RD Sharma Class 9 Chapter 15 Areas of Parallelograms and Triangles

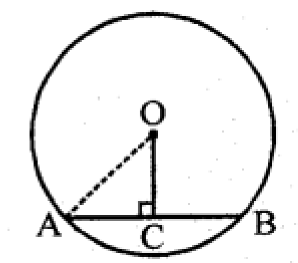
Question 1.
Fill in the blanks:

(i) All points lying inside / outside a circle are called …….. points / ………. points.
(ii) Circles having the same centre and different radii are called …….. circles.
(iii) A point whose distance from the centre of a circle is greater than its radius lies in …….. of the circle.
(iv) A continuous piece of a circle is …….. of the circle.
(v) The longest chord of a circle is a ……… of the circle.
(vi) An arc is a …….. when its ends are the ends of a diameter.
(vii) Segment of a circle is the region between an are and ……..of the circle.
(viii)A circle divides the plane, on which it lies, in …….. parts.
Solution:
(i) All points lying inside / outside a circle are called interior points / exterior points.
(ii) Circles having the same centre and different radii are called concentric circles.
(iii) A point whose distance from the centre of a circle is greater than its radius lies in exterior of the circle.
(iv) A continuous piece of a circle is arc of the circle.
(v) The longest chord of a circle is a diameter of the circle.
(vi) An arc is a semi-circle when its ends are the ends of a diameter.
(vii) Segment of a circle is the region between an arc and centre of the circle.
(viii) A circle divides the plane, on which it lies, in three parts.

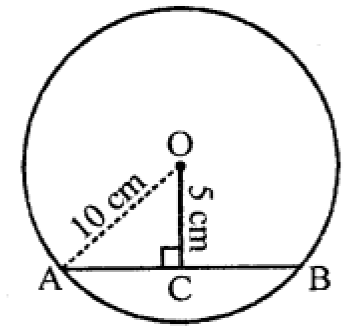
Question 2.
Write the truth value (T/F) of the following with suitable reasons: [NCERT]
(i) A circle is a plane figure.
(ii) Line segment joining the centre to any point on the circle is a radius of the circle.
(iii) If a circle is divided into three equal arcs each is a major arc.
(iv) A circle has only finite number of equal chords.
(v) A chord of a cirlce, which is twice as long is its radius is a diameter of the circle.
(vi) Sector is the region between the chord and its corresponding arc.
(vii) The degree measure of an arc is the complement of the central angle containing the arc.
(viii)The degree measure of a semi-circle is 180°.
Solution:
(i) True.
(ii) True.
(iii) True.

(iv) False. As it has infinite number of equal chords.
(v) True.
(vi) False. It is a segment not sector.
(vii) False. As total degree measure of a circle is 360°.
(viii) True.

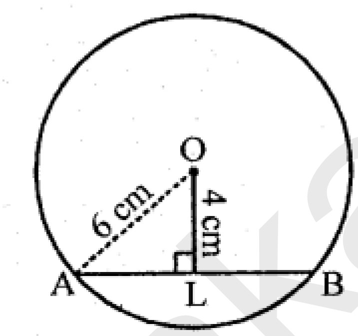
Question 1.
The radius of a circle is 8 cm and the length of one of its chords is 12 cm. Find the distance of the chord from the centre.
Solution:
Radius of circle with centre O is OA = 8 cm
Length of chord AB = 12 cm
OC ⊥ AB which bisects AB at C



∴ AC = CB = 12 x 12 = 6 cm
In ∆OAC,
OA2 = OC2 + AC2 (Pythagoras Theorem)
⇒ (8)2 = OC2 + (6)2
⇒ 64 = OC2 + 36
OC2 = 64 – 36 = 28
∴ OC = $\sqrt{28}$ = $\sqrt{4\times7}$ cm
= 2 x 2.6457 = 5.291 cm
Question 2.
Find the length of a chord which is at a distance of 5 cm from the centre of a circle of radius 10 cm.
Solution:
Let AB be a chord of a circle with radius 10 cm. OC ⊥ AB
∴ OA = 10 cm
OC = 5 cm

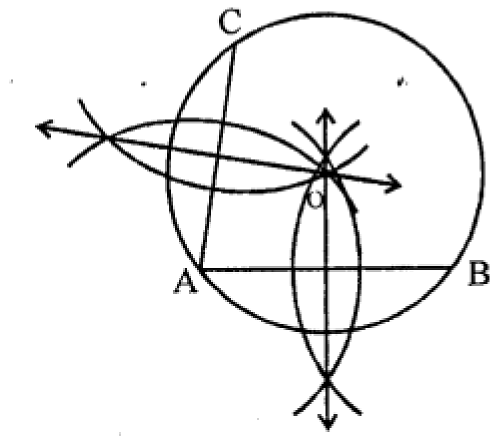


∵ OC divides AB into two equal parts
i.e. AC = CB
Now in right AOAC,
$OA^2 = OC^2 + AC^2$ (Pythagoras Theorem)
⇒ $(10)^2 = (5)^2 + AC^2$
⇒ $100 = 25 + AC^2$
⇒ $AC^2 = 100 – 25 = 75$
∴ AC = $\sqrt{75} = \sqrt{25\times3}$ = 5 x 1.732
∴ AB = 2 x AC = 2 x 5 x 1.732 = 10 x 1.732 = 17.32 cm
Question 3.
Find the length of a chord which is at a distance of 4 cm from the centre of the circle of radius 6 cm.
Solution:
In a circle with centre O and radius 6 cm and a chord AB at a distance of 4 cm from the centre of the circle
i.e. OA = 6 cm and OL ⊥ AB, OL = 4 cm



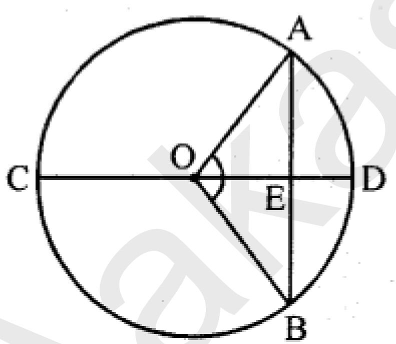
∵ Perpendicular OL bisects the chord AB at L 1
∴ AL = LB=$\frac{1}{2}$ AB
Now in right ΔOAL,
$OA^2 = OL^2 + AL^2$ (Pythagoras Theorem)
$(6)^2 = (4)^2 + AL^2$
⇒ $36=16+AL^2$
⇒ $AL^2 = 36 – 16 = 20$
∴ AL = $\sqrt{20}$ = $\sqrt{4\times5}$ = 2 x 2.236 = 4.472 cm
∴ Chord AB = 4.472 x 2 = 8.944 = 8.94 cm
Question 4.
Give a method to find the centre of a given circle.

Solution:
Steps of construction :



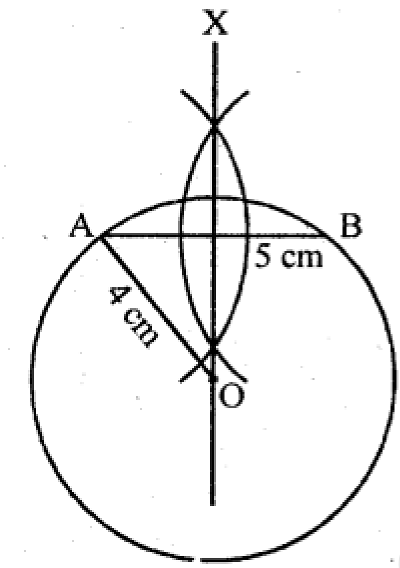
(i) Take three distinct points on the circle say A, B and C.
(ii) Join AB and AC.
(iii) Draw the perpendicular bisectors of AB and AC which intersect each other at O.
O is the required centre of the given circle

Question 5.
Prove that a diameter of a circle which bisects a chord of the circle also bisects the angle subtended by the chord at the centre of the circle.
Solution:
Given : In circle with centre O
CD is the diameter and AB is the chord
which is bisected by diameter at E
OA and OB are joined

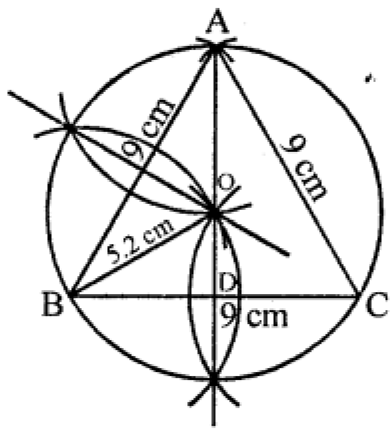


To prove : ∠AOB = ∠BOA
Proof : In ΔOAE and ΔOBE
OA = OB (Radii of the circle)
OE = OE (Common)
AE = EB (Given)
∴ ΔOAE = ΔOBE (SSS criterian)
∴ ∠AOE = ∠BOE (c.p.c.t.)
Hence diameter bisect the angle subtended by the chord AB.

Question 6.
A line segment AB is of length 5 cm. Draw a circle of radius 4 cm passing through A and B. Can you draw a circle of radius 2 cm passing through A and B? Give reason in support of your answer.
Solution:
Steps of construction :
(i) Draw a line segment AB = 5 cm.
(ii) Draw a perpendicular bisector of AB.
(iii) With centre A and radius 4 cm, draw an arc which intersects the perpendicular bisector at O.
(iv) With centre O and radius 4 cm, draw a circle which passes through A and B.
With radius 2 cm, we cannot draw the circle passing through A and B as diameter i. e. 2 + 2 = 4 cm is shorder than 5 cm.



Question 7.
An equilateral triangle of side 9 cm is inscribed in a circle. Find the radius of the circle.
Solution:
Steps of construction :
(i) Draw a line segment BC = 9 cm.
(ii) With centres B and C, draw arcs of 9 cm radius which intersect each other at A.
(iii) Join AB and AC.
ΔABC is the required triangle.
(iv) Draw perpendicular bisectors of sides AB and BC which intersect each other at O.
(v) With centre O and radius OB, draw a circle which passes through A, B and C.
This is the require circle in which ΔABC is inscribed.



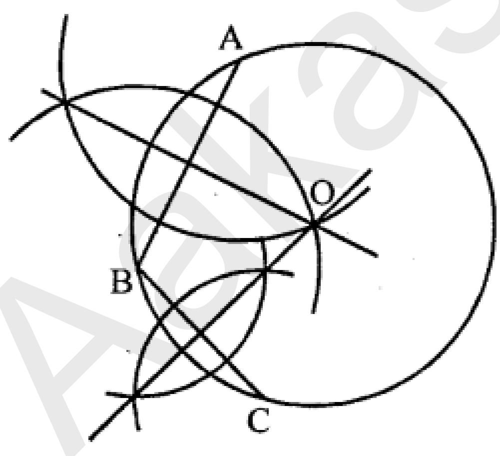
On measuring its radius, it is 5.2 cm

$$\text{Radius} = \frac{2}{3}\text{AD} = \frac{2}{3} \times \frac{\sqrt{3}}{2} \times \text{side}$$

$$= \frac{2}{3} \times \frac{\sqrt{3}}{2} \times 9 = 3\sqrt{3} \text{ cm}$$

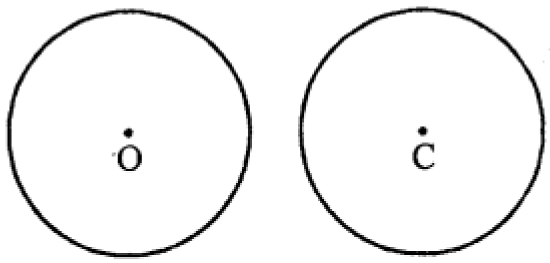
$$= 3 \times 1.732 = 5.196 = 5.2 \text{ cm}$$

Question 8.
Given an arc of a circle, complete the circle.
Solution:
Steps of construction :
(i) Take three points A, B and C on the arc and join AB and BC.
(ii) Draw the perpendicular bisector of AB and BC which intersect each other at O.

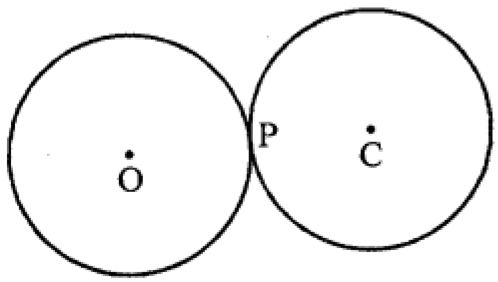


(iii) With centre O and radius OA or OB, complete the circle.
This is the required circle.

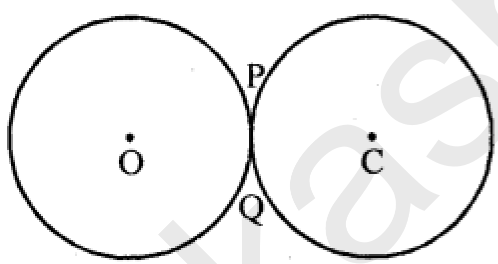
Question 9.
Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?
Solution:
Below, three different pairs of circles are drawn:



(i)



(ii)



(iii)

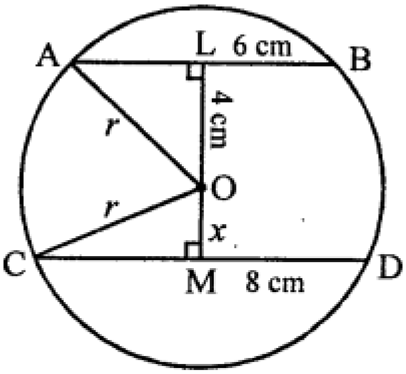
(i) In the first pair, two circles do not intersect each other. Therefore they have no point in common. .
(ii) In the second pair, two circles intersect (touch) each other at one point P. Therefore they have one point in common.
(iii) In the third pair, two circles intersect each other at two points. Therefore they have two points in common.
There is no other possibility of two circles intersecting each other.
Therefore, two circles have at the most two points in common.

