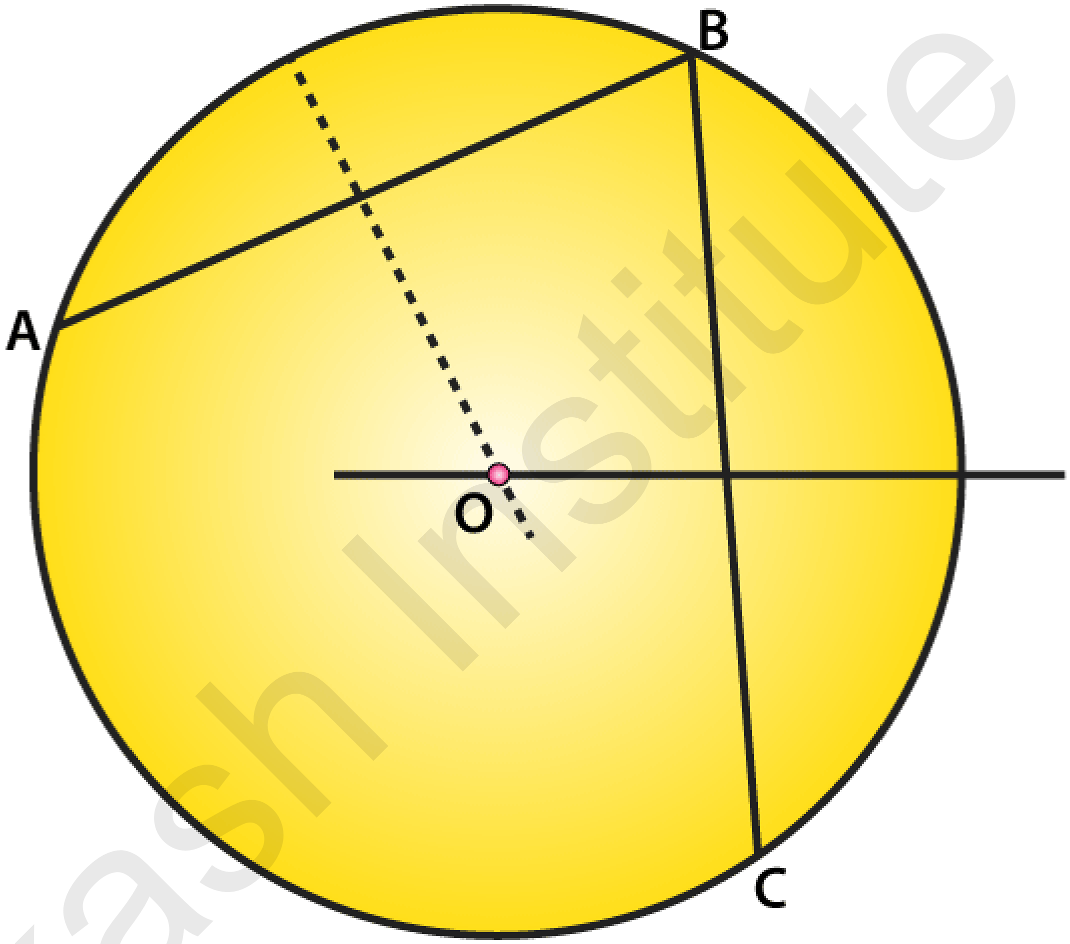
Steps of Construction:

Step 1: Consider three points A, B and C on a circle.

Step 2: Join AB and BC.

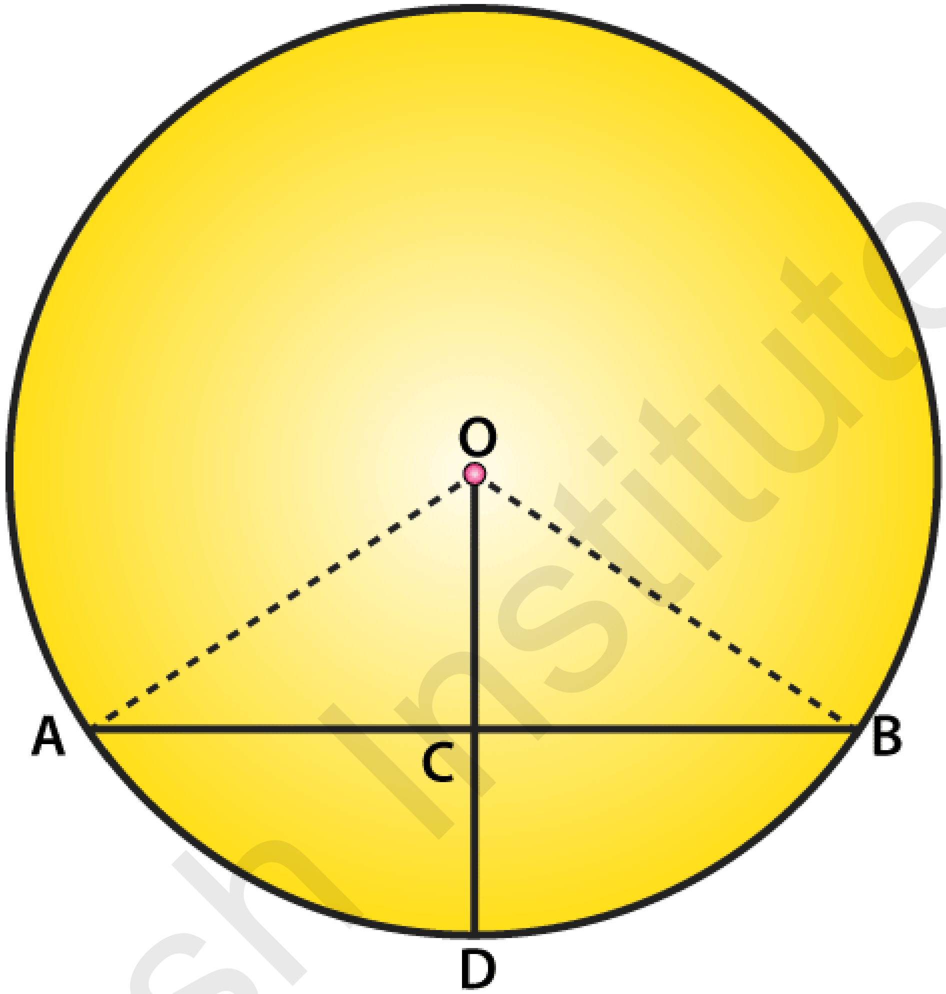
Step 3: Draw perpendicular bisectors of chord AB and BC which intersect each other at a point, say O.

Step 4: This point O is a centre of the circle, because we know that, the Perpendicular bisectors of chord always pass through the centre.



Question 6: Prove that the line joining the mid-point of a chord to the centre of the circle passes through the mid-point of the corresponding minor arc.

Solution:



From figure, Let C is the mid-point of chord AB.

To prove: D is the mid-point of arc AB.

Now, In ΔOAC and ΔOBC

OA = OB [Radius of circle]

OC = OC [Common]

AC = BC [C is the mid-point of chord AB (given)]

So, by SSS condition: ΔOAC ≅ ΔOBC

So, ∠AOC = ∠BOC (BY CPCT)

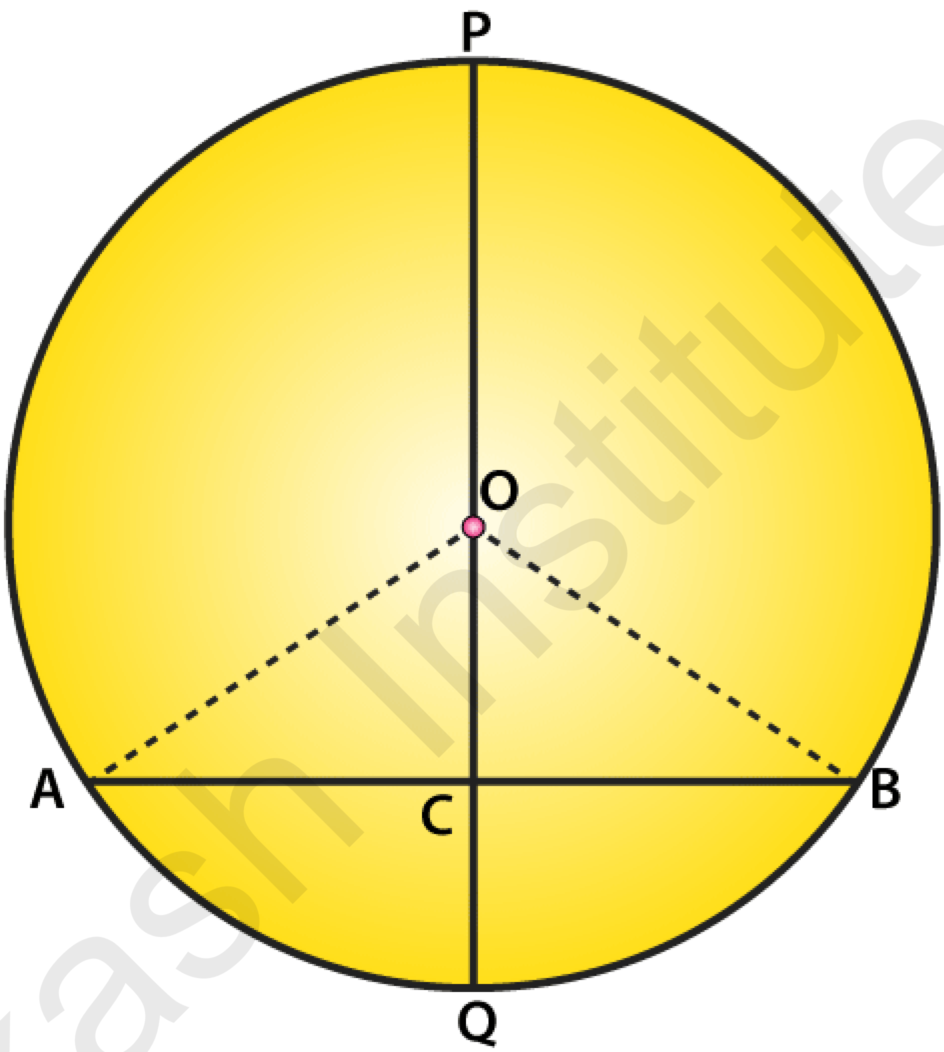
$$\Rightarrow m\bar{A}\bar{D} \cong m\bar{B}\bar{D}$$

$$\Rightarrow \bar{A}\bar{D} \cong \bar{B}\bar{D}$$

Therefore, D is the mid-point of arc AB. Hence Proved.

Question 7: Prove that a diameter of a circle which bisects a chord of the circle also bisects the angle subtended by the chord at the centre of the circle.

Solution:



Form figure: PQ is a diameter of circle which bisects the chord AB at C. (Given)

To Prove: PQ bisects ∠AOB

Now,

In ΔBOC and ΔAOC

OA = OB [Radius]

OC = OC [Common side]

AC = BC [Given]

Then, by SSS condition: ΔAOC ≅ ΔBOC

So, ∠AOC = ∠BOC [By c.p.c.t.]

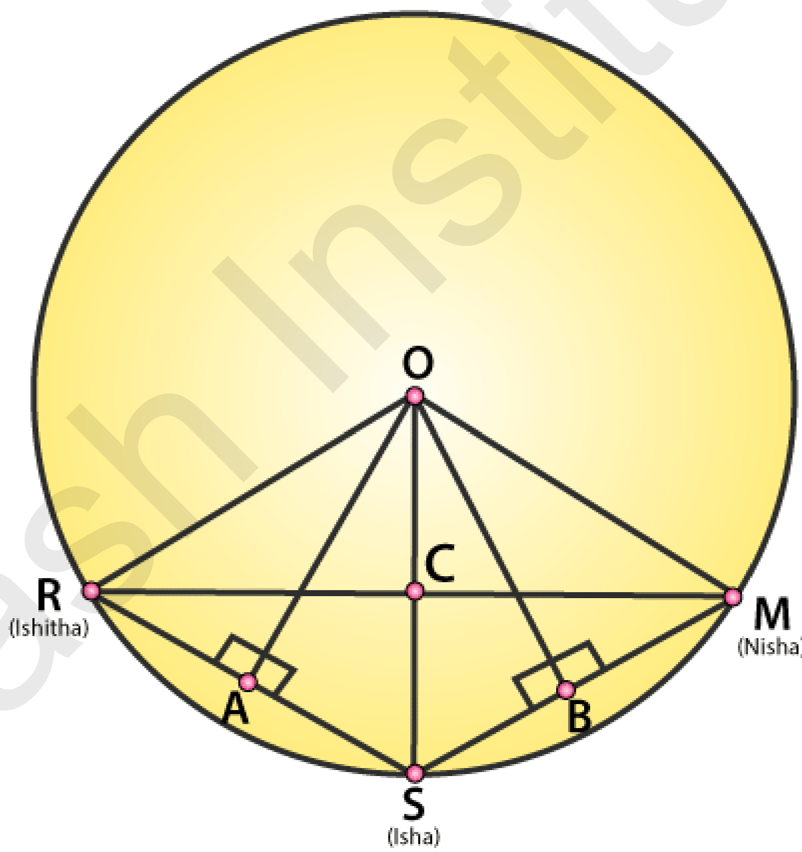
Therefore, PQ bisects ∠AOB. Hence proved.

Exercise 16.3 Page No: 16.40

Question 1: Three girls Ishita, Isha and Nisha are playing a game by standing on a circle of radius 20 m drawn in a park. Ishita throws a ball to Isha, Isha to Nisha and Nisha to Ishita. If the distance between Ishita and Isha and between Isha and Nisha is 24 m each, what is the distance between Ishita and Nisha.

Solution:

Let R, S and M be the position of Ishita, Isha and Nisha respectively.



Since OA is a perpendicular bisector on RS, so AR = AS = 24/2 = 12 cm

Radii of circle = OR = OS = OM = 20 cm (Given)

In ΔOAR:

By Pythagoras theorem,

$OA^2+AR^2=OR^2$

$OA^2+12^2=20^2$

$OA^2 = 400 – 144 = 256$

Or OA = 16 m …(1)

From figure, OABC is a kite since OA = OC and AB = BC. We know that, diagonals of a kite are perpendicular and the diagonal common to both the isosceles triangles is bisected by another diagonal.