Question 10.
Suppose you are given a circle. Give a construction to find its centre.

Solution:
See Q. No. 4 of this exercise.

Question 11.
The length of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance of 4 cm from the centre, what is the distance of the other chord from the centre? [NCERT]
Solution:
A circle with centre O and two parallel chords
AB and CD are AB = 6 cm, CD = 8 cm
Let OL $\perp$ AB and OM $\perp$ CD
$\therefore$ OL = 4 cm
Let OM = x cm

Let r be the radius of the circle



Now in right ΔOAL

$$OA^2 = OL^2 + AL^2 = 4^2 + \left(\frac{6}{2}\right)^2$$

$r^2 = 16 + 9 = 25$ ...(*i*)

and in right ΔOMC,

$OC^2 = OM^2 + CM^2$

$$r^2 = x^2 + \left(\frac{8}{2}\right)^2$$

$= x^2 + (4)^2 = x^2 + 16$ ...(*ii*)

From (*i*) and (*ii*),

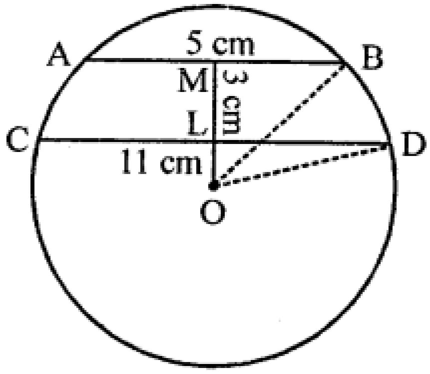
$x^2 + 16 = 25 \Rightarrow x^2 = 25 - 16 = 9$

$\Rightarrow x^2 = (3)^2$

$\therefore x = 3$ cm

$\therefore$ Distance = 3 cm

Question 12.
Two chords AB, CD of lengths 5 cm and 11 cm respectively of a circle are parallel. If the distance between AB and CD is 3 cm, find the radius of the circle.
Solution:
Let two chords AB and CD of length 5 cm and 11 cm are parallel to each other AB = 5 cm, CD = 11 cm
Distance between AB and LM = 3 cm
Join OB and OD
OL and OM are the perpendicular on CD and AB respectively. Which bisects AB and CD.
Let OL = x, then OM = (x + 3)



Now in right ΔOLD,
OD2 = OL2 + LD2
= x2 + (5.5)2
Similarly in right ΔOMB,
OB2 = OM2 + MB2 = (x + 3)2 + (2.5)2
But OD = OB (Radii of the circle)
∴ (x + 3)2 + (2.5)2 = x2 + (5.5)2
x2 + 6x + 9 + 6.25 = x2 + 30.25
6x = 30.25 – 6.25 – 9 = 15

$$x = \frac{15}{6} = \frac{5}{2}$$

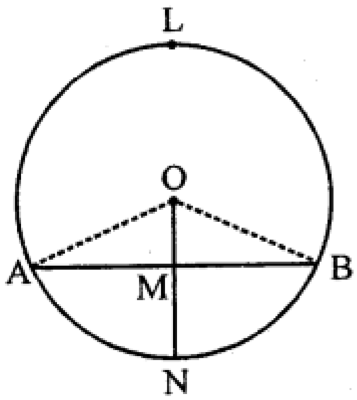
Now, $OD^2 = x^2 + 30.25$

$= (2.5)^2 + 30.25 = 6.25 + 30.25 = 36.50$

$$OD = \sqrt{36.50} = \sqrt{\frac{3650}{100}} = \sqrt{\frac{146}{4}}$$

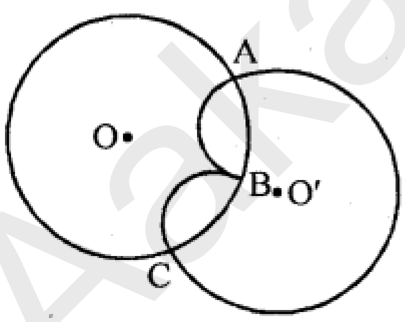
$$\therefore \text{ Radius} = \frac{\sqrt{146}}{2} \text{ cm}$$

Question 13.
Prove that the line joining the mid-point of a chord to the centre of the circle passes through the mid-point of the corresponding minor arc.
Solution:
Given : A circle with centre O and a chord AB
Let M be the mid point of AB and OM is joined and produced to meet the minor arc AB at N
To prove : M is the mid point of arc AB
Construction : Join OA, OB



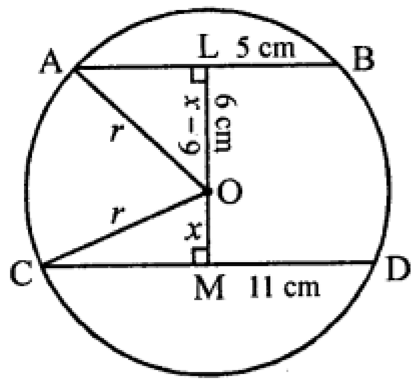
Proof: ∵ M is mid point of AB
∴ OM ⊥ AB
In AOAM and OBM,
OA = OB (Radii of the circle)
OM = OM (common)
AM = BM (M is mid point of AB)
∴ ∆OAM = ∆OBM (SSS criterian)
∴ ∠AOM = ∠BOM (c.p.c.t.)
⇒ ∠AOM = ∠BOM
But these are centre angles at the centre made by arcs AN and BN
∴ Arc AN = Arc BN
Hence N divides the arc in two equal parts

Question 14.
Prove that two different circles cannot intersect each other at more than two points.
Solution:
Given : Two circles
To prove : They cannot intersect each other more than two points
Construction : Let two circles intersect each other at three points A, B and C



Proof : Since two circles with centres O and O' intersect at A, B and C
∴ A, B and C are non-collinear points
∴ Circle with centre O passes through three points A, B and C
and circle with centre O' also passes through three points A, B and C
But one and only one circle can be drawn through three points
∴Our supposition is wrong
∴ Two circle cannot intersect each other not more than two points.

Question 15.
Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are opposite side of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle. [NCERT]
Solution:
Let r be the radius of the circle with centre O.
Two parallel chords AB = 5 cm, CD = 11 cm
Let $OL \perp AB$ and $OM \perp CD$
$\therefore LM = 6$ cm
Let OM = x, then
OL = 6 – x



Now in right ΔOAL,

$OA^2 = OL^2 + AL^2$

$$r^2 = (6 - x)^2 + \left(\frac{5}{2}\right)^2 \quad \begin{Bmatrix} \because OL \perp AB \\ \therefore L \text{ is mid point} \end{Bmatrix}$$

$$= 36 - 12x + x^2 + \frac{25}{4} \qquad ...(i)$$

Similarly in right ΔOCM,

$$r^2 = x^2 + \left(\frac{11}{2}\right)^2 = x^2 + \frac{121}{4} \qquad ...(ii)$$

From (*i*) and (*ii*),

$$x^2 + \frac{121}{4} = 36 - 12x + x^2 + \frac{25}{4}$$

$$\Rightarrow \frac{121}{4} - \frac{25}{4} - 36 = -12x$$

$$\Rightarrow \frac{96}{4} - \frac{36}{1} = -12x$$

$$\Rightarrow 12x = 36 - 24 = 12$$

$$x = \frac{12}{12} = 1$$

$$\therefore r^2 = CM^2 + OM^2$$

$$= \left(\frac{11}{2}\right)^2 + (1)^2$$

$$= \frac{121}{4} + 1 = \frac{125}{4} \text{ cm}$$

$$\therefore r = \sqrt{\frac{125}{4}} = \frac{\sqrt{125}}{2} = \frac{\sqrt{25 \times 5}}{2} \text{ cm}$$

$$= \frac{5}{2}\sqrt{5} \text{ cm}$$

Class 9 Maths Chapter 15 Areas of Parallelograms and Triangles RD Sharma Solutions Ex 15.3

Question 1.
Three girls Ishita, Isha and Nisha are playing a game by standing on a circle of radius 20 m drawn in a park. Ishita throws a ball to Isha, Isha to Nisha and Nisha to Ishita. If the distance between Ishita and Isha and between Isha and Nisha is 24 m each, what is the distance between Ishita and Nisha. [NCERT]
Solution:
∵ Distance between Isha and Ishita and Ishita and Nisha is same
∴ RS = SM = 24 m
∴They are equidistant from the centre

In right $\triangle ORL$,

$OL^2 = OR^2 - RL^2$

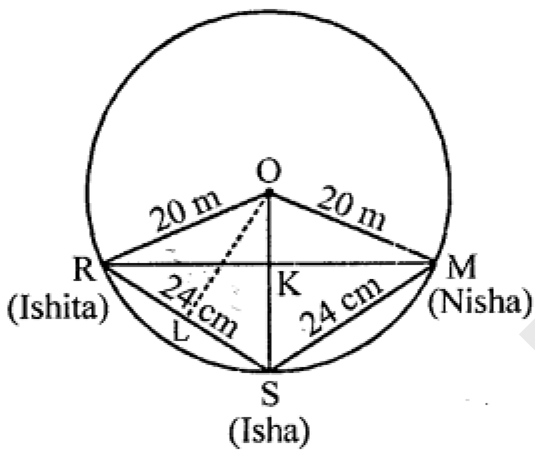
$$= 20^2 - \left(\frac{24}{2}\right)^2$$

$$= 400 - 144 = 256 = 16^2$$

$\therefore$ OL = 16 cm

Let $OL \perp RS$

OS bisects RM



Let RK = x

Now, $ar\Delta ORS = \frac{1}{2}$ base × altitude

$$= \frac{1}{2} \times 24 \times 16 \text{ m}^2 = 192 \text{ m}^2$$

and $ar\Delta ORS = \frac{1}{2} \times OS \times RK$

$$= \frac{1}{2} \times 20 \times x = 10x$$

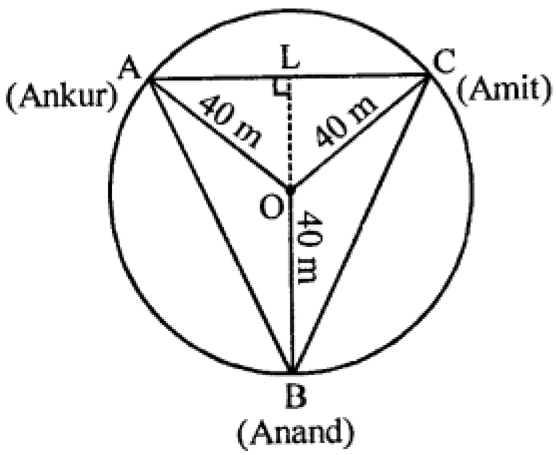
$\therefore 10x = 192 \Rightarrow x = \frac{192}{10} = 19.2$ m

Now RM = 2 × RK = $2x$

= 2 × 19.2 m = 38.4 m

Hence distance between Ishita and Nisha = 38.4 m

Question 2.
A circular park of radius 40 m is situated in a colony. Three beys Ankur, Amit and Anand are sitting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. Find thelength of the string of each phone. [NCERT]
Solution:
Radius of circular park = 40 m
Ankur, Amit and Anand are sitting at equal distance to each other By joining them, an equilateral triangle ABC is formed produce BO to L which is perpendicular bisector of AC



∴ BL = 40 + 20 = 60 m (∵ O is centroid of ∆ABC also)
Let a be the side of ∆ABC

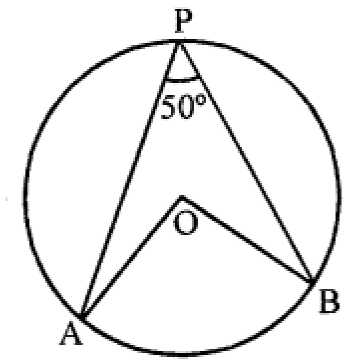
$$\therefore\ \frac{\sqrt{3}}{2}a = 60 \Rightarrow a = \frac{60 \times 2}{\sqrt{3}} = \frac{120 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$a = \frac{120 \times \sqrt{3}}{3} = 40\sqrt{3}\ \text{m}$$

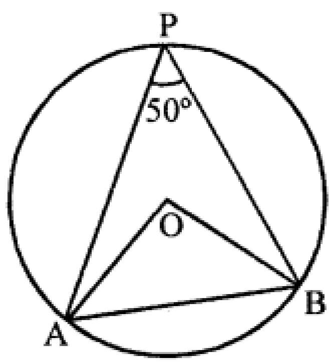
Hence the distance between each other = $40\sqrt{3}$ mRD Sharma Mathematics Class 9 Solutions Chapter 15 Areas of Parallelograms and Triangles

RD Sharma Mathematics Class 9 Solutions Chapter 15 Areas of Parallelograms and Triangles Ex 15.4

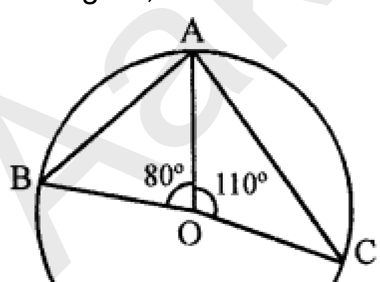
Question 1.
In the figure, O is the centre of the circle. If ∠APB = 50°, find ∠AOB and ∠OAB.