So in ΔRSM, ∠RCS = 90^0 and RC = CM …(2)

Now, Area of ΔORS = Area of ΔORS

=>1/2×OA×RS = 1/2 x RC x OS

=> OA ×RS = RC x OS

=> 16 x 24 = RC x 20

=> RC = 19.2

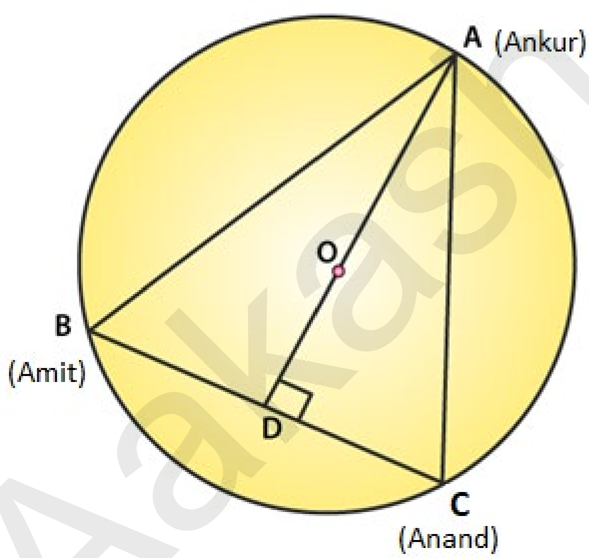
Since RC = CM (from (2), we have

RM = 2(19.2) = 38.4

So, the distance between Ishita and Nisha is 38.4 m.

Question 2: A circular park of radius 40 m is situated in a colony. Three boys Ankur, Amit and Anand are sitting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.

Solution:



Since, AB = BC = CA. So, ABC is an equilateral triangle

Radius = OA = 40 m (Given)

We know, medians of equilateral triangle pass through the circumcentre and intersect each other at the ratio 2 : 1.

Here AD is the median of equilateral triangle ABC, we can write:

$OA/OD = 2/1$

or $40/OD = 2/1$

or $OD = 20$ m

Therefore, $AD = OA + OD = (40 + 20)$ m $= 60$ m

Now, In ΔADC:

By Pythagoras theorem,

$AC^2 = AD^2 + DC^2$

$AC^2 = 60^2 + (AC/2)^2$

$AC^2 = 3600 + AC^2 / 4$

$3/4\ AC^2 = 3600$

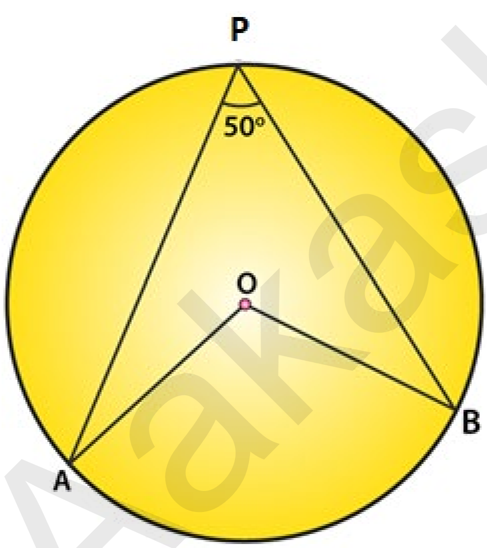
$AC^2 = 4800$

or $AC = 40\sqrt{3}$ m

Therefore, length of string of each phone will be $40\sqrt{3}$ m.

Exercise 16.4 Page No: 16.60

Question 1: In figure, O is the centre of the circle. If $\angle APB = 50^0$, find $\angle AOB$ and $\angle OAB$.



Solution:

$\angle APB = 50^0$ (Given)

By degree measure theorem: $\angle AOB = 2\angle APB$

$\angle AOB = 2 \times 50^0 = 100^0$

Again, $OA = OB$ [Radius of circle]

Then ∠OAB = ∠OBA [Angles opposite to equal sides]

Let ∠OAB = m

In ΔOAB,

By angle sum property: ∠OAB+∠OBA+∠AOB=180^0

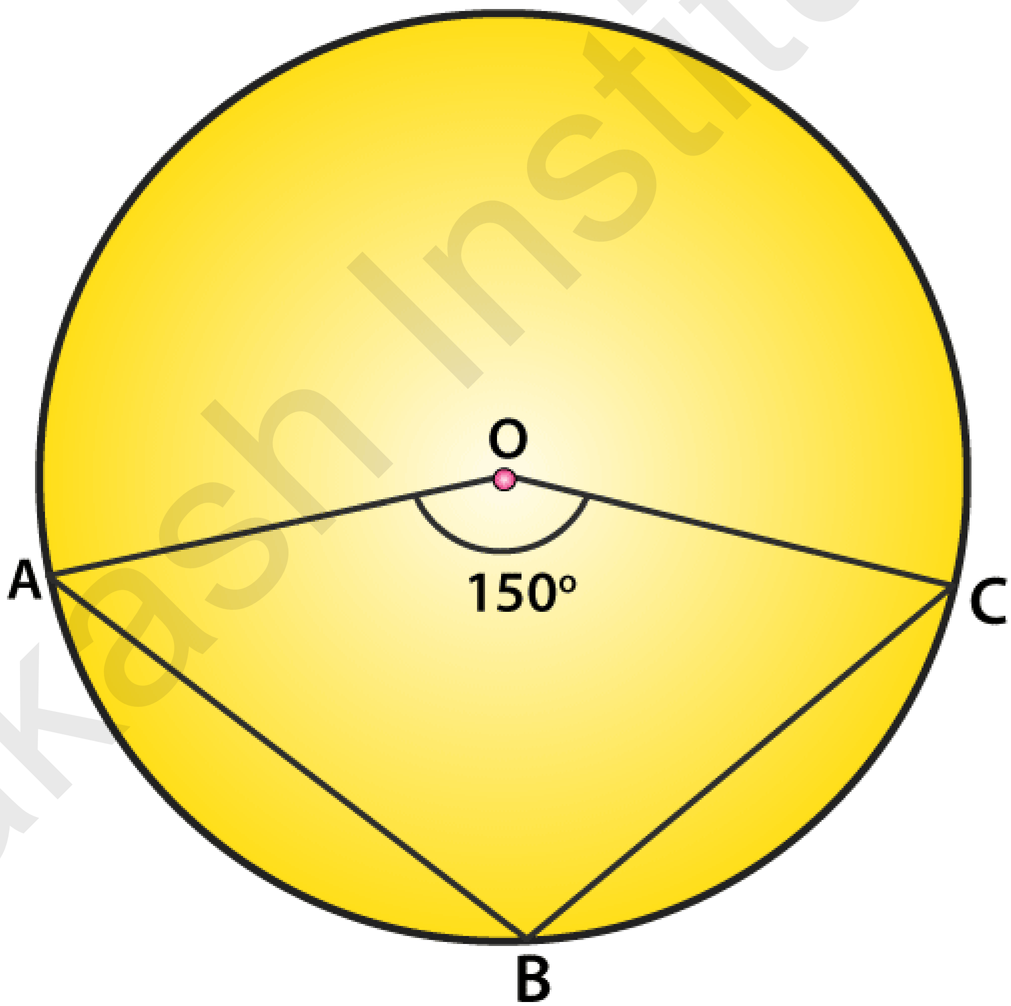
=> m + m + 100^0 = 180^0

=>2m = 180^0 – 100^0 = 80^0

=>m = 80^0/2 = 40^0

∠OAB = ∠OBA = 40^0

Question 2: In figure, it is given that O is the centre of the circle and ∠AOC = 150^0. Find ∠ABC.



Solution:

∠AOC = 150^0 (Given)

By degree measure theorem: ∠ABC = (reflex∠AOC)/2 …(1)

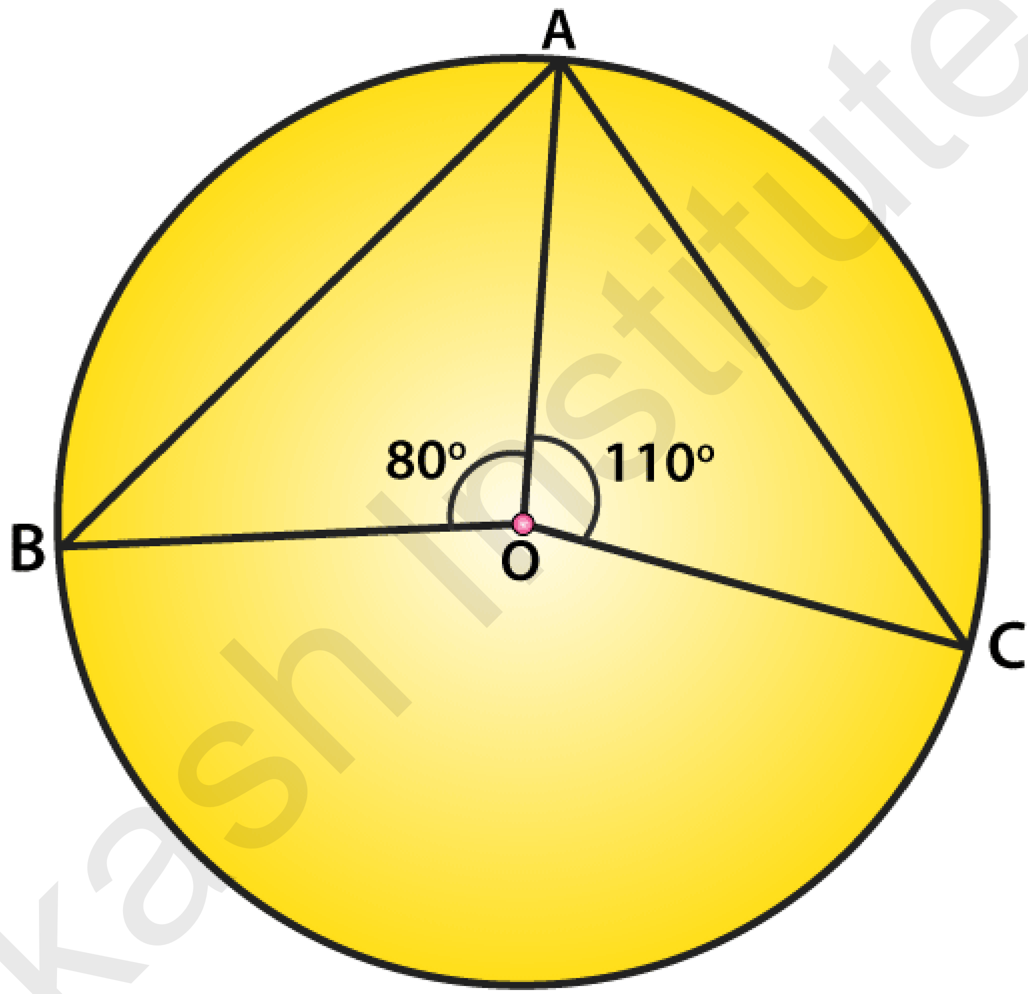
We know, ∠AOC + reflex(∠AOC) = 360^0 [Complex angle]

150^0 + reflex∠AOC = 360^0

or reflex ∠AOC = $360^0 - 150^0$ = 210^0

From (1) => ∠ABC = 210° /2 = 105°

Question 3: In figure, O is the centre of the circle. Find ∠BAC.



Solution:

Given: ∠AOB = 80^0 and ∠AOC = 110^0

Therefore, ∠AOB+∠AOC+∠BOC=360^0 [Completeangle]

Substitute given values,

80^0 + 100^0 + ∠BOC = 360^0

∠BOC = 360^0 – 80^0 – 110^0 = 170^0

or ∠BOC = 170^0

Now, by degree measure theorem

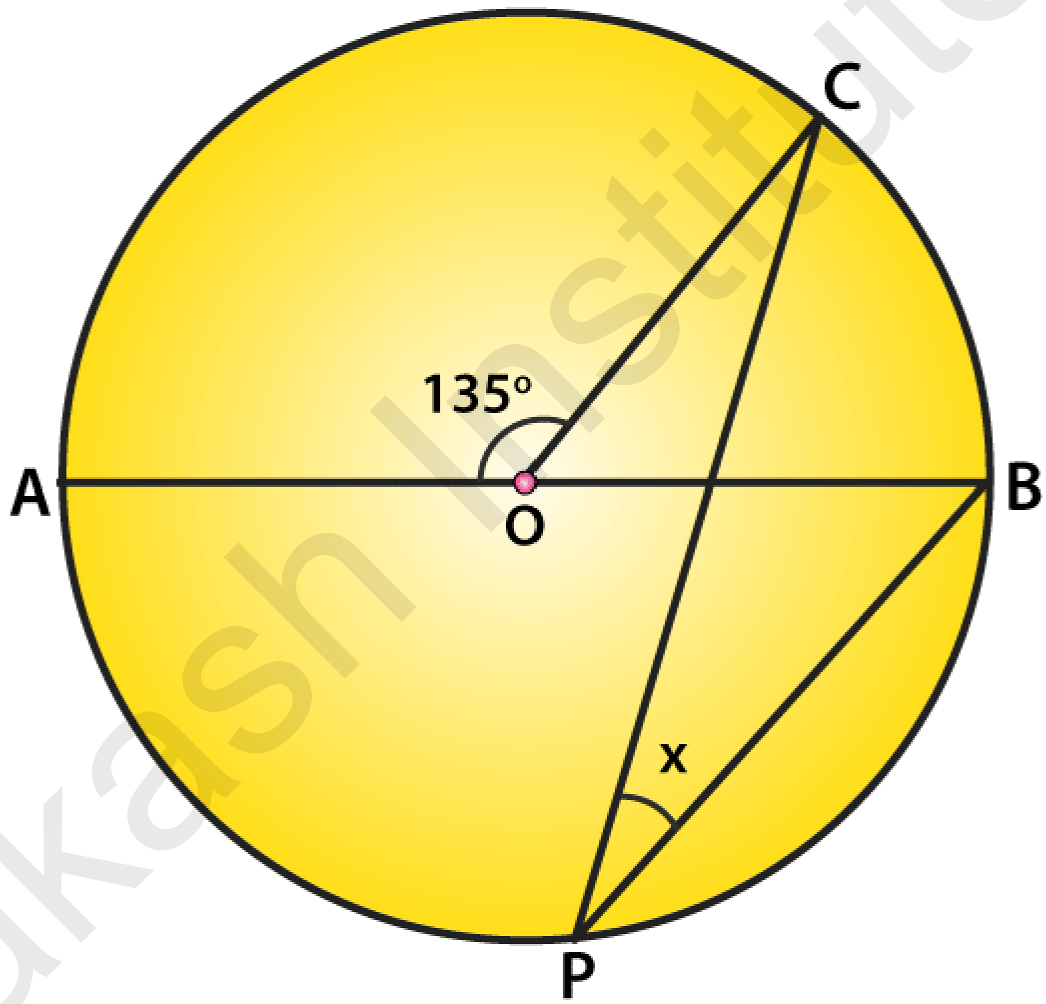
$\angle BOC = 2\angle BAC$

$170^0 = 2\angle BAC$

Or $\angle BAC = 170^0/2 = 85^0$

Question 4: If O is the centre of the circle, find the value of x in each of the following figures.