Solution:
Arc AB, subtends $\angle AOB$ at the centre and $\angle APB$ at the remaining part of the circle
$\therefore \angle AOB = 2\angle APB = 2 \times 50° = 100°$
Join AB

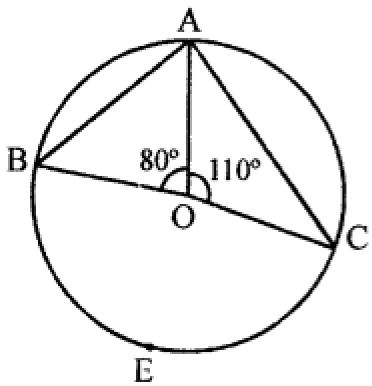


ΔAOB is an isosceles triangle in which
OA = OB
$\therefore \angle OAB = \angle OBA$ But $\angle AOB = 100°$
$\therefore \angle OAB + \angle OBA = 180° - 100° = 80°$
$\Rightarrow 2\angle OAB = 80°$
80°
$\therefore \angle OAB = \frac{80°}{2} = 40°$
Question 2.
In the figure, O is the centre of the circle. Find $\angle BAC$.

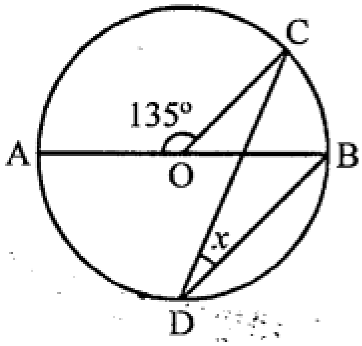


Solution:
In the circle with centre O

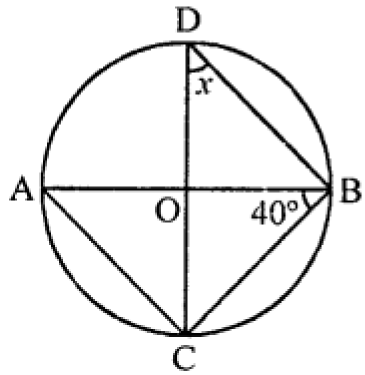
$\angle AOB = 80^\circ$ and $\angle AOC = 110^\circ$
$\therefore \angle BOC = \angle AOB + \angle AOC$
$= 80^\circ + 110^\circ = 190^\circ$
$\therefore$ Reflex $\angle BOC = 360^\circ - 190^\circ = 170^\circ$
Now arc BEC subtends $\angle BOC$ at the centre and $\angle BAC$ at the remaining part of the circle.



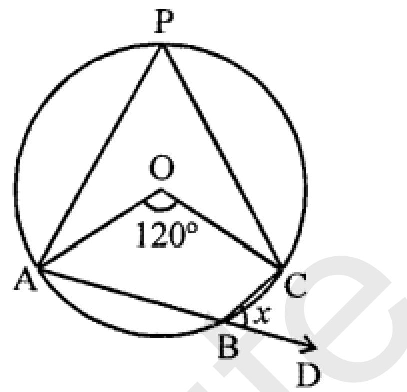
$\therefore \angle BOC = 2\angle BAC$
$\Rightarrow 170^\circ = 2\angle BAC$
$\Rightarrow \angle BAC = \frac{170^\circ}{2} = 85^\circ$
$\therefore \angle BAC = 85^\circ$
Question 3.
If O is the centre of the circle, find the value of x in each of the following figures:



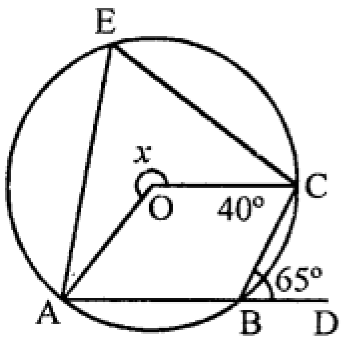
(*i*)



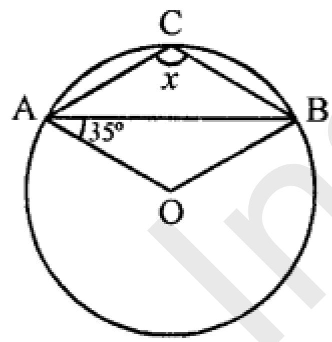
(*ii*)



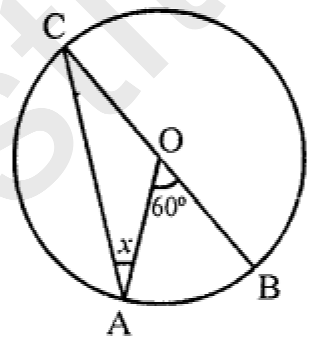
(*iii*)



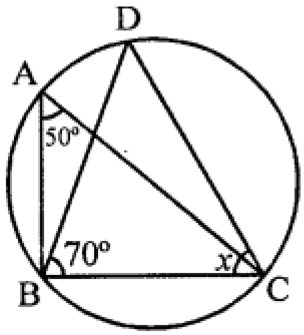
(*iv*)



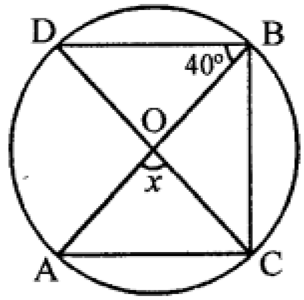
(*v*)



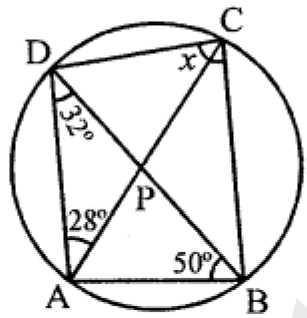
(*vi*)



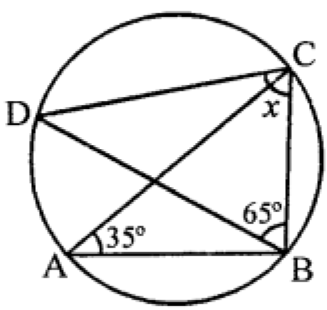
(vii)



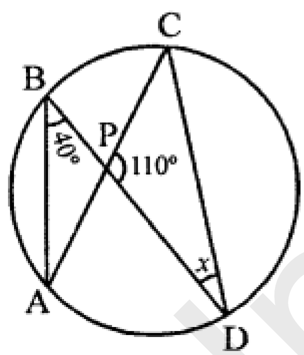
(viii)



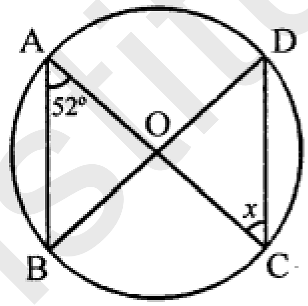
(ix)



(x)



(xi)



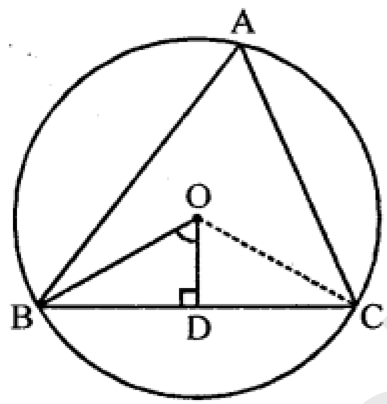
(xii)

Solution:
(i) A circle with centre O
∠AOC = 135°
But ∠AOC + ∠COB = 180° (Linear pair)
⇒ 135° + ∠COB = 180°
⇒ ∠COB = 180°- 135° = 45°
Now arc BC subtends ∠BOC at the centre and ∠BPC at the remaining part of the circle
∴ ∠BOC = 2∠BPC
⇒ ∠BPC = $\frac{1}{2}$∠BOC = $\frac{1}{2}$ x 45° = $\frac{45^\circ}{2}$
∴ ∠BPC = $22\frac{1}{2}$° or x = $22\frac{1}{2}$°
(ii) ∵ CD and AB are the diameters of the circle with centre O
∠ABC = 40°
But in ∆OBC,
OB = OC (Radii of the circle)
∠OCB = ∠OBC – 40°
Now in ABCD,
∠ODB + ∠OCB + ∠CBD = 180° (Angles of a triangle)
⇒ x + 40° + 90° = 180°
⇒ x + 130° = 180°
⇒ x = 180° – 130° = 50°
∴ x = 50°

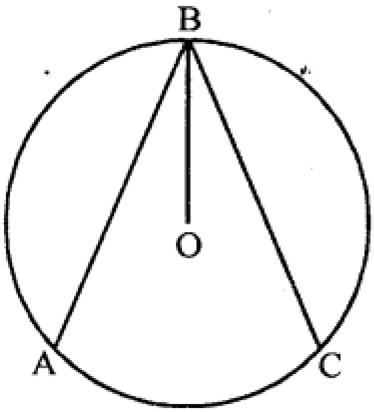
(iii) In circle with centre O,
∠AOC = 120°, AB is produced to D
∵ ∠AOC = 120°
and ∠AOC + convex ∠AOC = 360°
⇒ 120° + convex ∠AOC = 360°
∴ Convex ∠AOC = 360° – 120° = 240°
∴ Arc APC Subtends ∠AOC at the centre and ∠ABC at the remaining part of the circle
∴ ∠ABC = 12∠AOC = 12x 240° = 120°
But ∠ABC + ∠CBD = 180° (Linear pair)
⇒ 120° + x = 180°
⇒ x = 180° – 120° = 60°
∴ x = 60°
(iv) A circle with centre O and ∠CBD = 65°
But ∠ABC + ∠CBD = 180° (Linear pair)
⇒ ∠ABC + 65° = 180°
⇒ ∠ABC = 180°-65°= 115°
Now arc AEC subtends ∠x at the centre and ∠ABC at the remaining part of the circle
∴ ∠AOC = 2∠ABC
⇒ x = 2 x 115° = 230°
∴ x = 230°
(v) In circle with centre O
AB is chord of the circle, ∠OAB = 35°
In ∆OAB,
OA = OB (Radii of the circle)
∠OBA = ∠OAB = 35°
But in ∆OAB,
∠OAB + ∠OBA + ∠AOB = 180° (Angles of a triangle)
⇒ 35° + 35° + ∠AOB = 180°
⇒ 70° + ∠AOB = 180°
⇒ ∠AOB = 180°-70°= 110°
∴ Convex ∠AOB = 360° -110° = 250°
But arc AB subtends ∠AOB at the centre and ∠ACB at the remaining part of the circle.
∴∠ACB = 12∠AOB
⇒ x = 12 x 250° = 125°
∴ x= 125°
(vi) In the circle with centre O,
BOC is its diameter, ∠AOB = 60°
Arc AB subtends ∠AOB at the centre of the circle and ∠ACB at the remaining part of the circle
∴ ∠ACB = 12 ∠AOB
= 12 x 60° = 30°
But in ∆OAC,
OC = OA (Radii of the circle)
∴ ∠OAC = ∠OCA = ∠ACB
⇒ x = 30°
(vii) In the circle, ∠BAC and ∠BDC are in the same segment

$\therefore \angle BDC = \angle BAC = 50°$
Now in ABCD,
$\angle DBC + \angle BCD + \angle BDC = 180°$ (Angles of a triangle)
$\Rightarrow 70° + x + 50° = 180°$
$\Rightarrow x + 120° = 180° \Rightarrow x = 180° – 120° = 60°$
$\therefore x = 60°$
(viii) In circle with centre O,
$\angle OBD = 40°$
AB and CD are diameters of the circle
$\angle DBA$ and $\angle ACD$ are in the same segment
$\therefore \angle ACD = \angle DBA = 40°$
In AOAC, OA = OC (Radii of the circle)
$\therefore \angle OAC = \angle OCA = 40°$
and $\angle OAC + \angle OCA + \angle AOC = 180°$ (Angles in a triangle)
$\Rightarrow 40° + 40° + x = 180°$
$\Rightarrow x + 80° = 180° \Rightarrow x = 180° – 80° = 100°$
$\therefore x = 100°$
(ix) In the circle, ABCD is a cyclic quadrilateral $\angle ADB = 32°$, $\angle DAC = 28°$ and $\angle ABD = 50°$
$\angle ABD$ and $\angle ACD$ are in the same segment of a circle
$\therefore \angle ABD = \angle ACD \Rightarrow \angle ACD = 50°$
Similarly, $\angle ADB = \angle ACB$
$\Rightarrow \angle ACB = 32°$
Now, $\angle DCB = \angle ACD + \angle ACB$
$= 50° + 32° = 82°$
$\therefore x = 82°$
(x) In a circle,
$\angle BAC = 35°$, $\angle CBD = 65°$
$\angle BAC$ and $\angle BDC$ are in the same segment
$\therefore \angle BAC = \angle BDC = 35°$
In ΔBCD,
$\angle BDC + \angle BCD + \angle CBD = 180°$ (Angles in a triangle)
$\Rightarrow 35° + x + 65° = 180°$
$\Rightarrow x + 100° = 180°$
$\Rightarrow x = 180° – 100° = 80°$
$\therefore x = 80°$
(xi) In the circle,
$\angle ABD$ and $\angle ACD$ are in the same segment of a circle
$\therefore \angle ABD = \angle ACD = 40°$
Now in ΔCPD,
$\angle CPD + \angle PCD + \angle PDC = 180°$ (Angles of a triangle)
$110° + 40° + x = 180°$
$\Rightarrow x + 150° = 180°$
$\therefore x = 180° - 150° = 30°$
(xii) In the circle, two diameters AC and BD intersect each other at O
$\angle BAC = 50°$

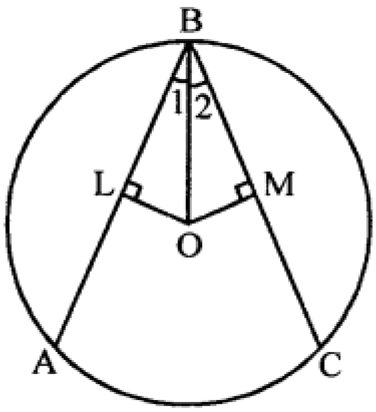
In ∆OAB,
OA = OB (Radii of the circle)
∴ ∠OBA = ∠OAB = 52°
⇒ ∠ABD = 52°
But ∠ABD and ∠ACD are in the same segment of the circle
∴ ∠ABD = ∠ACD ⇒ 52° = x
∴ x = 52°
Question 4.
O is the circumcentre of the triangle ABC and OD is perpendicular on BC. Prove that ∠BOD = ∠A.
Solution:
Given : O is the circumcentre of ∆ABC.
OD ⊥ BC
OB is joined
To prove : ∠BOD = ∠A
Construction : Join OC.