(i)



Solution:

$\angle AOC = 135^0$ (Given)

From figure, $\angle AOC + \angle BOC = 180^0$ [Linear pair of angles]

$135^0 + \angle BOC = 180^0$

or $\angle BOC = 180^0 - 135^0$

or $\angle BOC = 45^0$

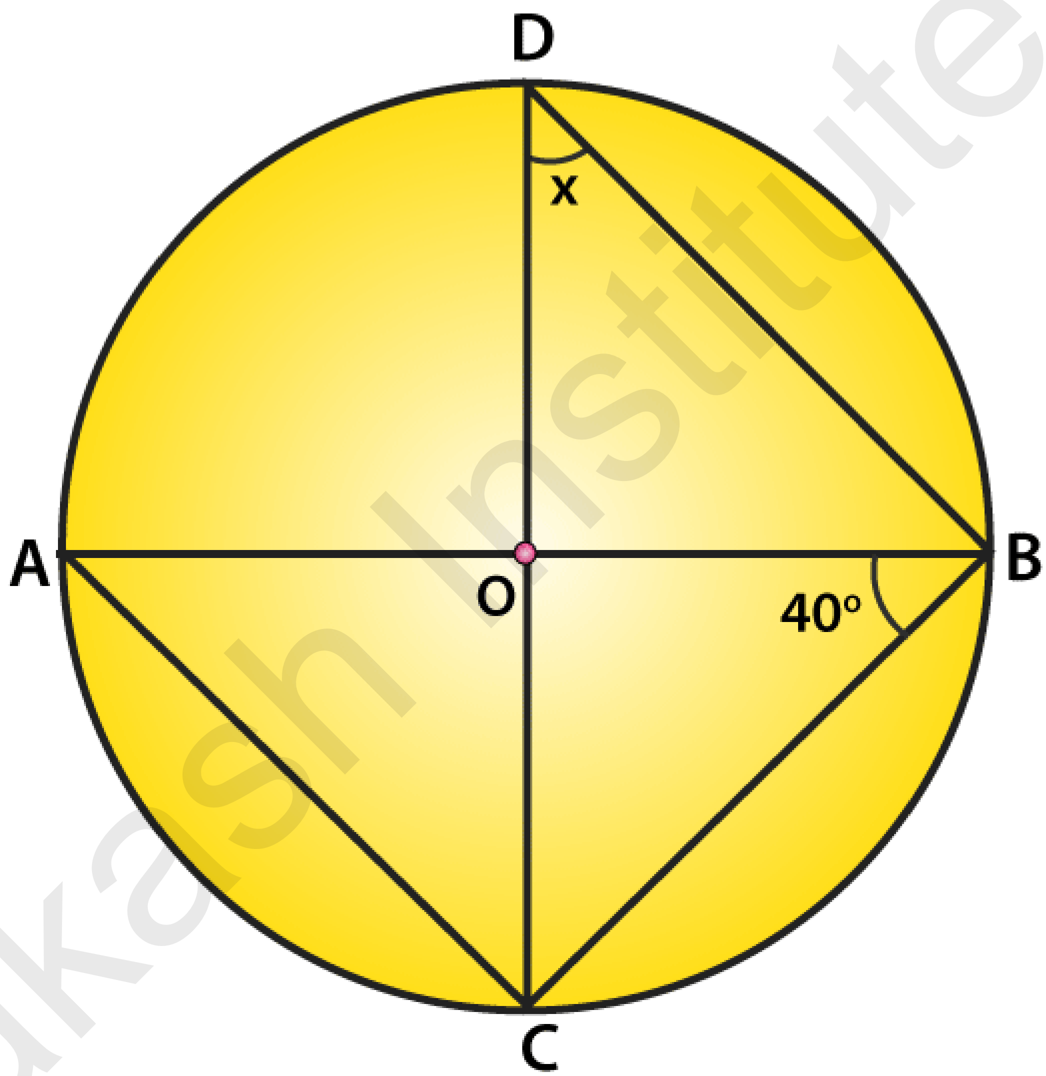
Again, by degree measure theorem

$\angle BOC = 2\angle CPB$

$45^0 = 2x$

$x = 45^0/2$

(ii)



Solution:

$\angle ABC = 40^0$ (given)

$\angle ACB = 90^0$ [Angle in semicircle]

In ΔABC,

$\angle CAB + \angle ACB + \angle ABC = 180^0$ [angle sum property]

$\angle CAB + 90^0 + 40^0 = 180^0$

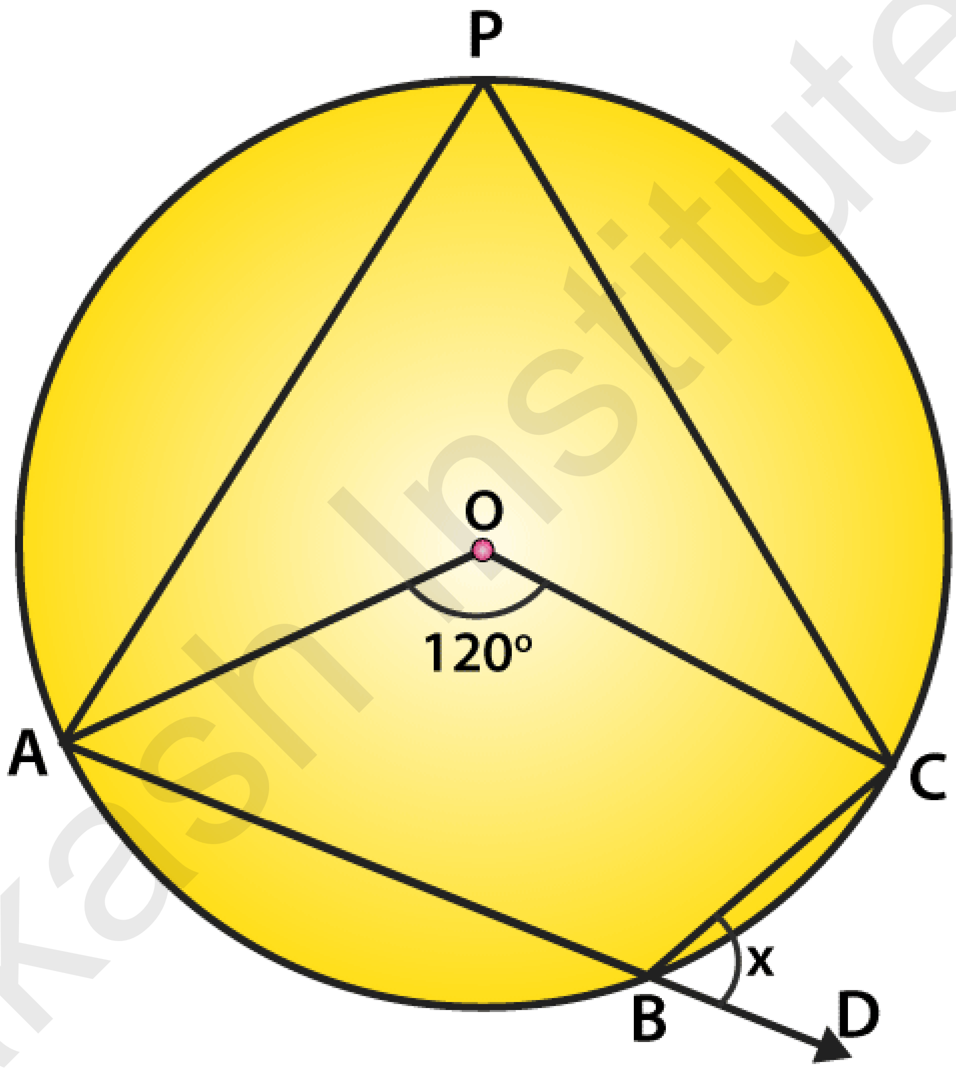
$\angle CAB = 180^0 - 90^0 - 40^0$

$\angle CAB = 50^0$

Now, $\angle CDB = \angle CAB$ [Angle is on same segment]

This implies, $x = 50^0$

(iii)



Solution:

$\angle AOC = 120^0$ (given)

By degree measure theorem: $\angle AOC = 2\angle APC$

$120^0 = 2\angle APC$

$\angle APC = 120^0/2 = 60^0$

Again, $\angle APC + \angle ABC = 180^0$ [Sum of opposite angles of cyclic quadrilaterals = 180°]

$60^0 + \angle ABC = 180^0$

$\angle ABC = 180^0 - 60^0$

$\angle ABC = 120^0$

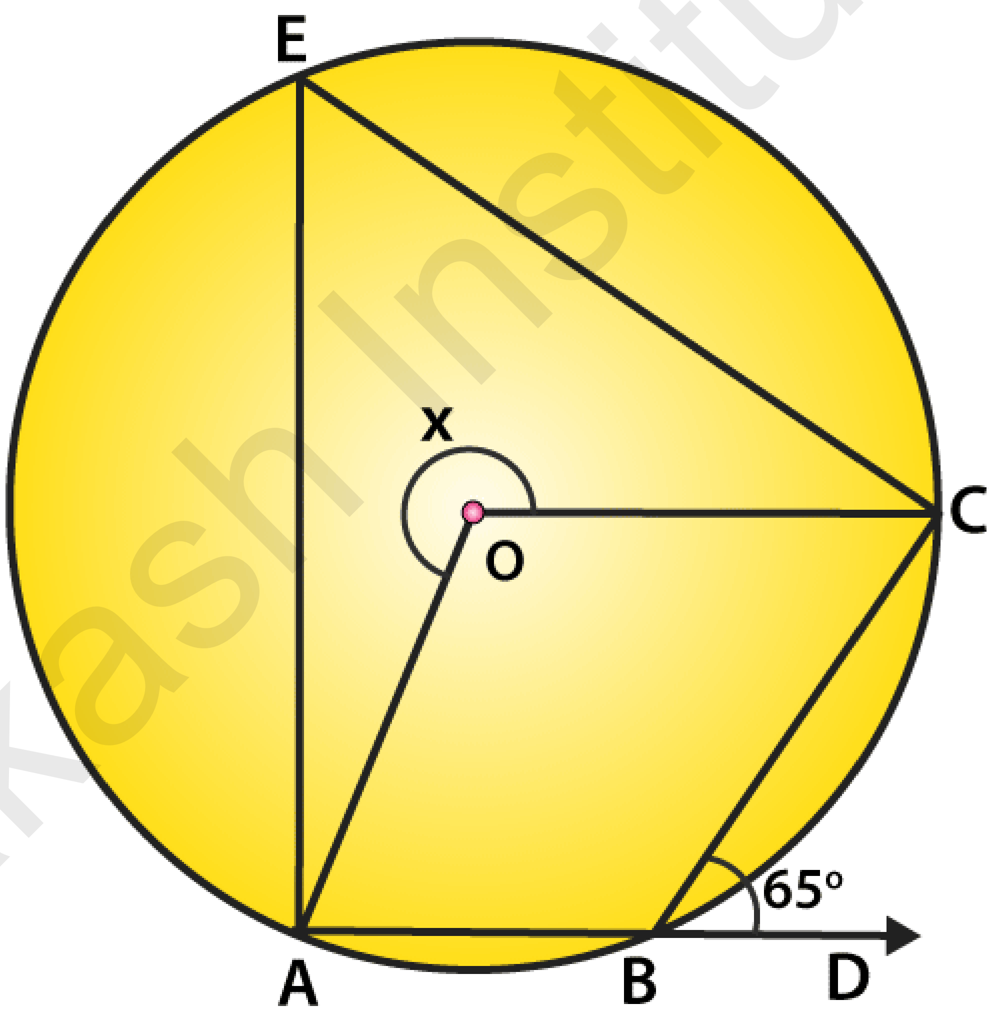
$\angle ABC + \angle DBC = 180^0$ [Linear pair of angles]

$120^0 + x = 180^0$

$x = 180^0 - 120^0 = 60^0$

The value of x is 60^0

(iv)



Solution:

$\angle CBD = 65^0$ (given)

From figure:

$\angle ABC + \angle CBD = 180^{0}$ [Linear pair of angles]

$\angle ABC + 65^{0} = 180^{0}$

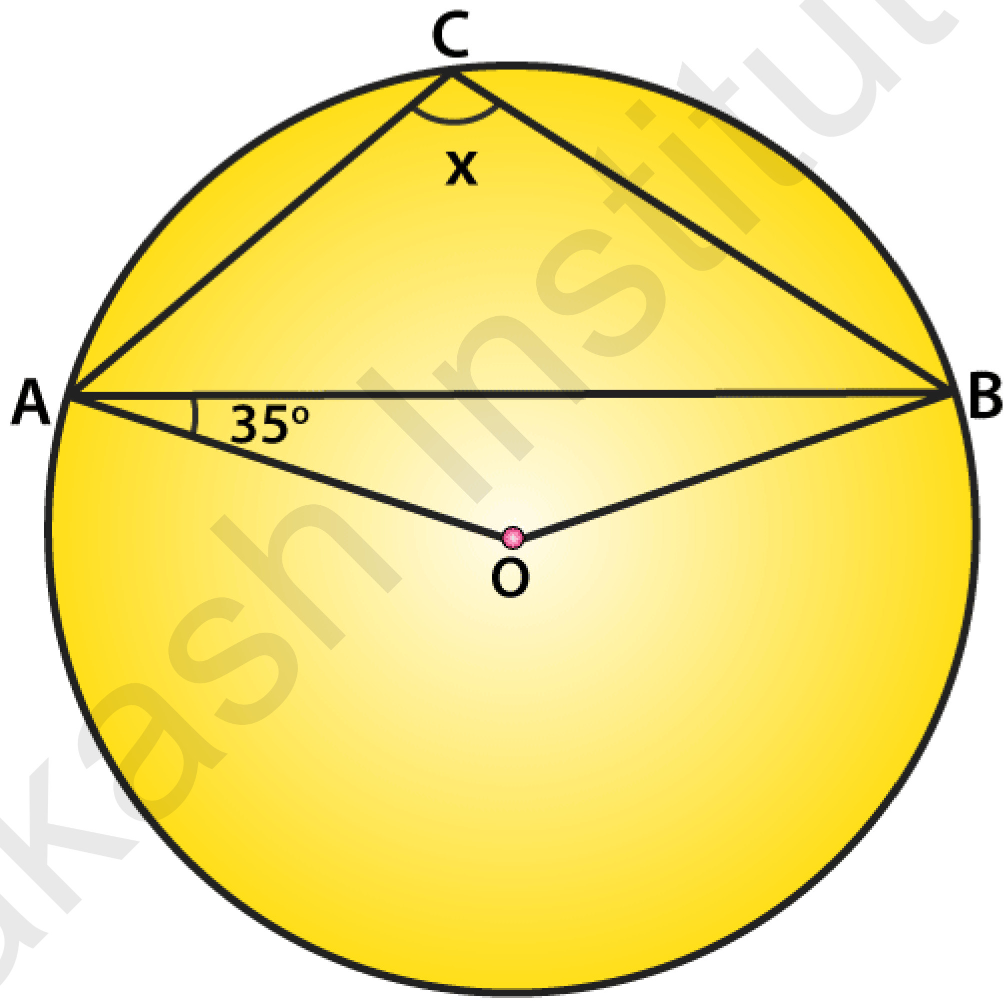
$\angle ABC = 180^{0} - 65^{0} = 115^{0}$

Again, reflex $\angle AOC = 2\angle ABC$ [Degree measure theorem]

$x = 2(115^{0}) = 230^{0}$

The value of x is 230^{0}

(v)



Solution:

$\angle OAB = 35^{0}$ (Given)

From figure:

$\angle OBA = \angle OAB = 35^{0}$ [Angles opposite to equal radii]

InΔAOB:

$\angle AOB + \angle OAB + \angle OBA = 180^0$ [angle sum property]

$\angle AOB + 35^0 + 35^0 = 180^0$

$\angle AOB = 180^0 - 35^0 - 35^0 = 110^0$

Now, $\angle AOB$ + reflex$\angle AOB = 360^0$ [Complex angle]

110^0 + reflex$\angle AOB = 360^0$

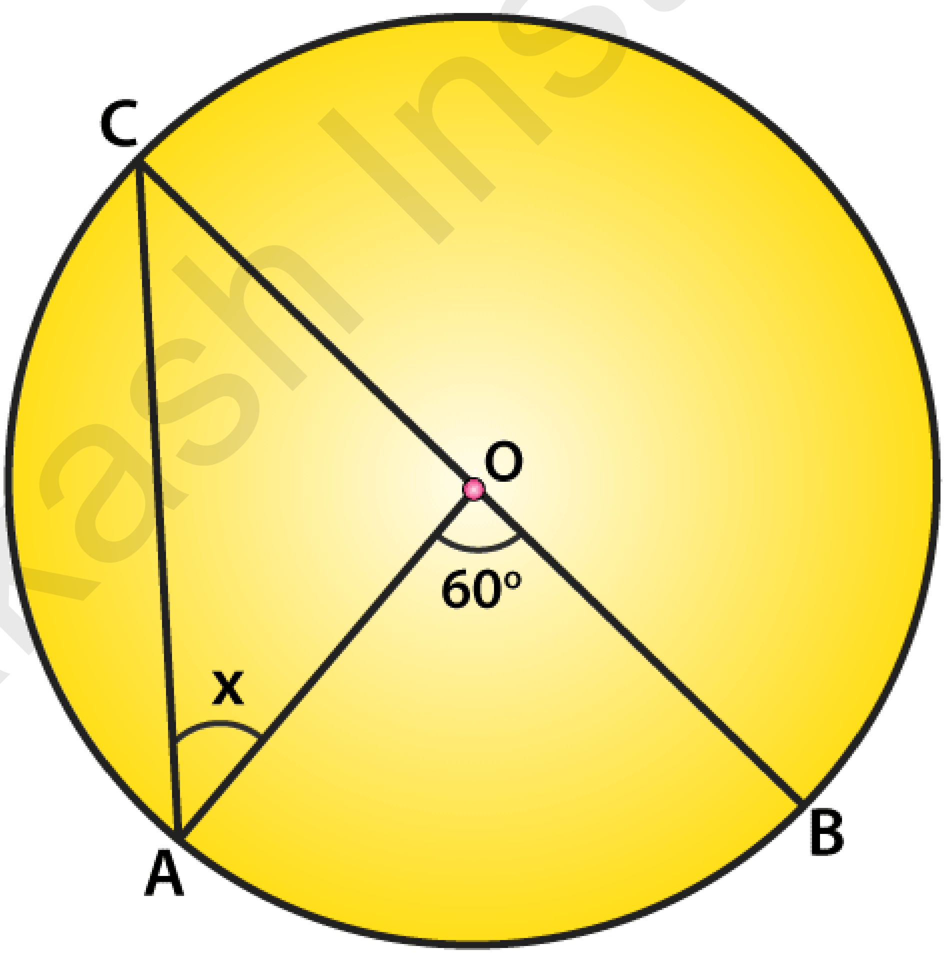
reflex$\angle AOB = 360^0 - 110^0 = 250^0$

By degree measure theorem: reflex $\angle AOB = 2\angle ACB$

$250^0 = 2x$

$x = 250^0/2 = 125^0$

(vi)



Solution:

$\angle AOB = 60^\circ$ (given)

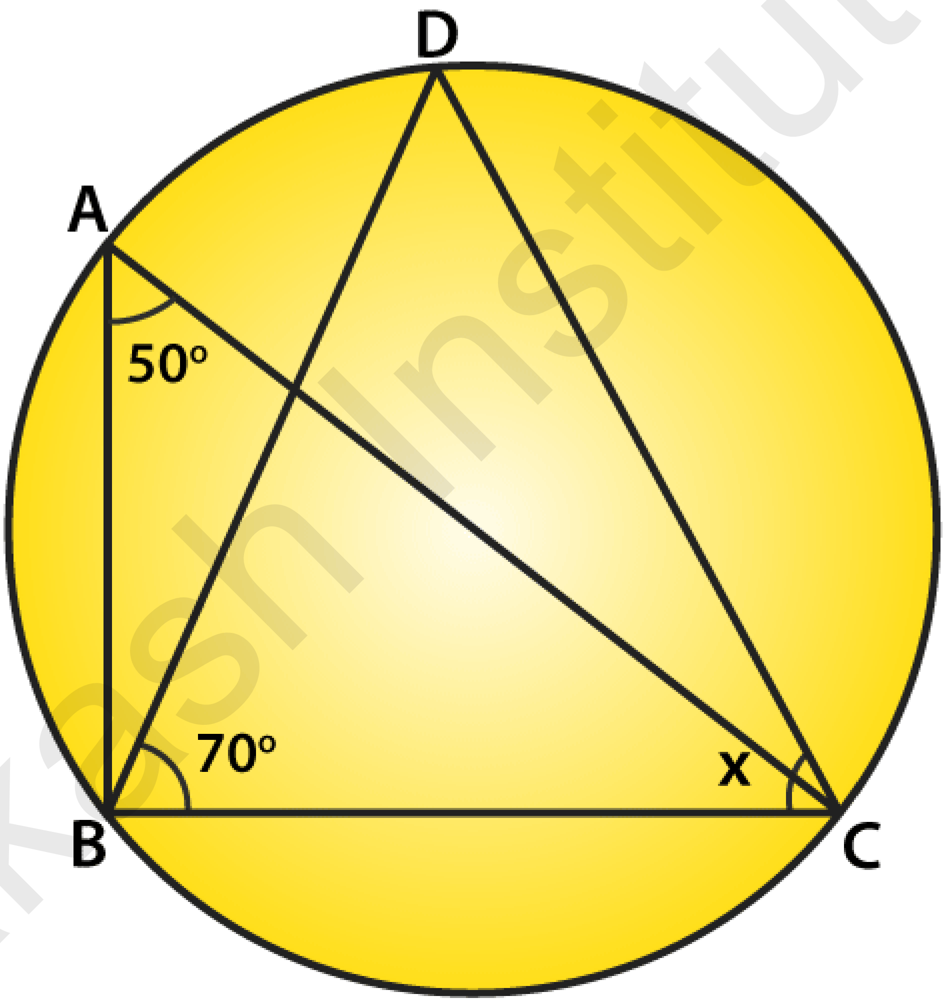
By degree measure theorem: reflex$\angle AOB = 2\angle OAC$

$60^\circ = 2\angle OAC$

$\angle OAC = 60^\circ / 2 = 30^\circ$ [Angles opposite to equal radii]

Or $x = 30^\circ$

(vii)



Solution:

$\angle BAC = 50^\circ$ and $\angle DBC = 70^\circ$ (given)

From figure:

$\angle BDC = \angle BAC = 50^\circ$ [Angle on same segment]

Now,

In ΔBDC:

Using angle sum property, we have

$\angle BDC+\angle BCD+\angle DBC=180^0$

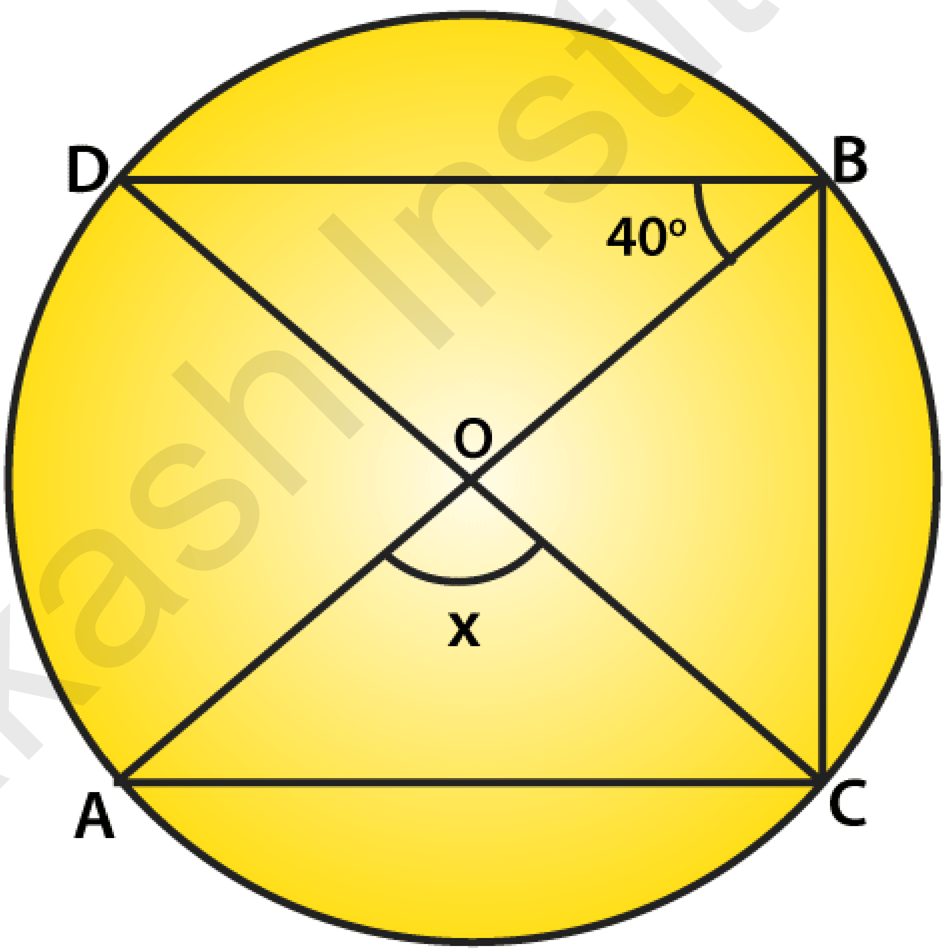
Substituting given values, we get

$50^0 + x^0 + 70^0 = 180^0$

$x^0 = 180^0-50^0-70^0=60^0$

or $x = 60^\circ$

(viii)



Solution:

$\angle DBO = 40^0$ (Given)

Form figure:

$\angle DBC = 90^0$ [Angle in a semicircle]

$\angle DBO + \angle OBC = 90^0$

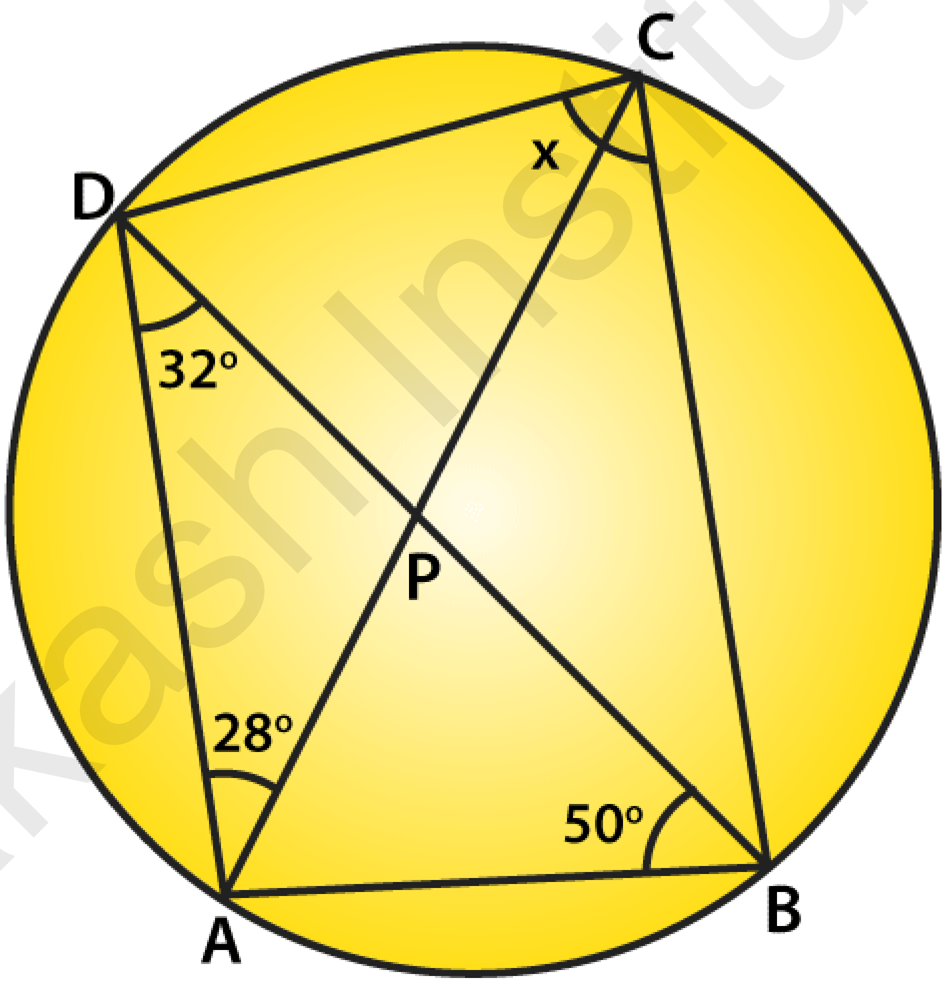
$40^0 + \angle OBC = 90^0$

or $\angle OBC = 90^0 - 40^0 = 50^0$

Again, By degree measure theorem: $\angle AOC = 2\angle OBC$

or $x = 2 \times 50^0 = 100^0$

(ix)