Proof : Arc BC subtends ∠BOC at the centre and ∠BAC at the remaining part of the circle
∴ ∠BOC = 2∠A …(i)
In right ∆OBD and ∆OCD Side OD = OD (Common)
Hyp. OB = OC (Radii of the circle)
∴ ∆OBD ≅ ∆OCD (RHS criterion)
∴ ∠BOD = ∠COD = 12 ∠BOC
⇒ ∠BOC = 2∠BOD …(ii)
From (i) and (ii)
2∠BOD = 2∠A
∴∠BOD = ∠A
Question 5.
In the figure, O is the centre of the circle, BO is the bisector of ∠ABC. Show that AB = BC.

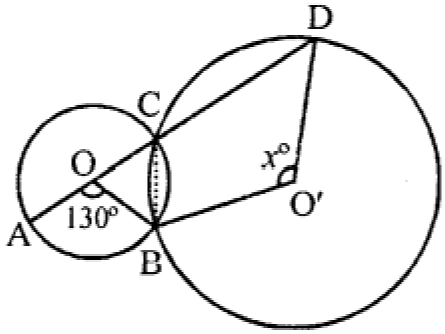


Solution:
Given : In the figure, a circle with centre O OB is the bisector of ∠ABC
To prove : AB = BC
Construction : Draw OL ⊥ AB and OM ⊥ BC

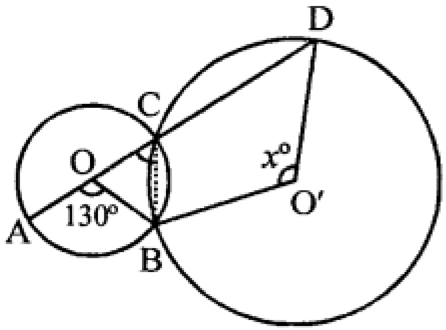


Proof: In ΔOLB and ΔOMB,
∠1 = ∠2 (Given)
∠L = ∠M (Each = 90°)
OB = OB (Common)
∴ ΔOLB ≅ ΔOMB (AAS criterion)
∴ OL = OM (c.p.c.t.)
But these are distance from the centre and chords equidistant from the centre are equal
∴ Chord BA = BC
Hence AB = BC

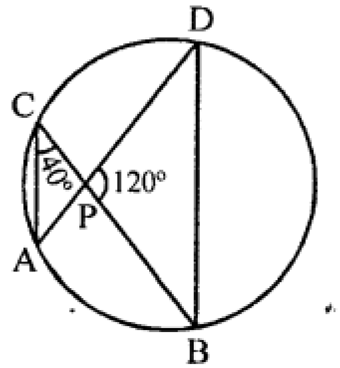
Question 6.
In the figure, O and O’ are centres of two circles intersecting at B and C. ACD is a straight line, find x.



Solution:
In the figure, two circles with centres O and O’ intersect each other at B and C.
ACD is a line, ∠AOB = 130°
Arc AB subtends ∠AOB at the centre O and ∠ACB at the remaining part of the circle.

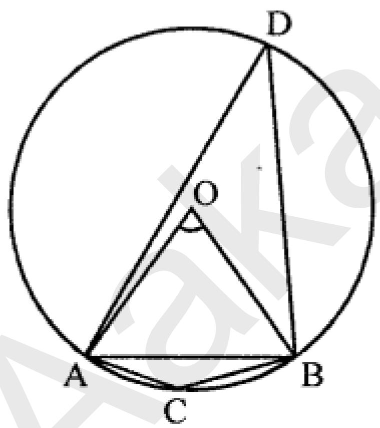


∴ ∠ACB = $\frac{1}{2}$∠AOB
= $\frac{1}{2}$ x 130° = 65°
But ∠ACB + ∠BCD = 180° (Linear pair)
⇒ 65° + ∠BCD = 180°
⇒ ∠BCD = 180°-65°= 115°
Now, arc BD subtends reflex ∠BO’D at the centre and ∠BCD at the remaining part of the circle
∴ ∠BO’D = 2∠BCD = 2 x 115° = 230°
But ∠BO’D + reflex ∠BO’D = 360° (Angles at a point)
⇒ x + 230° = 360°
⇒ x = 360° -230°= 130°
Hence x = 130°
Question 7.
In the figure, if ∠ACB = 40°, ∠DPB = 120°, find ∠CBD.



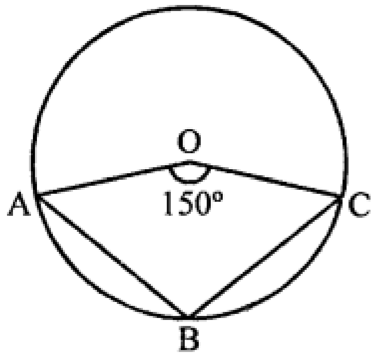
Solution:
Arc AB subtend ∠ACB and ∠ADB in the same segment of a circle
∴ ∠ACB = ∠ADB = 40°
In ∆PDB,
∠DPB + ∠PBD + ∠ADB = 180° (Sum of angles of a triangle)
⇒ 120° + ∠PBD + 40° = 180°
⇒ 160° + ∠PBD = 180°
⇒ ∠PBD = 180° – 160° = 20°
⇒ ∠CBD = 20°

Question 8.
A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.
Solution:
A circle with centre O, a chord AB = radius of the circle C and D are points on the minor and major arcs of the circle

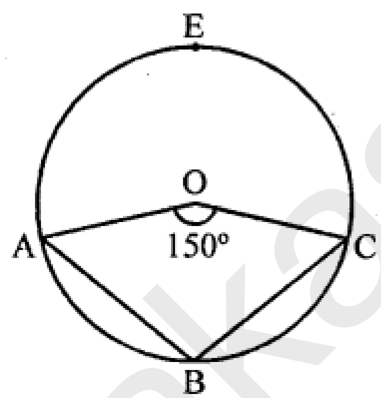


∴ ∠ACB and ∠ADB are formed Now in ∆AOB,
OA = OB = AB (∵ AB = radii of the circle)
∴ ∆AOB is an equilateral triangle,
∴ ∠AOB = 60°
Now arc AB subtends ∠AOB at the centre and ∠ADB at the remainder part of the circle.
∴ ∠ADB = 12 ∠AOB = 12x 60° = 30°
Now ACBD is a cyclic quadrilateral,

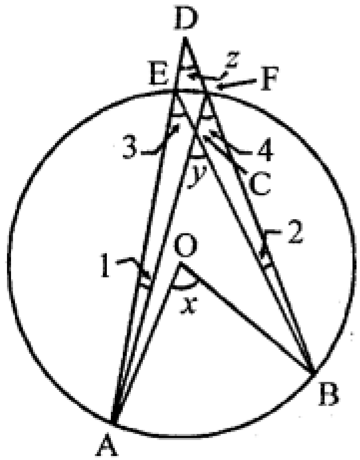
∴ ∠ADB + ∠ACB = 180° (Sum of opposite angles of cyclic quad.)
⇒ 30° + ∠ACB = 180°
⇒ ∠ACB = 180° – 30° = 150°
∴ ∠ACB = 150°
Hence angles are 150° and 30°
Question 9.
In the figure, it is given that O is the centre of the circle and ∠AOC = 150°. Find ∠ABC.



Solution:
In circle with centre O and ∠AOC = 150°
But ∠AOC + reflex ∠AOC = 360°
∴ 150° + reflex ∠AOC = 360°
⇒ Reflex ∠AOC = 360° – 150° = 210°
Now arc AEC subtends ∠AOC at the centre and ∠ABC at the remaining part of the circle.



Reflex ∠AOC = 2∠ABC
⇒ 210° = 2∠ABC
∴ ∠ABC = $\frac{210^\circ}{2}$ = 105°
Question 10.
In the figure, O is the centre of the circle, prove that ∠x = ∠y + ∠z.
Solution:
Given : In circle, O is centre



To prove : $\angle x = \angle y + \angle z$

Proof : $\because$ $\angle 3$ and $\angle 4$ are in the same segment of the circle

$\therefore \angle 3 = \angle 4$ …(i)

$\because$ Arc AB subtends $\angle AOB$ at the centre and $\angle 3$ at the remaining part of the circle

$\therefore \angle x = 2\angle 3 = \angle 3 + \angle 3 = \angle 3 + \angle 4$ ($\because \angle 3 = \angle 4$) …(ii)

In $\triangle ACE$,

Ext. $\angle y = \angle 3 + \angle 1$

(Ext. is equal to sum of its interior opposite angles)

$\Rightarrow \angle 3 - \angle y - \angle 1$ …(ii)

From (i) and (ii),

$\angle x = \angle y - \angle 1 + \angle 4$ …(iii)

Similarly in $\triangle ADF$,

Ext. $\angle 4 = \angle 1 + \angle z$ …(iv)

From (iii) and (iv)

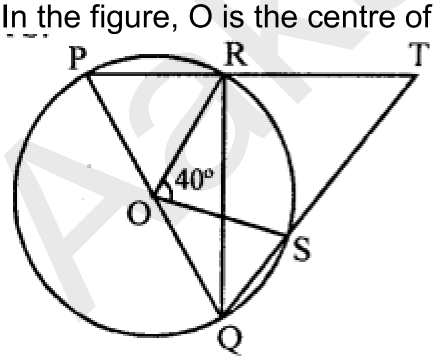
$\angle x = \angle y - \angle l + (\angle 1 + \angle z)$

$= \angle y - \angle 1 + \angle 1 + \angle z = \angle y + \angle z$

Hence $\angle x = \angle y + \angle z$

Question 11.

In the figure, O is the centre of a circle and PQ is a diameter. If $\angle ROS = 40°$, find $\angle RTS$.



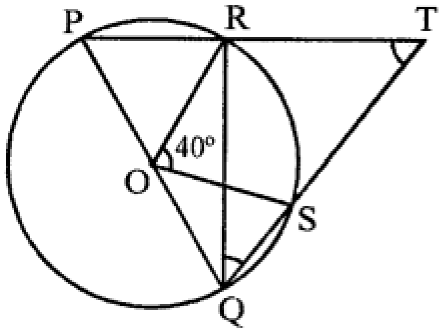
Solution:

In the figure, O is the centre of the circle,

PQ is the diameter and $\angle ROS = 40°$

Now we have to find $\angle RTS$

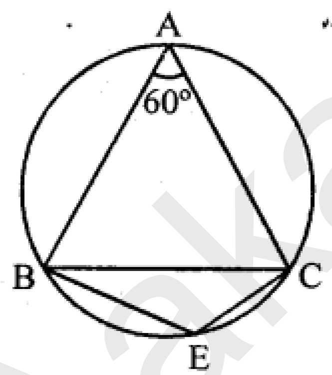
Arc RS subtends ∠ROS at the centre and ∠RQS at the remaining part of the circle



∴ ∠RQS = 12 ∠ROS
= 12 x 40° = 20°
∵ ∠PRQ = 90° (Angle in a semi circle)
∴ ∠QRT = 180° – 90° = 90° (∵ PRT is a straight line)
Now in ∆RQT,
∠RQT + ∠QRT + ∠RTQ = 180° (Angles of a triangle)
⇒ 20° + 90° + ∠RTQ = 180°
⇒ ∠RTQ = 180° – 20° – 90° = 70° or ∠RTS = 70°
Hence ∠RTS = 70°

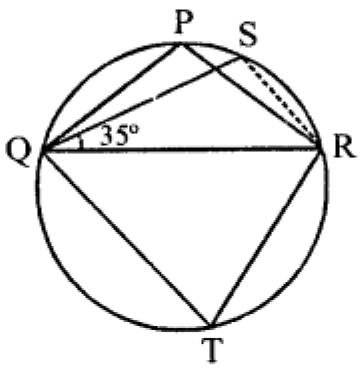
Class 9 Maths Chapter 15 Areas of Parallelograms and Triangles RD Sharma Solutions Ex 15.5

Question 1.
In the figure, ∆ABC is an equilateral triangle. Find m ∠BEC.



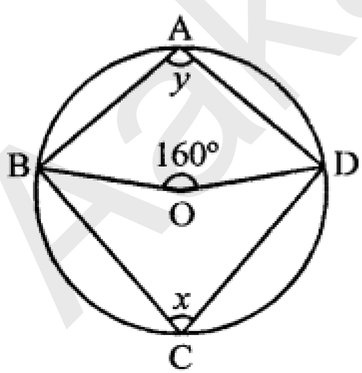
Solution:
∵ ∆ABC is an equilateral triangle
∴ A = 60°
∵ ABEC is a cyclic quadrilateral
∴ ∠A + ∠E = 180° (Sum of opposite angles)
⇒ 60° + ∠E = 180°
⇒ ∠E = 180° – 60° = 120°
∴ m ∠BEC = 120°

Question 2.
In the figure, ΔPQR is an isosceles triangle with PQ = PR and m $\angle PQR = 35°$. Find m $\angle QSR$ and m $\angle QTR$.