Solution:

$\angle CAD = 28$, $\angle ADB = 32$ and $\angle ABC = 50$ (Given)

From figure:

In ΔDAB:

Angle sum property: $\angle ADB + \angle DAB + \angle ABD = 180^0$

By substituting the given values, we get

$32^0 + \angle DAB + 50^0 = 180^0$

$\angle DAB = 180^0 - 32^0 - 50^0$

$\angle DAB = 98^0$

Now,

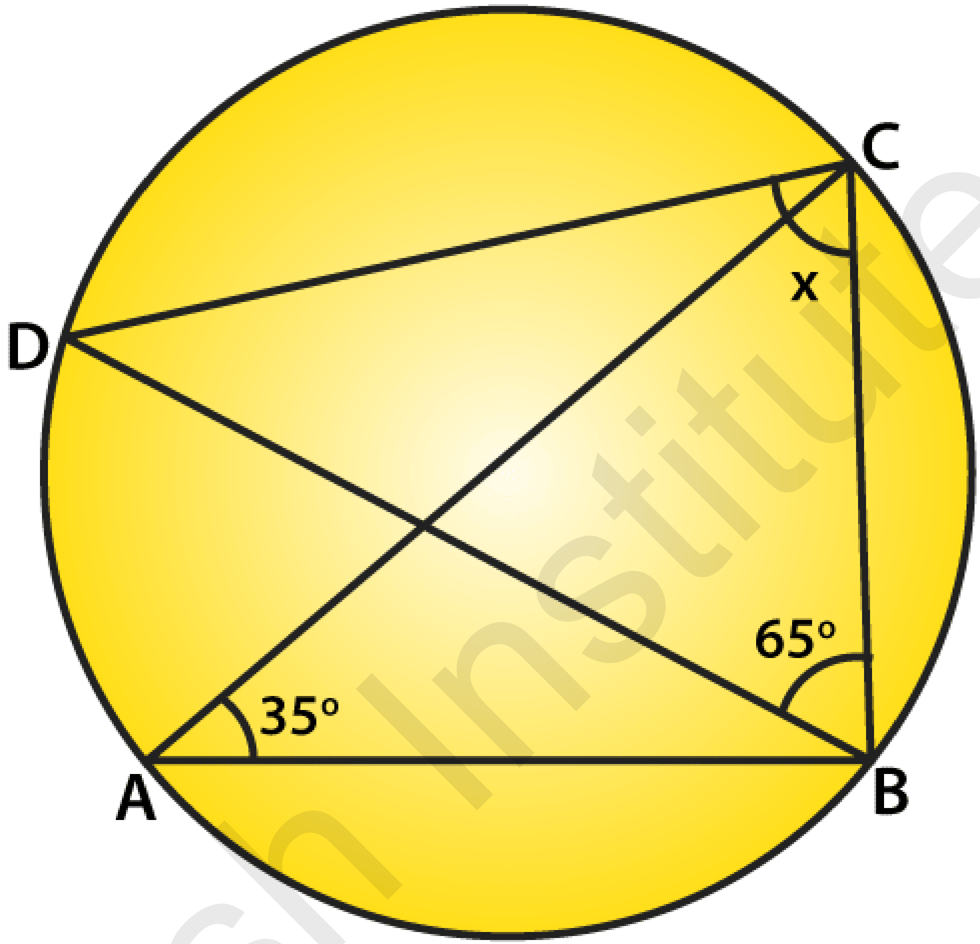
$\angle DAB + \angle DCB = 180^0$ [Opposite angles of cyclic quadrilateral, their sum = 180 degrees]

$98^0 + x = 180^0$

or $x = 180^0 - 98^0 = 82^0$

The value of x is 82 degrees.

(x)



Solution:

$\angle BAC = 35^0$ and $\angle DBC = 65^0$

From figure:

$\angle BDC = \angle BAC = 35^0$ [Angle in same segment]

In ΔBCD:

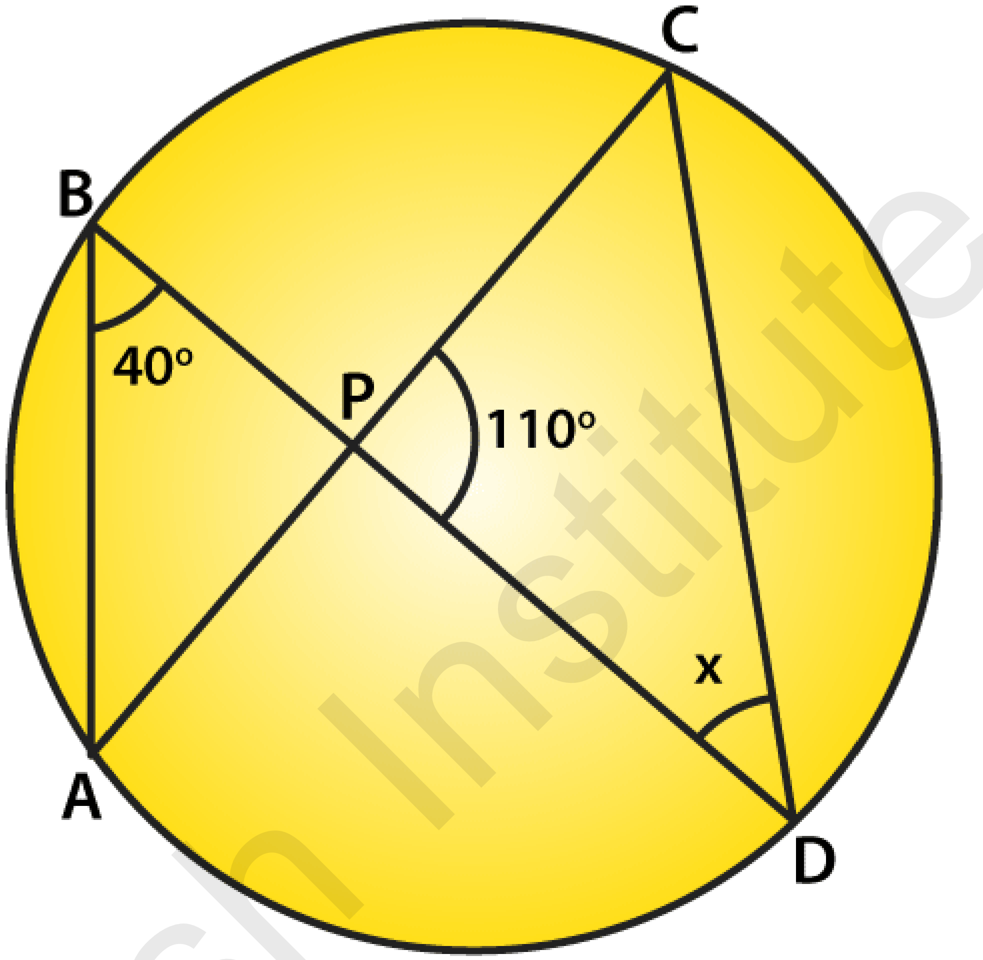
Angle sum property, we have

$\angle BDC + \angle BCD + \angle DBC = 180^0$

$35^0 + x + 65^0 = 180^0$

or $x = 180^0 - 35^0 - 65^0 = 80^0$

(xi)



Solution:

$\angle ABD = 40^0$, $\angle CPD = 110^0$ (Given)

Form figure:

$\angle ACD = \angle ABD = 40^0$ [Angle in same segment]

In ΔPCD,

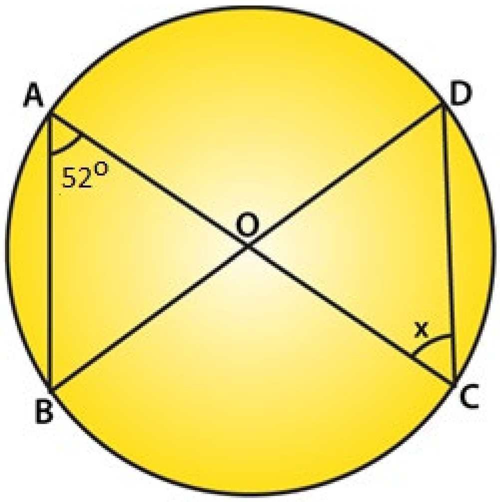
Angle sum property: $\angle PCD+\angle CPO+\angle PDC=180^0$

$400 + 110^0 + x = 180^0$

$x=180^0-150^0=30^0$

The value of x is 30 degrees.

(xii)



Solution:

∠BAC = 52° (Given)

From figure:

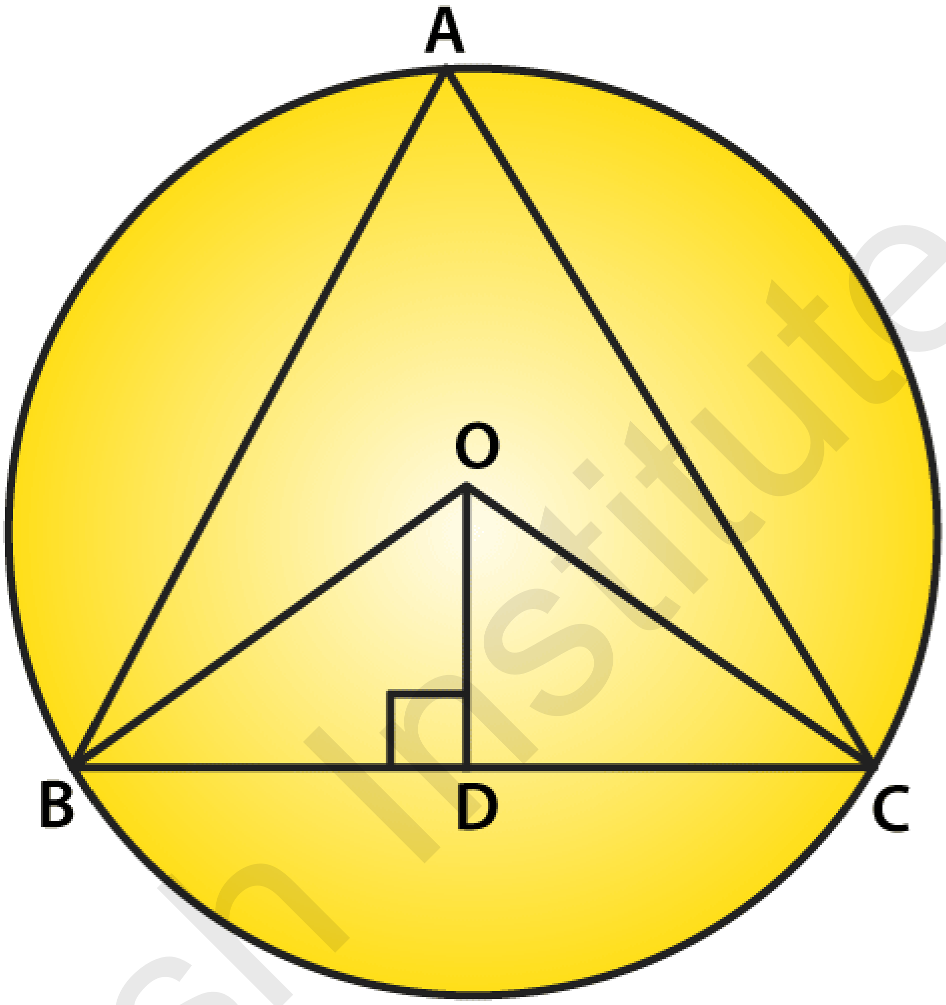
∠BDC = ∠BAC = 52° [Angle in same segment]

Since OD = OC (radii), then ∠ODC = ∠OCD [Opposite angle to equal radii]

So, x = 52°

Question 5: O is the circumcentre of the triangle ABC and OD is perpendicular on BC. Prove that ∠BOD = ∠A.

Solution:



In ΔOBD and ΔOCD:

OB = OC [Radius]

∠ODB = ∠ODC [Each 90^0]

OD = OD [Common]

Therefore, By RHS Condition

ΔOBD ≅ ΔOCD

So, ∠BOD = ∠COD…..(i)[By CPCT]

Again,

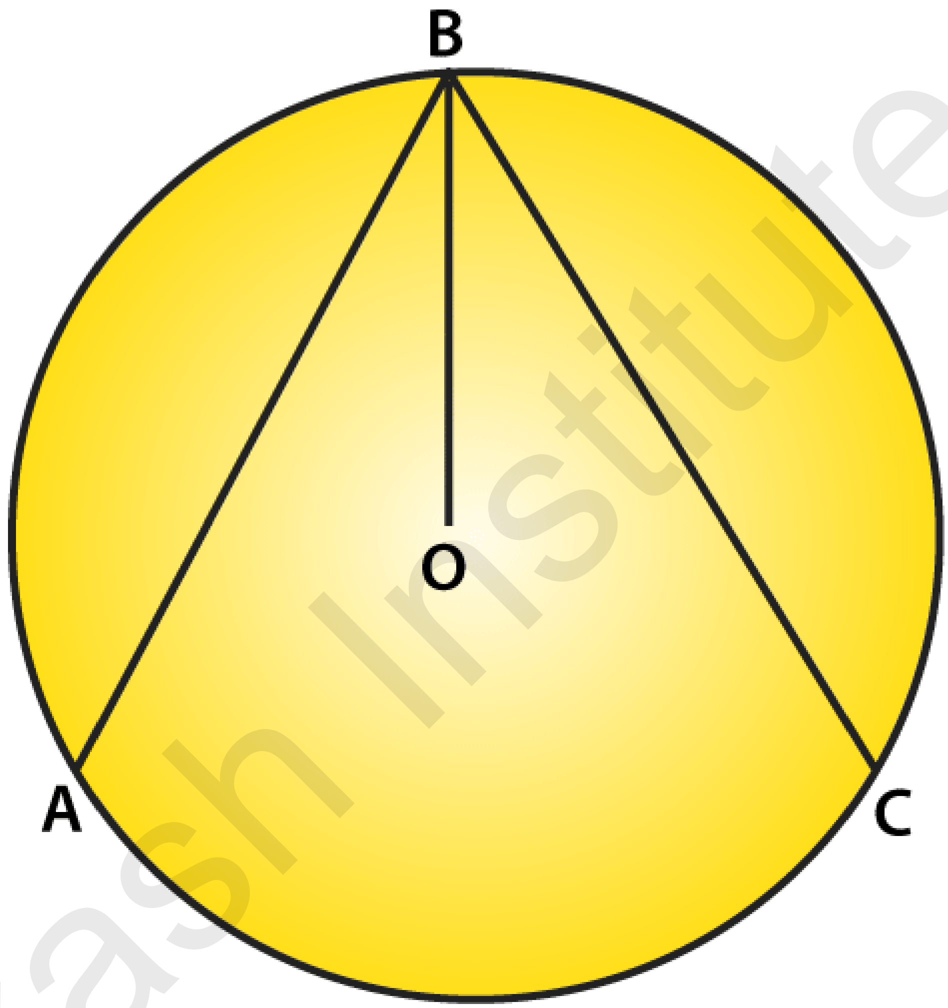
By degree measure theorem: ∠BOC = 2∠BAC

2∠BOD = 2∠BAC [Using(i)]

∠BOD = ∠BAC

Hence proved.