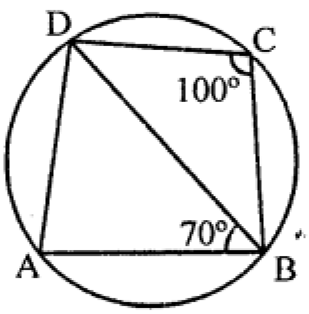
Solution:
In the figure, ΔPQR is an isosceles PQ = PR
$\angle PQR = 35°$
$\therefore \angle PRQ = 35°$
But $\angle PQR + \angle PRQ + \angle QPR = 180°$ (Sum of angles of a triangle)
$\Rightarrow 35° + 35° + \angle QPR = 180°$
$\Rightarrow 70° + \angle QPR = 180°$
$\therefore \angle QPR = 180° – 70° = 110°$
$\because \angle QSR = \angle QPR$ (Angle in the same segment of circles)
$\therefore \angle QSR = 110°$
But PQTR is a cyclic quadrilateral
$\therefore \angle QTR + \angle QPR = 180°$
$\Rightarrow \angle QTR + 110° = 180°$
$\Rightarrow \angle QTR = 180° -110° = 70°$
Hence $\angle QTR = 70°$

Question 3.
In the figure, O is the centre of the circle. If $\angle BOD = 160°$, find the values of x and y.

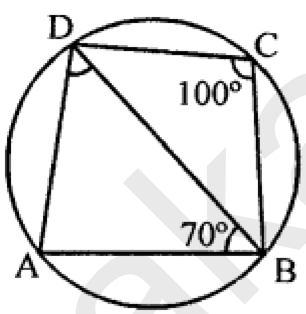


Solution:
In the figure, O is the centre of the circle $\angle BOD =160°$
ABCD is the cyclic quadrilateral

∵ Arc BAD subtends ∠BOD is the angle at the centre and ∠BCD is on the other part of the circle
∴ ∠BCD = 12 ∠BOD
⇒ x = 12 x 160° = 80°
∵ ABCD is a cyclic quadrilateral,
∴ ∠A + ∠C = 180°
⇒ y + x = 180°
⇒ y + 80° = 180°
⇒ y =180°- 80° = 100°
∴ x = 80°, y = 100°
Question 4.
In the figure, ABCD is a cyclic quadrilateral. If ∠BCD = 100° and ∠ABD = 70°, find ∠ADB.

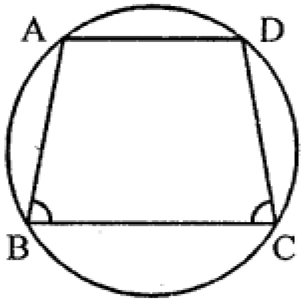


Solution:
In a circle, ABCD is a cyclic quadrilateral ∠BCD = 100° and ∠ABD = 70°
∵ ABCD is a cyclic quadrilateral,
∴ ∠A + ∠C = 180° (Sum of opposite angles)

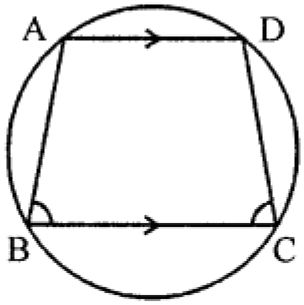


⇒ ∠A + 100°= 180°
∠A = 180°- 100° = 80°
Now in ∆ABD,
∠A + ∠ABD + ∠ADB = 180°
⇒ 80° + 70° + ∠ADB = 180°
⇒ 150° +∠ADB = 180°
∴ ∠ADB = 180°- 150° = 30°
Hence ∠ADB = 30°

Question 5.
If ABCD is a cyclic quadrilateral in which AD || BC. Prove that ∠B = ∠C.

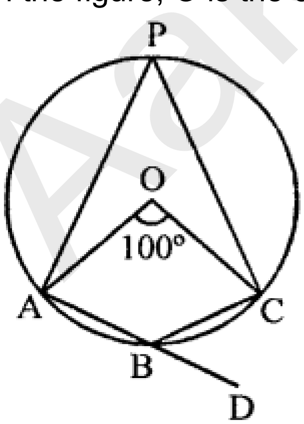


Solution:
Given : ABCD is a cyclic quadrilateral in which AD || BC



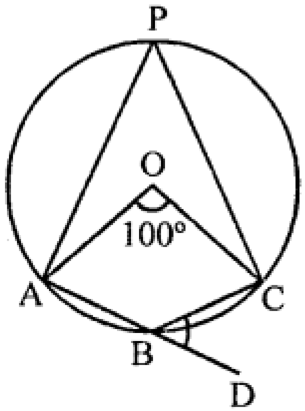
To prove : ∠B = ∠C
Proof : ∵ AD || BC
∴ ∠A + ∠B = 180°
(Sum of cointerior angles)
But ∠A + ∠C = 180°
(Opposite angles of the cyclic quadrilateral)
∴ ∠A + ∠B = ∠A + ∠C
⇒ ∠B = ∠C
Hence ∠B = ∠C

Question 6.
In the figure, O is the centre of the circle. Find ∠CBD.

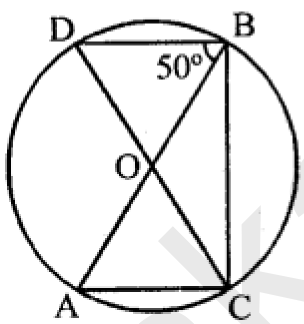


Solution:

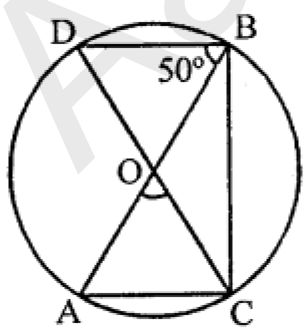
Arc AC subtends ∠AOC at the centre and ∠APC at the remaining part of the circle



∴ ∠APC = 12 ∠AOC
= 12 x 100° = 50°
∵ APCB is a.cyclic quadrilateral,
∴ ∠APC + ∠ABC = 180°
⇒ 50° + ∠ABC = 180° ⇒ ∠ABC =180°- 50°
∴ ∠ABC =130°
But ∠ABC + ∠CBD = 180° (Linear pair)
⇒ 130° + ∠CBD = 180°
⇒ ∠CBD = 180°- 130° = 50°
∴ ∠CBD = 50°
Question 7.
In the figure, AB and CD are diameiers of a circle with centre O. If ∠OBD = 50°, find ∠AOC.

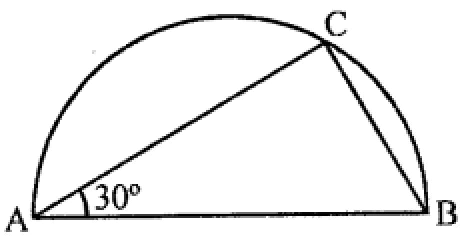


Solution:
Two diameters AB and CD intersect each other at O. AC, CB and BD are joined



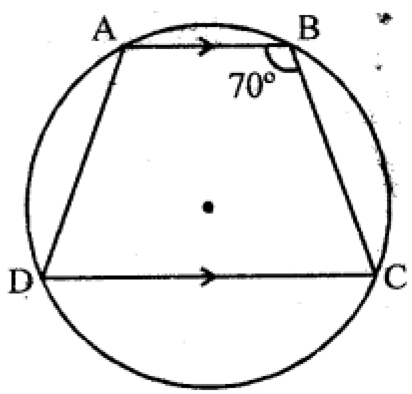
$\angle DBA = 50°$
$\angle DBA$ and $\angle DCA$ are in the same segment
$\therefore \angle DBA = \angle DCA = 50°$
In ΔOAC, OA = OC (Radii of the circle)
$\therefore \angle OAC = \angle OCA = \angle DCA = 50°$
and $\angle OAC + \angle OCA + \angle AOC = 180°$ (Sum of angles of a triangle)
$\Rightarrow 50° + 50° + \angle AOC = 180°$
$\Rightarrow 100° + \angle AOC = 180°$
$\Rightarrow \angle AOC = 180° – 100° = 80°$
Hence $\angle AOC = 80°$

Question 8.
On a semi circle with AB as diameter, a point C is taken so that m ($\angle CAB$) = 30°. Find m ($\angle ACB$) and m ($\angle ABC$).
Solution:
A semicircle with AB as diameter



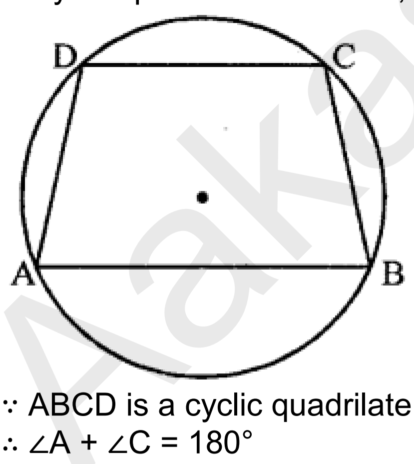
$\angle$ CAB = 30°
$\angle ACB = 90°$ (Angle in a semi circle)
But $\angle CAB + \angle ACB + \angle ABC = 180°$
$\Rightarrow 30° + 90° + \angle ABC – 180°$
$\Rightarrow 120° + \angle ABC = 180°$
$\therefore \angle ABC = 180°- 120° = 60°$
Hence m $\angle ACB = 90°$
and m $\angle ABC = 60°$

Question 9.
In a cyclic quadrilateral ABCD, if AB || CD and $\angle B = 70°$, find the remaining angles.
Solution:
In a cyclic quadrilateral ABCD, AB || CD and $\angle B = 70°$



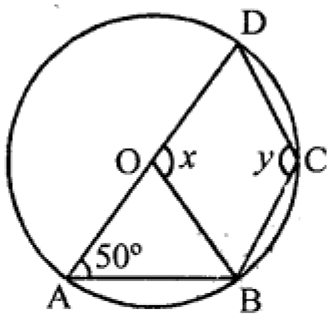
∵ ABCD is a cyclic quadrilateral
∴ ∠B + ∠D = 180°
⇒ 70° + ∠D = 180°
⇒ ∠D = 180°-70° = 110°
∵ AB || CD
∴ ∠A + ∠D = 180° (Sum of cointerior angles)
∠A+ 110°= 180°
⇒ ∠A= 180°- 110° = 70°
Similarly, ∠B + ∠C = 180°
⇒ 70° + ∠C- 180° ‘
⇒ ∠C = 180°-70°= 110°
∴ ∠A = 70°, ∠C = 110°, ∠D = 110°

Question 10.
In a cyclic quadrilateral ABCD, if m ∠A = 3(m ∠C). Find m ∠A.
Solution:
In cyclic quadrilateral ABCD, m ∠A = 3(m ∠C)



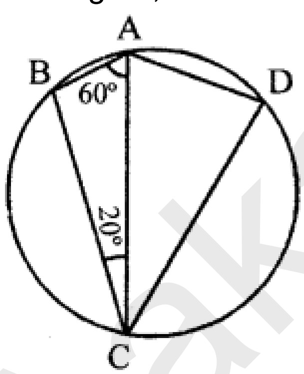
∵ ABCD is a cyclic quadrilateral,
∴ ∠A + ∠C = 180°
⇒ 3 ∠C + ∠C = 180° ⇒ 4∠C = 180°
⇒ ∠C = 180∘4 = 45°
∴ ∠A = 3 x 45°= 135°
Hence m ∠A =135°

Question 11.
In the figure, O is the centre of the circle and ∠DAB = 50°. Calculate the values of x and y.

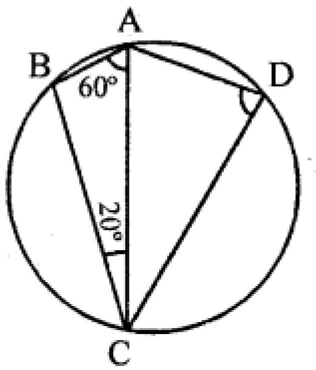


Solution:
In the figure, O is the centre of the circle ∠DAB = 50°
∵ ABCD is a cyclic quadrilateral
∴ ∠A + ∠C = 180°
⇒ 50° + y = 180°
⇒ y = 180° – 50° = 130°
In ∆OAB, OA = OB (Radii of the circle)
∴ ∠A = ∠OBA = 50°
∴ Ext. ∠DOB = ∠A + ∠OBA
x = 50° + 50° = 100°
∴ x= 100°, y= 130°

Question 12.
In the figure, if ∠BAC = 60° and ∠BCA = 20°, find ∠ADC.

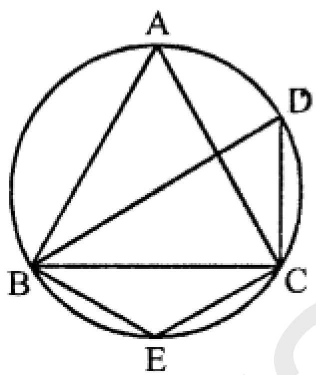


Solution:
In ∆ABC,
∠BAC + ∠ABC + ∠ACB = 180° (Sum of angles of a triangle)

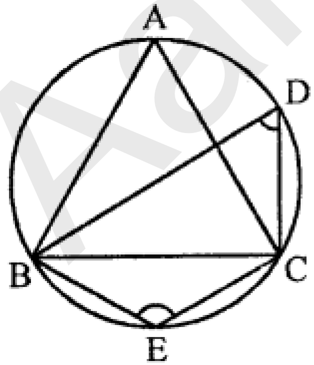


60° + ∠ABC + 20° = 180°
∠ABC + 80° = 180°
∴ ∠ABC = 180° -80°= 100°
∵ ABCD is a cyclic quadrilateral,
∴ ∠ABC + ∠ADC = 180°
100° + ∠ADC = 180°
∴ ∠ADC = 180°- 100° = 80°

Question 13.
In the figure, if ABC is an equilateral triangle. Find ∠BDC and ∠BEC.



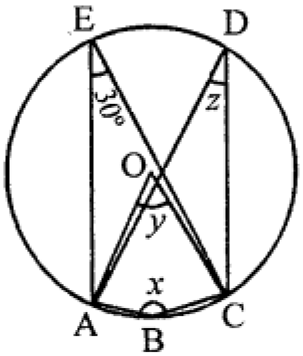
Solution:
In a circle, ΔABC is an equilateral triangle



∴ ∠A = 60°
∵ ∠BAC and ∠BDC are in the same segment

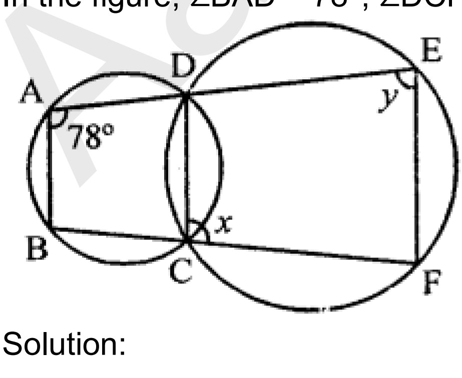
∴ ∠BAC = ∠BDC = 60°
∵ BECD is a cyclic quadrilateral
∴ ∠BDC + ∠BEC = 180°
⇒ 60° + ∠BEC = 180°
⇒ ∠BEC = 180°-60°= 120°
Hence ∠BDC = 60° and ∠BEC = 120°

Question 14.
In the figure, O is the centre of the circle. If ∠CEA = 30°, find the values of x, y and z.



Solution:
∠AEC and ∠ADC are in the same segment
∴ ∠AEC = ∠ADC = 30°
∴ z = 30°
ABCD is a cyclic quadrilateral
∴ ∠B + ∠D = 180°
⇒ x + z = 180°
⇒ x + 30° = 180°
⇒ x = 180° – 30° = 150°
Arc AC subtends ∠AOB at the centre and ∠ADC at the remaining part of the circle
∴ ∠AOC = 2∠D = 2 x 30° = 60°
∴ y = 60°
Hence x = 150°, y – 60° and z = 30°

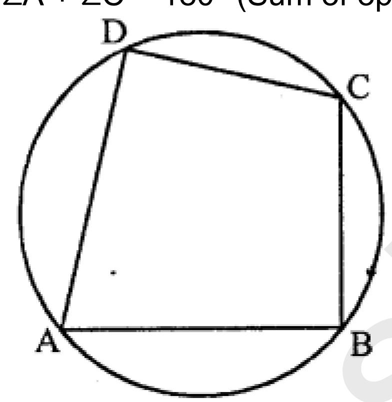
Question 15.
In the figure, ∠BAD = 78°, ∠DCF = x° and ∠DEF = y°. Find the values of x and y.



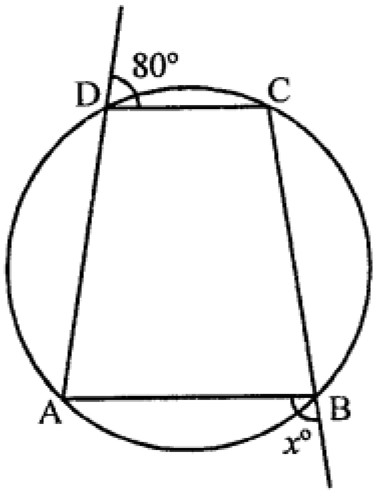
Solution:

In the figure, two circles intersect each other at C and D
$\angle BAD = 78°$, $\angle DCF = x$, $\angle DEF = y$
ABCD is a cyclic quadrilateral
∴ Ext. $\angle DCF$ = its interior opposite $\angle BAD$
$\Rightarrow x = 78°$
In cyclic quadrilateral CDEF,
$\angle DCF + \angle DEF = 180°$
$\Rightarrow 78° + y = 180°$
$\Rightarrow y = 180° - 78°$
$y = 102°$
Hence $x = 78°$, and y- 102°

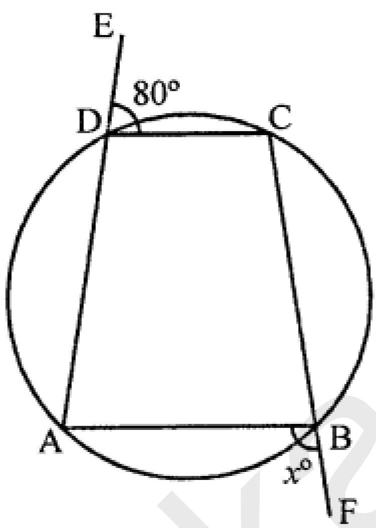
Question 16.
In a cyclic quadrilateral ABCD, if $\angle A - \angle C = 60°$, prove that the smaller of two is 60°.
Solution:
In cyclic quadrilateral ABCD,
$\angle A - \angle C = 60°$
But $\angle A + \angle C = 180°$ (Sum of opposite angles)



Adding, $2\angle A = 240° \Rightarrow \angle A =$ 62∘2 $= 120°$ and subtracting
$2\angle C = 120° \Rightarrow \angle C =$ 120∘2 $= 60°$
∴ Smaller angle of the two is 60°.
Question 17.
In the figure, ABCD is a cyclic quadrilateral. Find the value of x.

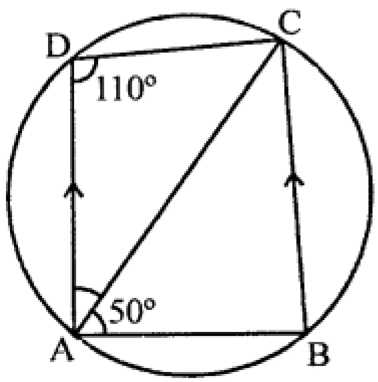


Solution:
∠CDE + ∠CDA = 180° (Linear pair)
⇒ 80° + ∠CDA = 180°
⇒ ∠CDA = 180° – 80° = 100°

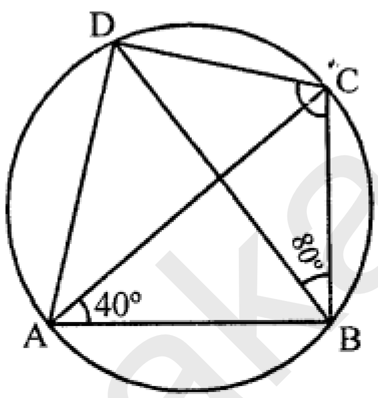


In cyclic quadrilateral ABCD,
Ext. ∠ABF = Its interior opposite angle ∠CDA = 100°
∴ x = 100°

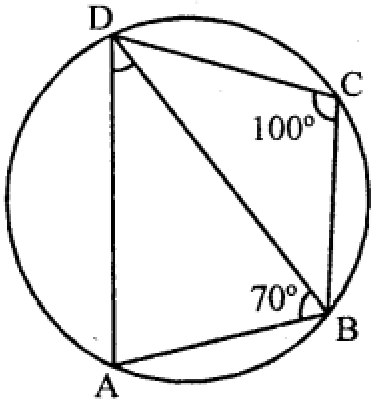
Question 18.
ABCD is a cyclic quadrilateral in which:
(i) BC || AD, ∠ADC =110° and ∠B AC = 50°. Find ∠DAC.
(ii) ∠DBC = 80° and ∠BAC = 40°. Find ∠BCD.
(iii) ∠BCD = 100° and ∠ABD = 70°, find ∠ADB.
Solution:
(i) In the figure,



ABCD is a cyclic quadrilateral and AD || BC, ∠ADC = 110°
∠BAC = 50°
∵ ∠B + ∠D = 180° (Sum of opposite angles)
⇒ ∠B + 110° = 180°
∴ ∠B = 180°- 110° = 70°
Now in △ABC,
∠CAB + ∠ABC + ∠BCA = 180° (Sum of angles of a triangle)
⇒ 50° + 70° + ∠BCA = 180°
⇒ 120° + ∠BCA = 180°
⇒ ∠BCA = 180° – 120° = 60°
But ∠DAC = ∠BCA (Alternate angles)
∴ ∠DAC = 60°
(ii) In cyclic quadrilateral ABCD,
Diagonals AC and BD are joined ∠DBC = 80°, ∠BAC = 40°

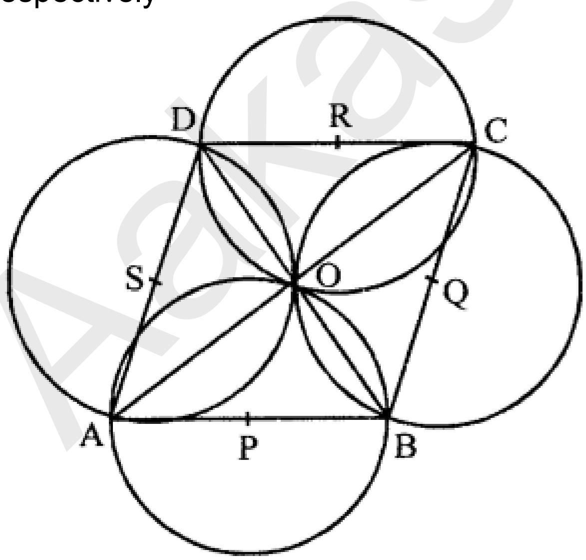


Arc DC subtends ∠DBC and ∠DAC in the same segment
∴ ∠DBC = ∠DAC = 80°
∴ ∠DAB = ∠DAC + ∠CAB = 80° + 40° = 120°
But ∠DAC + ∠BCD = 180° (Sum of opposite angles of a cyclic quad.)
⇒ 120° +∠BCD = 180°
⇒ ∠BCD = 180°- 120° = 60°
(iii) In the figure, ABCD is a cyclic quadrilateral BD is joined
∠BCD = 100°
and ∠ABD = 70°



$\angle A + \angle C = 180°$ (Sum of opposite angles of cyclic quad.)
$\angle A + 100° = 180°$
$\Rightarrow \angle A = 180° - 100°$
$\therefore \angle A = 80°$
Now in ΔABD,
$\angle A + \angle ABD + \angle ADB = 180°$ (Sum of angles of a triangle)
$\Rightarrow 80° + 70° + \angle ADB = 180°$
$\Rightarrow 150° + \angle ADB = 180°$
$\Rightarrow \angle ADB = 180° - 150° = 30°$
$\therefore \angle ADB = 30°$

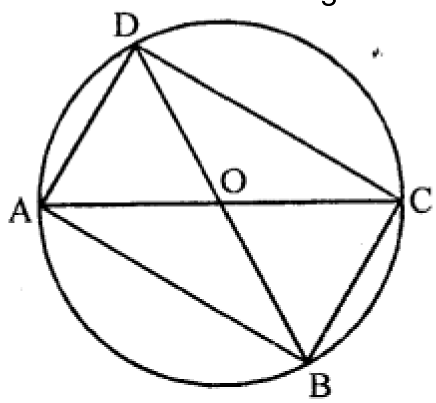
Question 19.
Prove that the circles described on the four sides of a rhombus as diameter, pass through the point of intersection of its diagonals.
Solution:
Given : ABCD is a rhombus. Four circles are drawn on the sides AB, BC, CD and DA respectively



To prove : The circles pass through the point of intersection of the diagonals of the rhombus ABCD

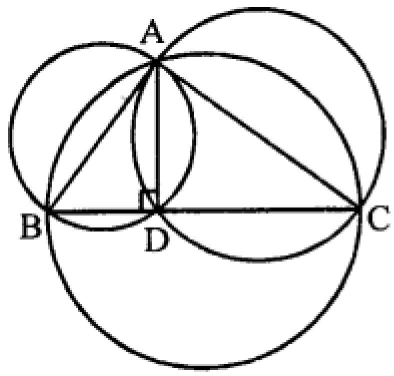
Proof: ABCD is a rhombus whose diagonals AC and BD intersect each other at O
∵ The diagonals of a rhombus bisect each other at right angles
∴ $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$
Now when $\angle AOB = 90°$
and a circle described on AB as diameter will pass through O
Similarly, the circles on BC, CD and DA as diameter, will also pass through O

Question 20.
If the two sides of a pair of opposite sides of a cyclic quadrilateral are equal, prove that is diagonals are equal.
Solution:
Given : In cyclic quadrilateral ABCD, AB = CD
AC and BD are the diagonals



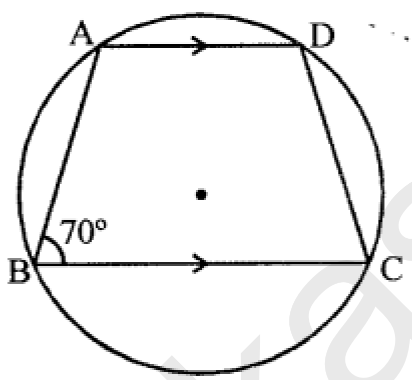
To prove : AC = BC
Proof: ∵ AB = CD
∴ arc AB = arc CD
Adding arc BC to both sides, then arc AB + arc BC = arc BC + arc CD
⇒ arc AC = arc BD
∴ AC = BD
Hence diagonal of the cyclic quadrilateral are equal.

Question 21.
Circles are described on the sides of a triangle as diameters. Prove that the circles on any two sides intersect each other on the third side (or third side produced).
Solution:
Given : In ΔABC, circles are drawn on sides AB and AC
To prove : Circles drawn on AB and AC intersect at D which lies on BC, the third side
Construction : Draw $AD \perp BC$



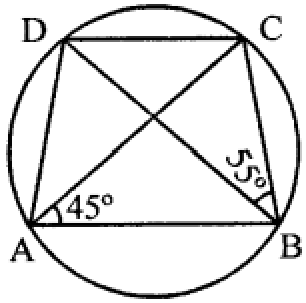
Proof: ∵ AD ⊥ BC
∴ ∠ADB = ∠ADC = 90°
So, the circles drawn on sides AB and AC as diameter will pass through D
Hence circles drawn on two sides of a triangle pass through D, which lies on the third side.