Question 6: In figure, O is the centre of the circle, BO is the bisector of ∠ABC. Show that AB = AC.



Solution:

Since, BO is the bisector of ∠ABC, then,

∠ABO = ∠CBO …..(i)

From figure:

Radius of circle = OB = OA = OB = OC

∠OAB = ∠OCB …..(ii) [opposite angles to equal sides]

∠ABO = ∠DAB …..(iii) [opposite angles to equal sides]

From equations (i), (ii) and (iii), we get

∠OAB = ∠OCB …..(iv)

In ΔOAB and ΔOCB:

∠OAB = ∠OCB [From (iv)]

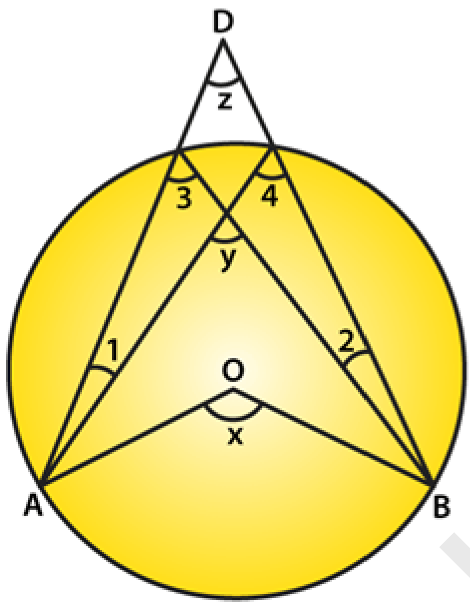
OB = OB [Common]

∠OBA = ∠OBC [Given]

Then, By AAS condition : ΔOAB ≅ ΔOCB

So, AB = BC [By CPCT]

Question 7: In figure, O is the centre of the circle, then prove that ∠x = ∠y + ∠z.



Solution:

From the figure:

∠3 = ∠4 ….(i) [Angles in same segment]

∠x = 2∠3 [By degree measure theorem]

∠x = ∠3 + ∠3

∠x = ∠3 + ∠4 (Using (i)) …..(ii)

Again, ∠y = ∠3 + ∠1 [By exterior angle property]

or ∠3 = ∠y − ∠1 …..(iii)

∠4 = ∠z + ∠1 …. (iv) [By exterior angle property]

Now, from equations (ii) , (iii) and (iv), we get

∠x = ∠y − ∠1 + ∠z + ∠1

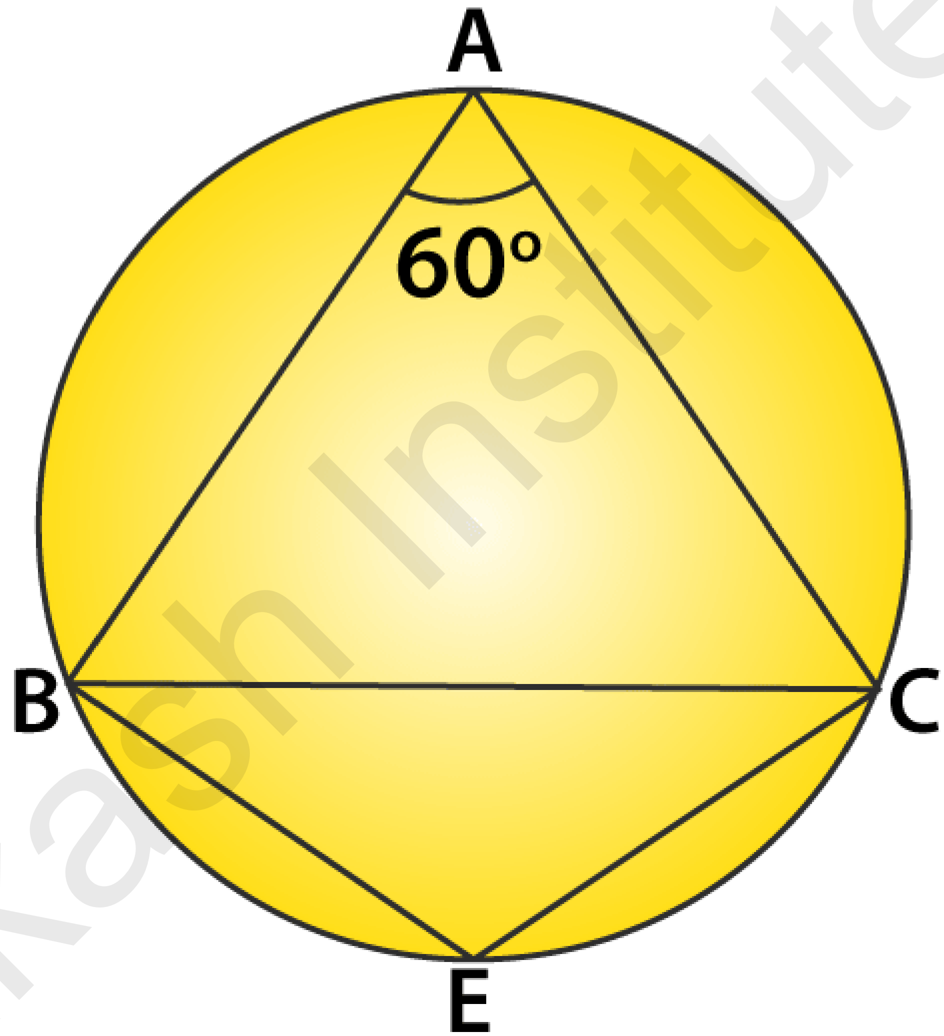
or ∠x = ∠y + ∠z + ∠1 − ∠1

or x = ∠y + ∠z

Hence proved.

Exercise 16.5 Page No: 16.83

Question 1: In figure, ΔABC is an equilateral triangle. Find m∠BEC.



Solution:

ΔABC is an equilateral triangle. (Given)

Each angle of an equilateral triangle is 60 degrees.

In quadrilateral ABEC:

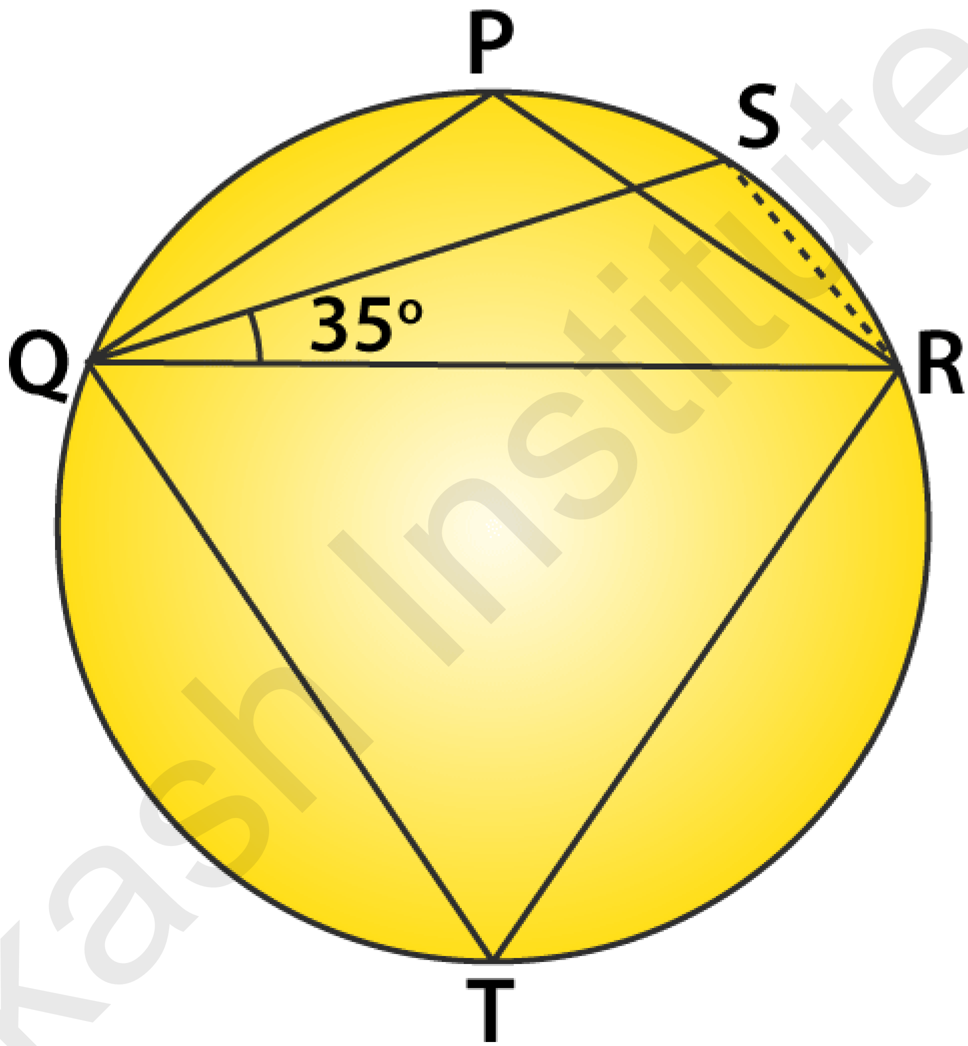
$\angle BAC + \angle BEC = 180^\circ$ (Opposite angles of quadrilateral)

$60^\circ + \angle BEC = 180^\circ$

$\angle BEC = 180^\circ - 60^\circ$

$\angle BEC = 120^\circ$

Question 2: In figure, Δ PQR is an isosceles triangle with PQ = PR and m∠PQR=35°. Find m∠QSR and m∠QTR.



Solution:

Given: ΔPQR is an isosceles triangle with PQ = PR and m∠PQR = 35°

In ΔPQR:

$\angle PQR = \angle PRQ = 35^\circ$ (Angle opposite to equal sides)

Again, by angle sum property

$\angle P + \angle Q + \angle R = 180^\circ$

$\angle P + 35^\circ + 35^\circ = 180^\circ$

$\angle P + 70^\circ = 180^\circ$

$\angle P = 180^\circ - 70^\circ$

$\angle P = 110^\circ$

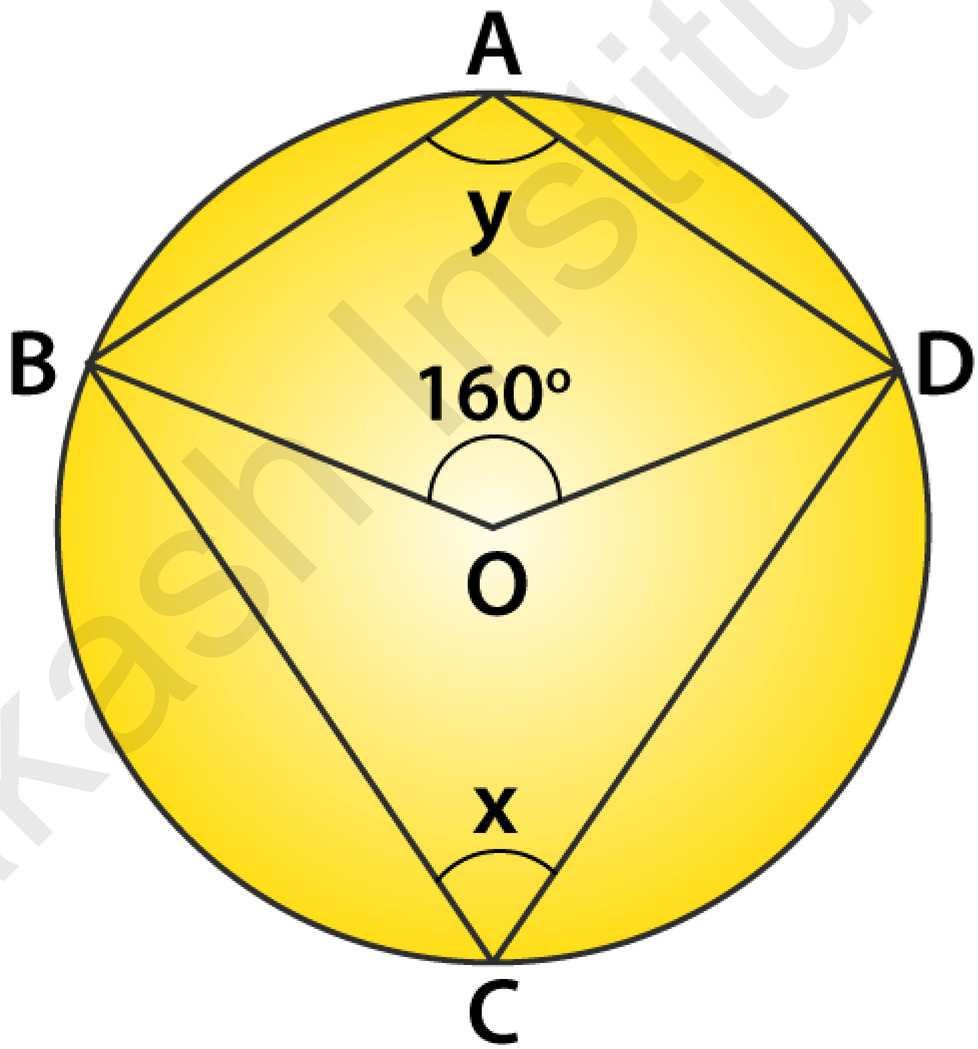
Now, in quadrilateral SQTR,

$\angle QSR + \angle QTR = 180^\circ$ (Opposite angles of quadrilateral)

$110^\circ + \angle QTR = 180^\circ$

$\angle QTR = 70^\circ$

Question 3: In figure, O is the centre of the circle. If $\angle BOD = 160^\circ$, find the values of x and y.