Question 22.
ABCD is a cyclic trapezium with AD || BC. If ∠B = 70°, determine other three angles of the trapezium.
Solution:
In the figure, ABCD is a trapezium in which AD || BC and ∠B = 70°

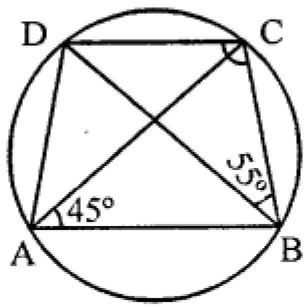


∵ AD || BC
∴ ∠A + ∠B = 180° (Sum of cointerior angles)
⇒ ∠A + 70° = 180°
⇒ ∠A= 180°- 70° = 110°
∴ ∠A = 110°
But ∠A + ∠C = 180° and ∠B + ∠D = 180° (Sum of opposite angles of a cyclic quadrilateral)
∴ 110° + ∠C = 180°
⇒ ∠C = 180°- 110° = 70°
and 70° + ∠D = 180°
⇒ ∠D = 180° – 70° = 110°
∴ ∠A = 110°, ∠C = 70° and ∠D = 110°

Question 23.
In the figure, ABCD is a cyclic quadrilateral in which AC and BD are its diagonals. If $\angle DBC = 55°$ and $\angle BAC = 45°$, find $\angle BCD$.

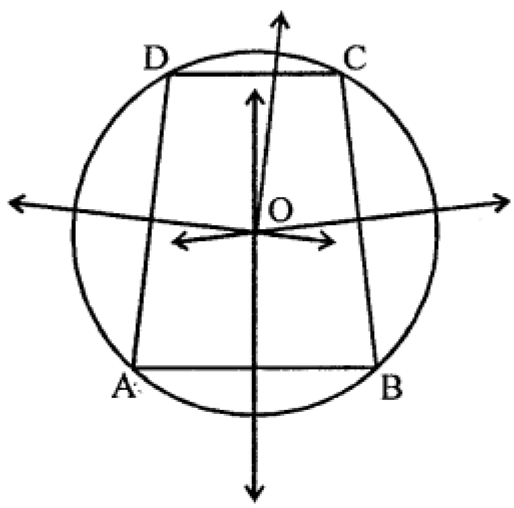


Solution:
In the figure, ABCD is a cyclic quadrilateral whose diagonals AC and BD are drawn
$\angle DBC = 55°$ and $\angle BAC = 45°$



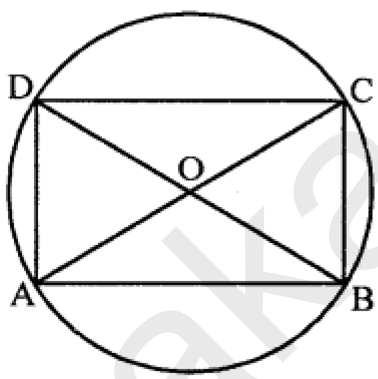
$\because$ $\angle BAC$ and $\angle BDC$ are in the same segment
$\therefore$ $\angle BAC = \angle BDC = 45°$
Now in ABCD,
$\angle DBC + \angle BDC + \angle BCD = 180°$ (Sum of angles of a triangle)
$\Rightarrow 55° + 45° + \angle BCD = 180°$
$\Rightarrow 100° + \angle BCD = 180°$
$\Rightarrow \angle BCD = 180° - 100° = 80°$
Hence $\angle BCD = 80°$

Question 24.
Prove that the perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent.
Solution:
Given : ABCD is a cyclic quadrilateral



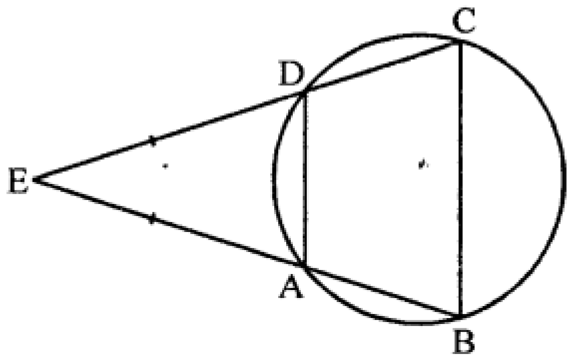
To prove : The perpendicular bisectors of the sides are concurrent
Proof : ∵ Each side of the cyclic quadrilateral is a chord of the circle and perpendicular of a chord passes through the centre of the circle
Hence the perpendicular bisectors of each side will pass through the centre O
Hence the perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent

Question 25.
Prove that the centre of the circle circumscribing the cyclic rectangle ABCD is the point of intersection of its diagonals.
Solution:
Given : ABCD is a cyclic rectangle and diagonals AC and BD intersect each other at O



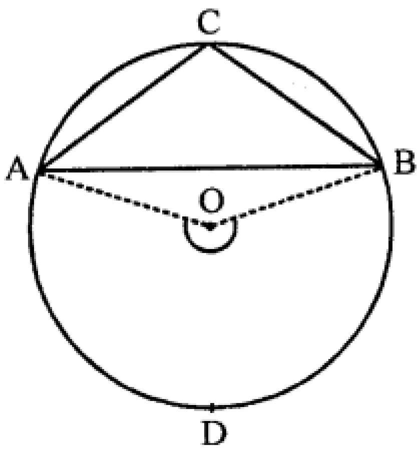
To prove : O is the point of intersection is the centre of the circle.
Proof : Let O be the centre of the circle- circumscribing the rectangle ABCD
Since each angle of a rectangle is a right angle and AC is the chord of the circle
∴ AC will be the diameter of the circle Similarly, we can prove that diagonal BD is also the diameter of the circle
∴ The diameters of the circle pass through the centre
Hence the point of intersection of the diagonals of the rectangle is the centre of the circle.

Question 26.
ABCD is a cyclic quadrilateral in which BA and CD when produced meet in E and EA = ED. Prove that:
(i) AD || BC
(ii) EB = EC.
Solution:
Given : ABCD is a cyclic quadrilateral in which sides BA and CD are produced to meet at E and EA = ED

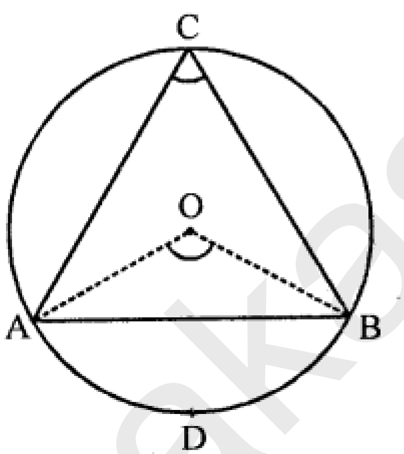


To prove :
(i) AD || BC
(ii) EB = EC
Proof: ∵ EA = ED
∴ In ΔEAD
∠EAD = ∠EDA (Angles opposite to equal sides)
In a cyclic quadrilateral ABCD,
Ext. ∠EAD = ∠C
Similarly Ext. ∠EDA = ∠B
∵ ∠EAD = ∠EDA
∴ ∠B = ∠C
Now in ΔEBC,
∵ ∠B = ∠C
∴ EC = EB (Sides opposite to equal sides)
and ∠EAD = ∠B
But these are corresponding angles
∴ AD || BC

Question 27.
Prove that the angle in a segment shorter than a semicircle is greater than a right angle.
Solution:
Given : A segment ACB shorter than a semicircle and an angle ∠ACB inscribed in it
To prove : ∠ACB < 90°
Construction : Join OA and OB

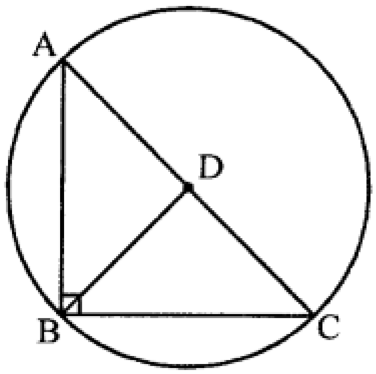


Proof : Arc ADB subtends ∠AOB at the centre and ∠ACB at the remaining part of the circle ∴ ∠ACB = 12∠AOB But ∠AOB > 180° (Reflex angle)
∴ ∠ACB > 12 x [80°
⇒ ∠ACB > 90°
Question 28.
Prove that the angle in a segment greater than a semi-circle is less than a right angle
Solution:
Given : A segment ACB, greater than a semicircle with centre O and ∠ACB is described in it
To prove : ∠ACB < 90°
Construction : Join OA and OB



Proof : Arc ADB subtends ∠AOB at the centre and ∠ACB at the remaining part of the circle
∴ ∠ACB =12 ∠AOB
But ∠AOB < 180° (A straight angle) 1
∴ ∠ACB < 12 x 180°
⇒ ∠ACB <90°
Hence ∠ACB < 90°
Question 29.
Prove that the line segment joining the mid-point of the hypotenuse of a rijght triangle to its opposite vertex is half of the hypotenuse.

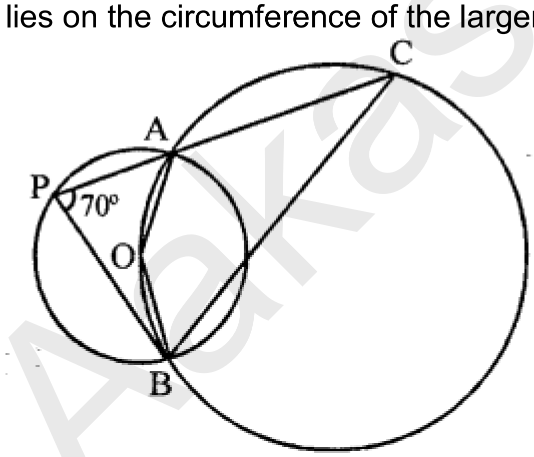
Solution:
Given : In a right angled ∆ABC
∠B = 90°, D is the mid point of hypotenuse AC. DB is joined.
To prove : BD = 12 AC
Construction : Draw a circle with centre D and AC as diameter



Proof: ∵ ∠ABC = 90°
∴ The circle drawn on AC as diameter will pass through B
∴ BD is the radius of the circle
But AC is the diameter of the circle and D is mid point of AC
∴ AD = DC = BD
∴ BD= 12 AC

RD Sharma Class 9 Book Chapter 15 Areas of Parallelograms and Triangles VSAQS

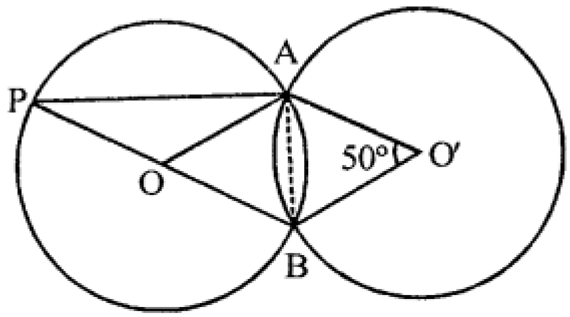
Question 1.
In the figure, two circles intersect at A and B. The centre of the smaller circle is O and it lies on the circumference of the larger circle. If ∠APB = 70°, find ∠ACB.



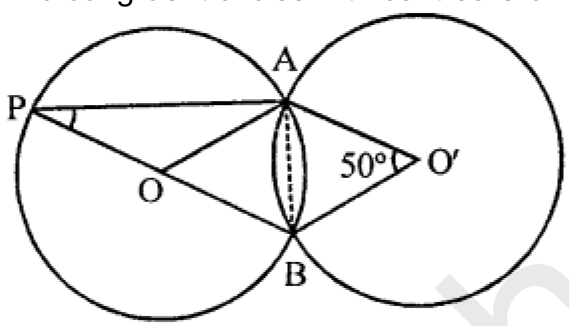
Solution:
Arc AB subtends ∠AOB at the centre and ∠APB at the remaining part of the circle
∴ ∠AOB = 2∠APB = 2 x 70° = 140°
Now in cyclic quadrilateral AOBC,
∠AOB + ∠ACB = 180° (Sum of the angles)
⇒ 140° +∠ACB = 180°

⇒ ∠ACB = 180° – 140° = 40°
∴ ∠ACB = 40°

Question 2.
In the figure, two congruent circles with centre O and O’ intersect at A and B. If ∠AO’B = 50°, then find ∠APB.



Solution:
Two congruent circles with centres O and O’ intersect at A and B

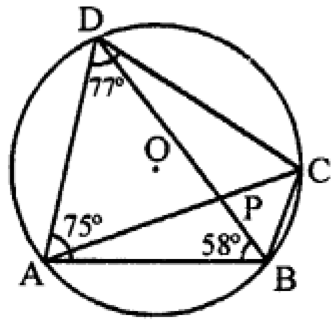


∠AO’B = 50°
∵ OA = OB = O’A = 04B (Radii of the congruent circles)

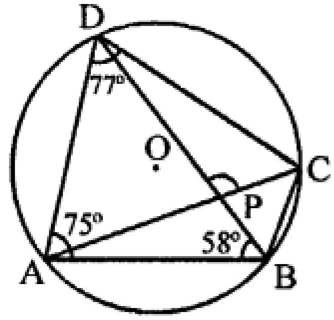
∵ OA = OB = O′A = O¢B
(Radii of the congruent circles)
∴ OAO¢B is a rhombus
∴ ∠AOB = ∠AO′B = 50°
Now arc AB subtends ∠AOB at the centre and ∠APB at the remaining part of the circle

$$\therefore \angle APB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 50^\circ = 25^\circ$$

Question 3.
In the figure, ABCD is a cyclic quadrilateral in which ∠BAD = 75°, ∠ABD = 58° and ∠ADC = IT, AC and BD intersect at P. Then, find ∠DPC.