Solution:

From figure: $\angle BOD = 160^\circ$

By degree measure theorem: $\angle BOD = 2\ \angle BCD$

$160^\circ = 2x$

or $x = 80^\circ$

Now, in quadrilateral ABCD,

$\angle BAD + \angle BCD = 180^\circ$ (Opposite angles of Cyclic quadrilateral)

$y + x = 180^\circ$

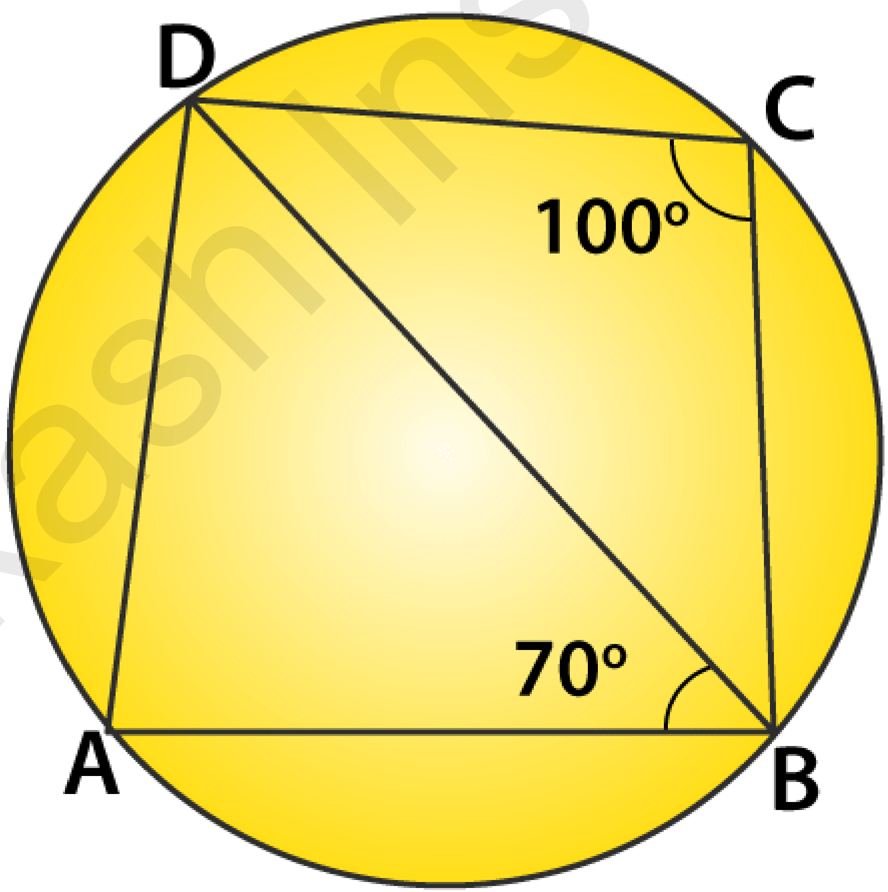
Putting value of x,

$y + 80^\circ = 180^\circ$

$y = 100^\circ$

Answer: $x = 80^\circ$ and $y = 100^\circ$.

Question 4: In figure, ABCD is a cyclic quadrilateral. If $\angle BCD = 100^\circ$ and $\angle ABD = 70^\circ$, find $\angle ADB$.



Solution:

From figure:

In quadrilateral ABCD,

$\angle DCB + \angle BAD = 180^\circ$ (Opposite angles of Cyclic quadrilateral)

$100^\circ + \angle BAD = 180^\circ$

$\angle BAD = 80^0$

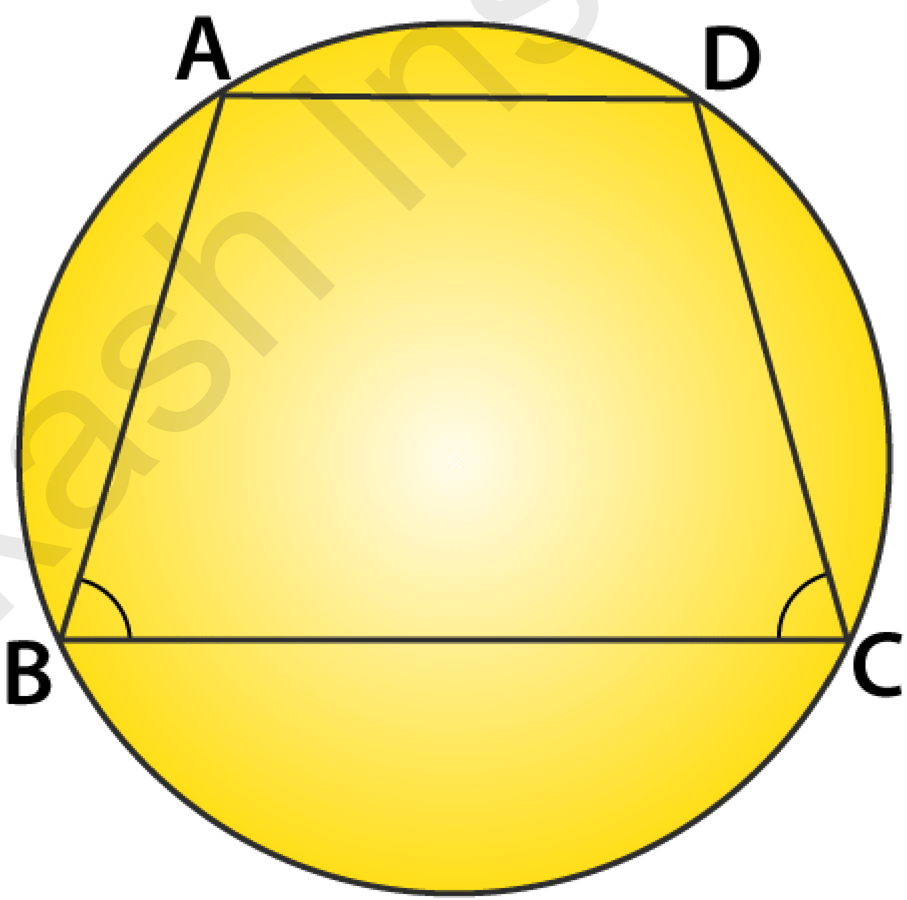
In Δ BAD:

By angle sum property: $\angle ADB + \angle DAB + \angle ABD = 180^\circ$

$\angle ADB + 80^\circ + 70^\circ = 180^\circ$

$\angle ADB = 30^\circ$

Question 5: If ABCD is a cyclic quadrilateral in which AD||BC (figure). Prove that ∠B = ∠C.



Solution:

Given: ABCD is a cyclic quadrilateral with AD ∥ BC

=> ∠A + ∠C = 180° ....(1)

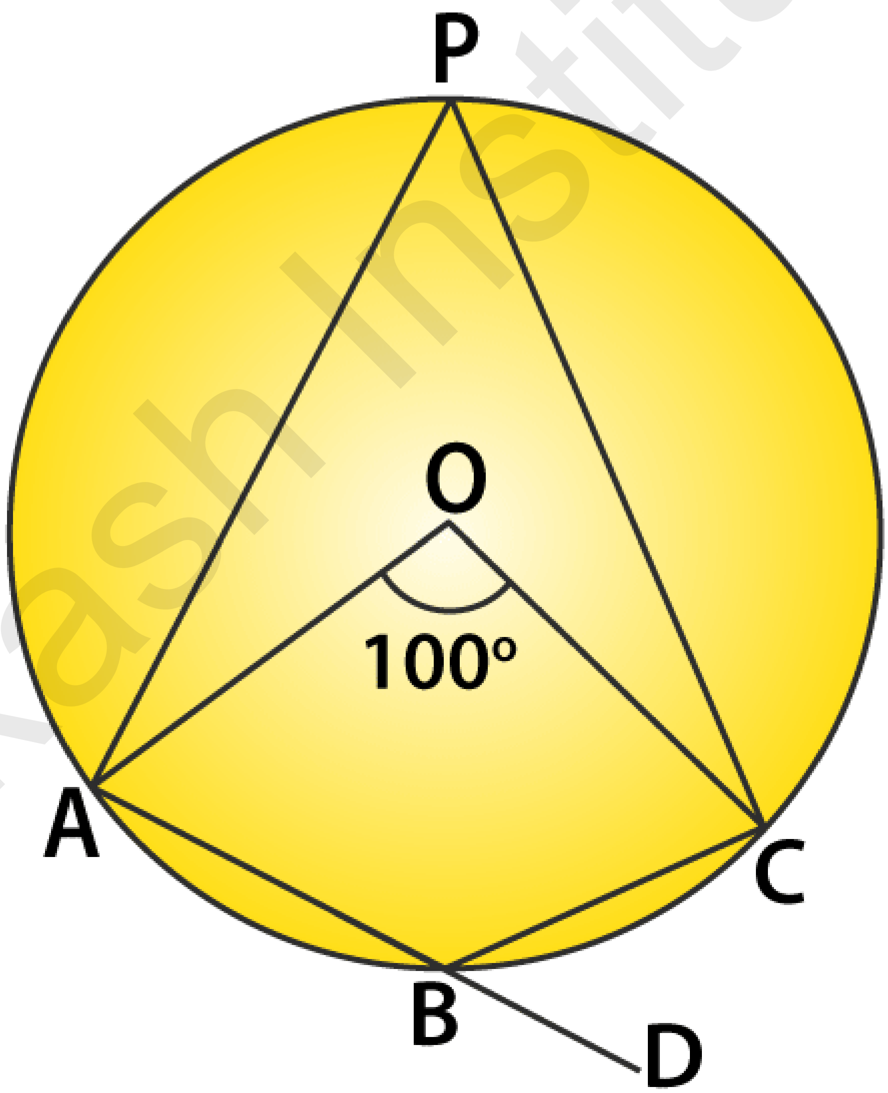
[Opposite angles of cyclic quadrilateral]
and ∠A + ∠B = 180° ....(2)

[Co-interior angles]
Form (1) and (2), we have

∠B = ∠C

Hence proved.

Question 6: In figure, O is the centre of the circle. Find ∠CBD.



Solution:

Given: ∠BOC = 100°

By degree measure theorem: ∠AOC = 2 ∠APC

100∘ = 2 ∠APC

or ∠APC = 50∘

Again,

∠APC + ∠ABC = 180∘ (Opposite angles of a cyclic quadrilateral)

50∘ + ∠ABC = 180∘

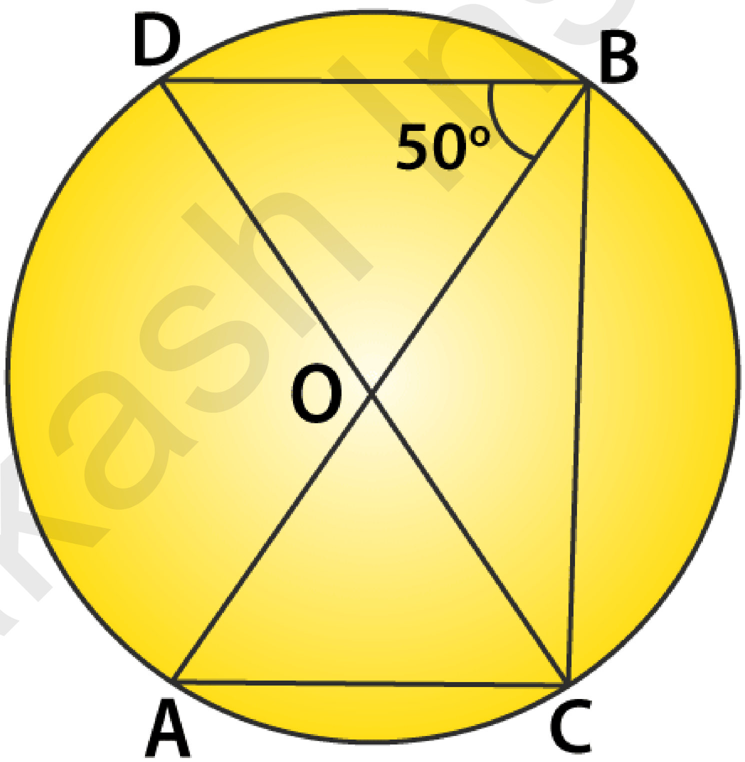
or ∠ABC = 130∘

Now, ∠ABC + ∠CBD = 180∘ (Linear pair)

130∘ + ∠CBD = 180∘

or ∠CBD = 50∘

Question 7: In figure, AB and CD are diameters of a circle with centre O. If ∠OBD = 50⁰, find ∠AOC.



Solution:

Given: ∠OBD = 50⁰

Here, AB and CD are the diameters of the circles with centre O.

$\angle DBC = 90^\circ$ ….(i)

[Angle in the semi-circle]
Also, $\angle DBC = 50^\circ + \angle OBC$

$90^\circ = 50^\circ + \angle OBC$

or $\angle OBC = 40^\circ$

Again, By degree measure theorem: $\angle AOC = 2\ \angle ABC$

$\angle AOC = 2\angle OBC = 2 \times 40^\circ = 80^\circ$

Question 8: On a semi-circle with AB as diameter, a point C is taken, so that m($\angle$CAB) = 30°. Find m($\angle$ACB) and m($\angle$ABC).

Solution:

Given: m($\angle$CAB)= 30°

To Find: m($\angle$ACB) and m($\angle$ABC).

Now,

$\angle ACB = 90^\circ$ (Angle in semi-circle)

Now,

In $\triangle ABC$, by angle sum property: $\angle CAB + \angle ACB + \angle ABC = 180^\circ$

$30^\circ + 90^\circ + \angle ABC = 180^\circ$

$\angle ABC = 60^\circ$

Answer: $\angle ACB = 90^\circ$ and $\angle ABC = 60^\circ$

Question 9: In a cyclic quadrilateral ABCD if AB||CD and $\angle B = 70^\circ$, find the remaining angles.

Solution:

A cyclic quadrilateral ABCD with AB||CD and $\angle B = 70^\circ$.

$\angle B + \angle C = 180^\circ$ (Co-interior angle)

$70^\circ + \angle C = 180^\circ$

$\angle C = 110^\circ$

And,

=> $\angle B + \angle D = 180^\circ$ (Opposite angles of Cyclic quadrilateral)

$70^\circ + \angle D = 180^\circ$

$\angle D = 110^\circ$

Again, $\angle A + \angle C = 180^\circ$ (Opposite angles of cyclic quadrilateral)

$\angle A + 110^\circ = 180^\circ$

$\angle A = 70^\circ$

Answer: $\angle A = 70^\circ$, $\angle C = 110^\circ$ and $\angle D = 110^\circ$

Question 10: In a cyclic quadrilateral ABCD, if m ∠A = 3(m∠C). Find m ∠A.
Solution:

$\angle A + \angle C = 180^\circ$ …..(1)

[Opposite angles of cyclic quadrilateral]
Since m ∠A = 3(m∠C) (given)

=> $\angle A = 3\angle C$ …(2)

Equation (1) => $3\angle C + \angle C = 180^\circ$

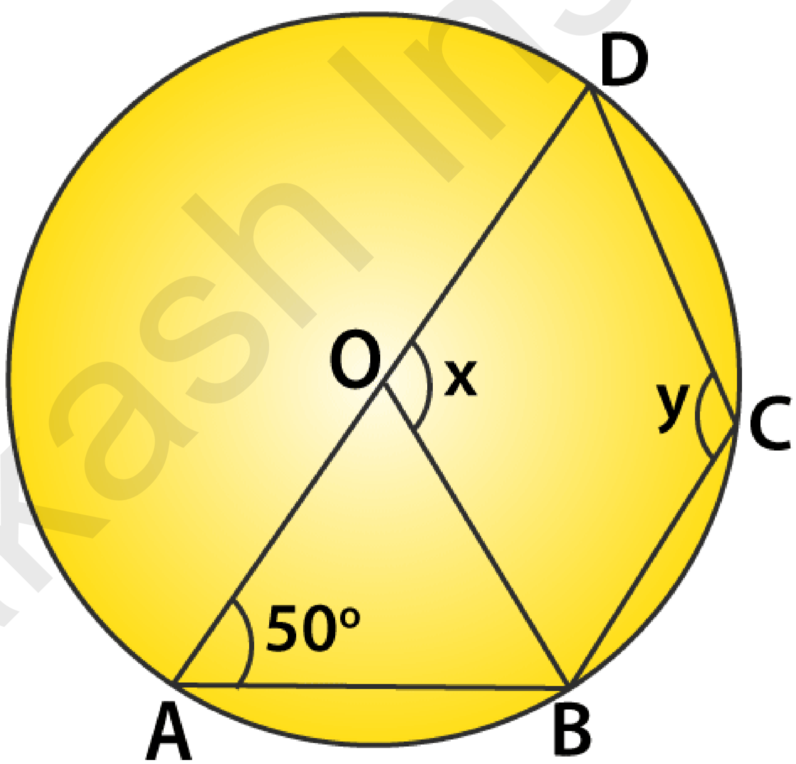
or $4\angle C = 180^\circ$

or $\angle C = 45^\circ$

From equation (2)

$\angle A = 3 \times 45^\circ = 135^\circ$

Question 11: In figure, O is the centre of the circle ∠DAB = 50°. Calculate the values of x and y.



Solution:

Given : $\angle DAB = 50^\circ$

By degree measure theorem: ∠BOD = 2 ∠BAD

so, x = 2(50^0) = 100^0

Since, ABCD is a cyclic quadrilateral, we have

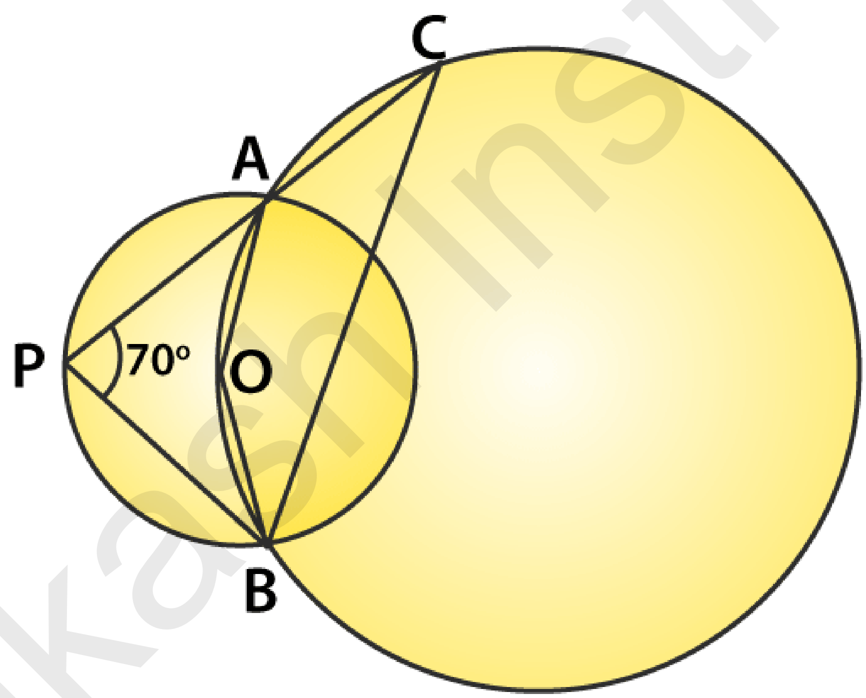
∠A + ∠C = 180^0

50^0 + y = 180^0

y = 130^0

Exercise VSAQs Page No: 16.89

Question 1: In figure, two circles intersect at A and B. The centre of the smaller circle is O and it lies on the circumference of the larger circle. If ∠APB = 70°, find ∠ACB.



Solution:

By degree measure theorem: ∠AOB = 2 ∠APB

so, ∠AOB = 2 × 70° = 140°

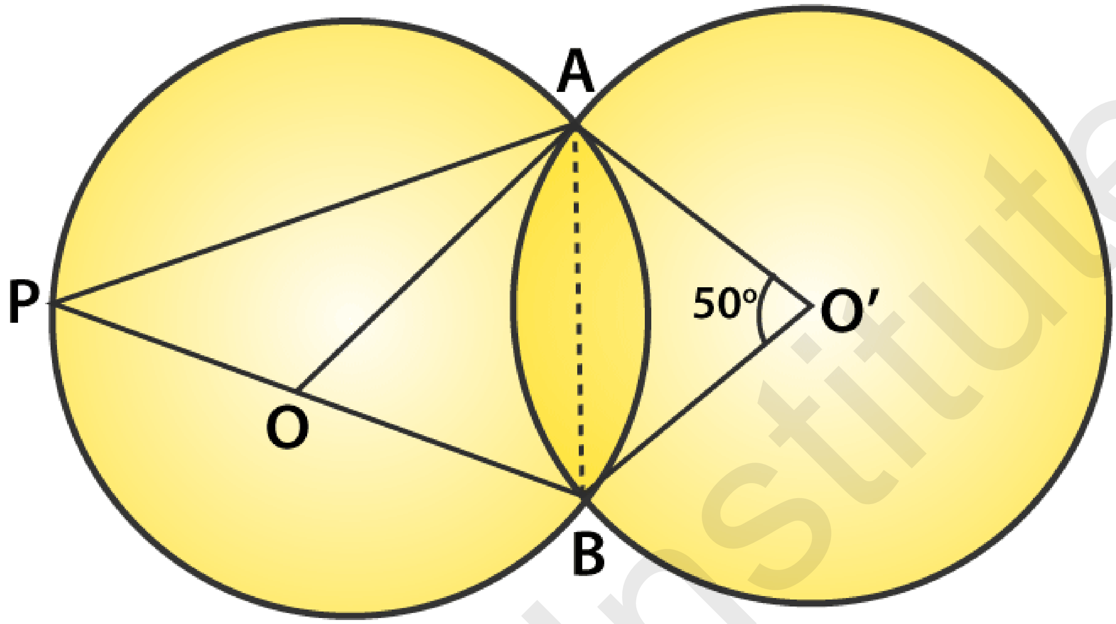
Since AOBC is a cyclic quadrilateral, we have

∠ACB + ∠AOB = 180°

∠ACB + 140° = 180°

∠ACB = 40°

Question 2: In figure, two congruent circles with centres O and O' intersect at A and B. If ∠AO'B = 50°, then find ∠APB.



Solution:

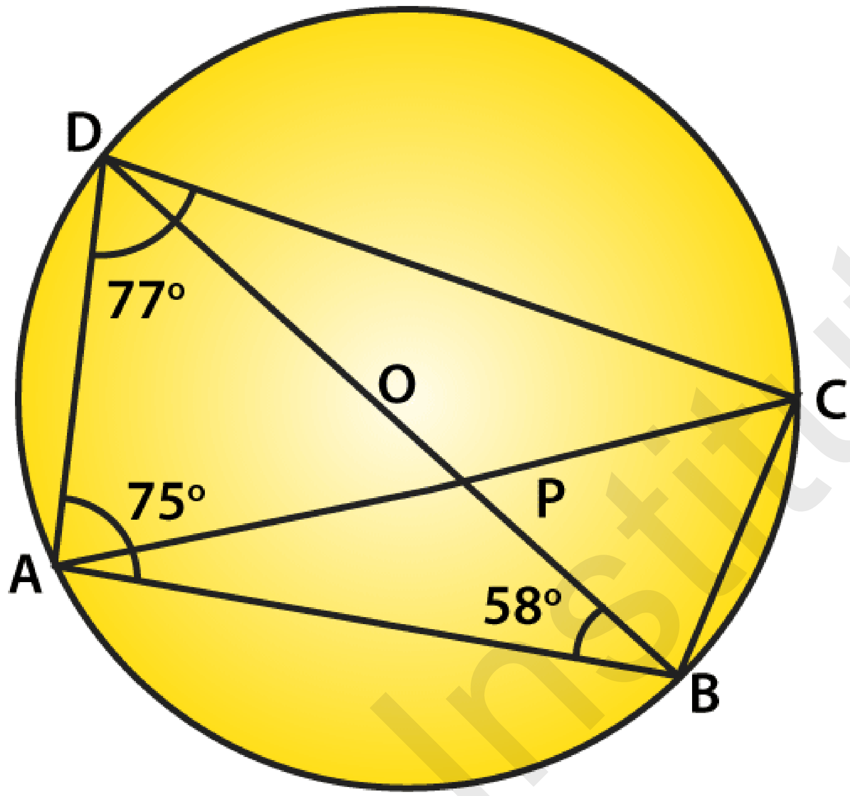
As we are given that, both the triangle are congruent which means their corresponding angles are equal.

Therefore, ∠AOB = AO'B = 50°

Now, by degree measure theorem, we have

∠APB = ∠AOB/2 = 25°

Question 3: In figure, ABCD is a cyclic quadrilateral in which ∠BAD=75°, ∠ABD=58° and ∠ADC=77°, AC and BD intersect at P. Then, find ∠DPC.



Solution:

∠DBA = ∠DCA = 58^0 …(1)

[Angles in same segment]
ABCD is a cyclic quadrilateral :

Sum of opposite angles = 180 degrees

∠A +∠C = 180^0

75^0 + ∠C = 180^0

∠C = 105^0

Again, ∠ACB + ∠ACD = 105^0

∠ACB + 58^0 = 105^0

or ∠ACB = 47^0 …(2)

Now, ∠ACB = ∠ADB = 47^0

[Angles in same segment]
Also, ∠D = 77^0 (Given)

Again From figure, ∠BDC + ∠ADB = 77^0

$\angle BDC + 47^0 = 77^0$

$\angle BDC = 30^0$

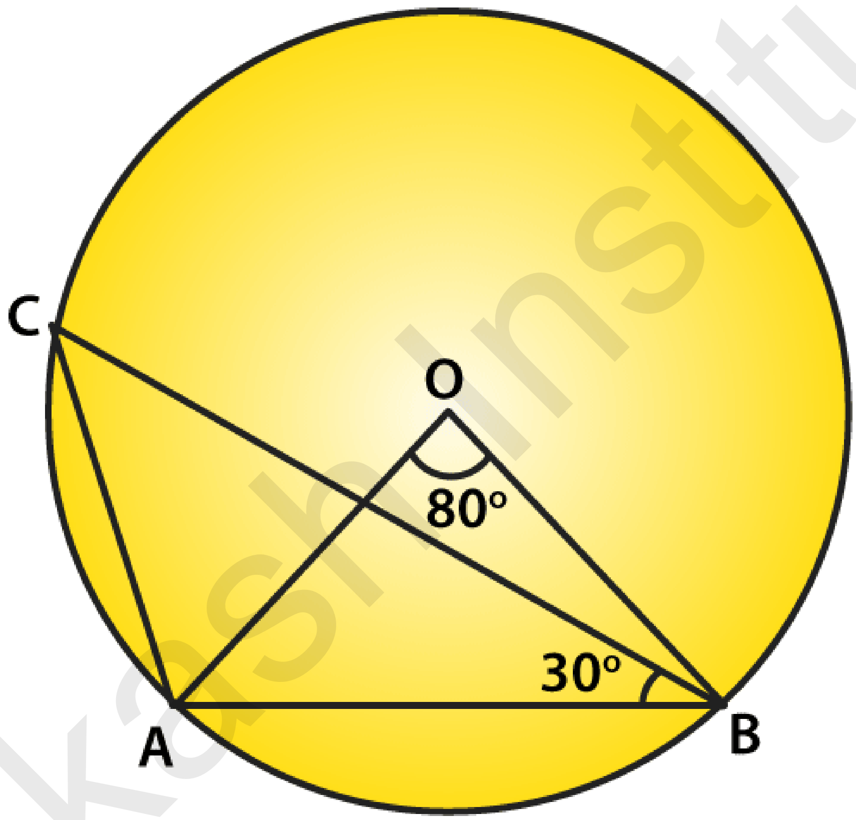
In triangle DPC

$\angle PDC + \angle DCP + \angle DPC = 180^0$

$30^0 + 58^0 + \angle DPC = 180^0$

or $\angle DPC = 92^0$

Question 4: In figure, if ∠AOB = 80° and ∠ABC=30°, then find ∠CAO.



Solution:

Given: $\angle AOB = 80^0$ and $\angle ABC = 30^0$

To find: $\angle CAO$

Join OC.

Central angle subtended by arc AC = $\angle COA$

then $\angle COA = 2 \times \angle ABC = 2 \times 30^0 = 60^0$ …(1)

In triangle OCA,

OC = OA

[same radii]
$\angle OCA = \angle CAO$ …(2)

[Angle opposite to equal sides]
In triangle COA,

$\angle OCA + \angle CAO + \angle COA = 180^0$

From (1) and (2), we get

$2\angle CAO + 60^0 = 180^0$

$\angle CAO = 60^0$