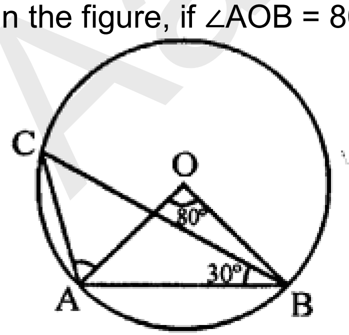


Solution:
∵ ABCD is a cyclic quadrilateral,



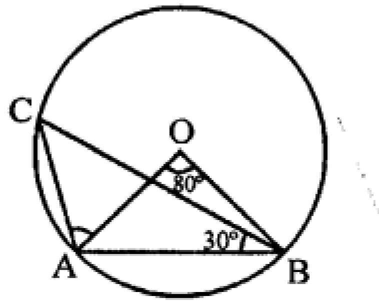
∴ ∠BAD + ∠BCD = 180°
⇒ 75° + ∠BCD – 180°
⇒ ∠BCD = 180°-75°= 105° and ∠ADC + ∠ABC = 180°
⇒ 77° + ∠ABC = 180°
⇒ ∠ABC = 180°-77°= 103°
∴ ∠DBC = ∠ABC – ∠ABD = 103° – 58° = 45°
∵ Arc AD subtends ∠ABD and ∠ACD in the same segment of the circle 3
∴ ∠ABD = ∠ACD = 58°
∴ ∠ACB = ∠BCD – ∠ACD = 105° – 58° = 47°
Now in ∆PBC,
Ext. ∠DPC = ∠PBC + ∠PCB
=∠DBC + ∠ACB = 45° + 47° = 92°
Hence ∠DPC = 92°

Question 4.
In the figure, if ∠AOB = 80° and ∠ABC = 30°, then find ∠CAO.

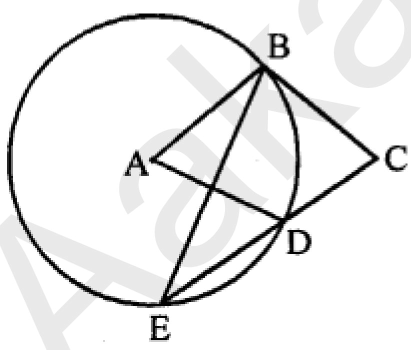


Solution:

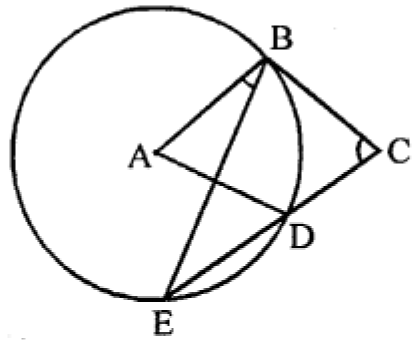
In the figure, ∠AOB = 80°, ∠ABC = 30°
∵ Arc AB subtends ∠AOB at the centre and
∠ACB at the remaining part of the circle



∴ ∠ACB = $\frac{1}{2}$∠AOB = $\frac{1}{2}$ x 80° = 40°
In ΔOAB, OA = OB
∴ ∠OAB = ∠OBA
But ∠OAB + ∠OBA + ∠AOB = 180°
∴ ∠OAB + ∠OBA + 80° = 180°
⇒ ∠OAB + ∠OAB = 180° – 80° = 100°
∴ 2∠OAB = 100°
⇒ ∠OAB = $\frac{100^\circ}{2}$ = 50°
Similarly, in ΔABC,
∠BAC + ∠ACB + ∠ABC = 180°
∠BAC + 40° + 30° = 180°
⇒ ∠BAC = 180°-30°-40°
= 180°-70°= 110°
∴ ∠CAO = ∠BAC – ∠OAB
= 110°-50° = 60°
Question 5.
In the figure, A is the centre of the circle. ABCD is a parallelogram and CDE is a straight line. Find ∠BCD : ∠ABE.



Solution:
In the figure, ABCD is a parallelogram and
CDE is a straight line



∵ ABCD is a ||gm
∴ ∠A = ∠C
and ∠C = ∠ADE (Corresponding angles)
⇒ ∠BCD = ∠ADE
Similarly, ∠ABE = ∠BED (Alternate angles)
∵ arc BD subtends ∠BAD at the centre and
∠BED at the remaining part of the circle

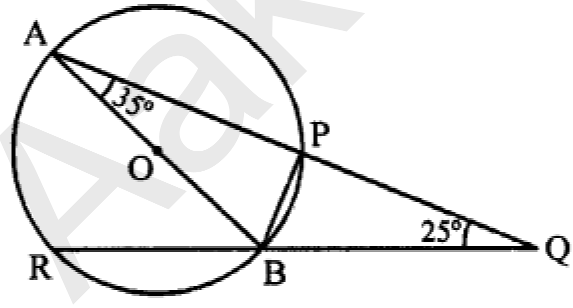
$$\therefore \angle BED = \frac{1}{2}\angle A = \frac{1}{2}\angle C = \frac{1}{2}\angle ADE$$

$$\text{Now } \frac{\angle BCD}{\angle ABE} = \frac{\angle BAD}{\angle BED} = \frac{\angle ADE}{\frac{1}{2}\angle ADE} = \frac{2}{1}$$

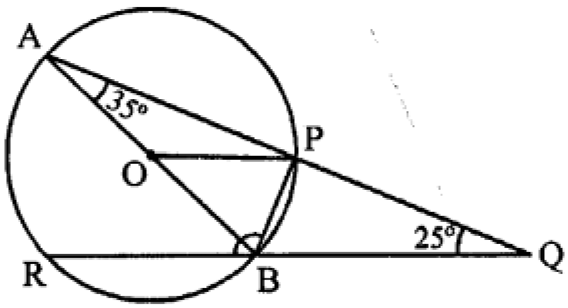
$$= \frac{2}{1}$$

$$\therefore \angle BCD : \angle ABE = 2 : 1$$

Question 6.
In the figure, AB is a diameter of the circle such that ∠A = 35° and ∠Q = 25°, find ∠PBR.

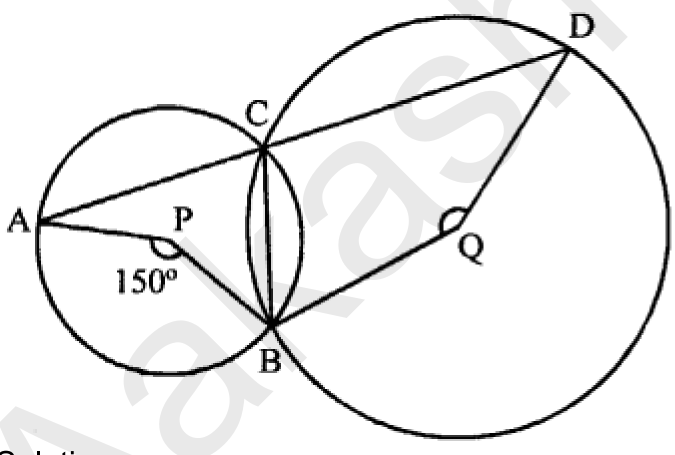


Solution:
In the figure, AB is the diameter of the circle such that ∠A = 35° and ∠Q = 25°, join OP.



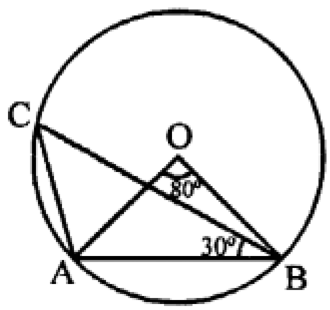
Arc PB subtends ∠POB at the centre and
∠PAB at the remaining part of the circle
∴ ∠POB = 2∠PAB = 2 x 35° = 70°
Now in ΔOP,
OP = OB radii of the circle
∴ ∠OPB = ∠OBP = 70° (∵ ∠OPB + ∠OBP = 140°)
Now ∠APB = 90° (Angle in a semicircle)
∴ ∠BPQ = 90°
and in ΔPQB,
Ext. ∠PBR = ∠BPQ + ∠PQB
= 90° + 25°= 115°
∴ ∠PBR = 115°

Question 7.
In the figure, P and Q are centres of two circles intersecting at B and C. ACD is a straight line. Then, ∠BQD =

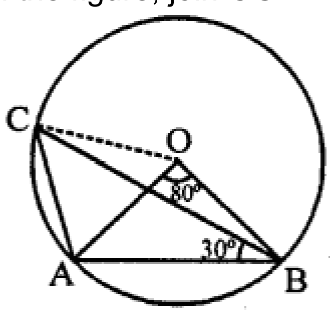


Solution:
In the figure, P and Q are the centres of two circles which intersect each other at C and B
ACD is a straight line ∠APB = 150°
Arc AB subtends ∠APB at the centre and
∠ACB at the remaining part of the circle
∴ ∠ACB = 12 ∠APB = 12 x 150° = 75°
But ∠ACB + ∠BCD = 180° (Linear pair)
⇒ 75° + ∠BCD = 180°

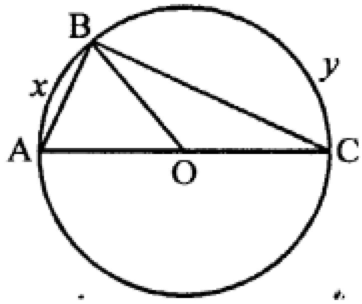
∠BCD = 180°-75°= 105°
Now arc BD subtends reflex ∠BQD at the centre and ∠BCD at the remaining part of the circle
Reflex ∠BQD = 2∠BCD = 2 x 105° = 210°
But ∠BQD + reflex ∠BQD = 360°
∴ ∠BQD+ 210° = 360°
∴ ∠BQD = 360° – 210° = 150°
Question 8.
In the figure, if O is circumcentre of ΔABC then find the value of ∠OBC + ∠BAC.



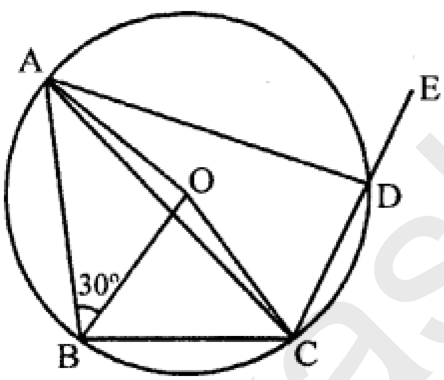
Solution:
In the figure, join OC



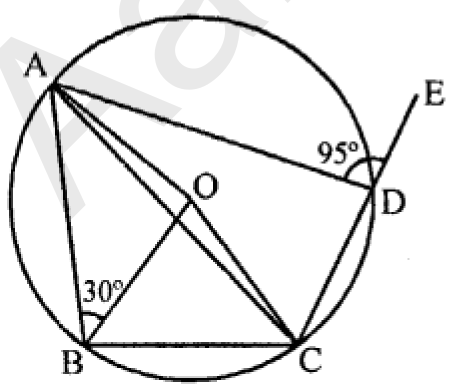
∵ O is the circumcentre of ΔABC
∴ OA = OB = OC
∵ ∠CAO = 60° (Proved)
∴ ΔOAC is an equilateral triangle
∴ ∠AOC = 60°
Now, ∠BOC = ∠BOA + ∠AOC
= 80° + 60° = 140°
and in ΔOBC, OB = OC
∠OCB = ∠OBC
But ∠OCB + ∠OBC = 180° – ∠BOC
= 180°- 140° = 40°
⇒ ∠OBC + ∠OBC = 40°
∴ ∠OBC = 40∘2 = 20°
∠BAC = OAB + ∠OAC = 50° + 60° = 110°
∴ ∠OBC + ∠BAC = 20° + 110° = 130°
Question 9.
In the AOC is a diameter of the circle and arc AXB = 1/2 arc BYC. Find ∠BOC.



Solution:
In the figure, AOC is diameter arc AxB = 12 arc BYC 1
∠AOB = 12 ∠BOC
⇒ ∠BOC = 2∠AOB
But ∠AOB + ∠BOC = 180°
⇒ ∠AOB + 2∠AOB = 180°
⇒ 3 ∠AOB = 180°
∴ ∠AOB = 180∘3 = 60°
∴ ∠BOC = 2 x 60° = 120°
Question 10.
In the figure, ABCD is a quadrilateral inscribed in a circle with centre O. CD produced to E such that ∠AED = 95° and ∠OBA = 30°. Find ∠OAC.



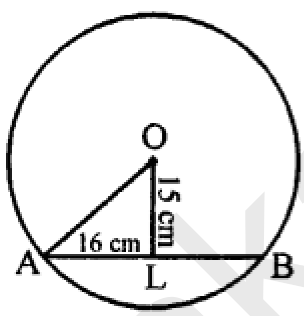
Solution:
In the figure, ABCD is a cyclic quadrilateral
CD is produced to E such that ∠ADE = 95°
O is the centre of the circle



∵ ∠ADC + ∠ADE = 180°
⇒ ∠ADC + 95° = 180°
⇒ ∠ADC = 180°-95° = 85°
Now arc ABC subtends ∠AOC at the centre and ∠ADC at the remaining part of the circle
∵ ∠AOC = 2∠ADC = 2 x 85° = 170°
Now in ΔOAC,
∠OAC + ∠OCA + ∠AOC = 180° (Sum of angles of a triangle)
⇒ ∠OAC = ∠OCA (∵ OA = OC radii of circle)
∴ ∠OAC + ∠OAC + 170° = 180°
2∠OAC = 180°- 170°= 10°
∴ ∠OAC = 10∘2 = 5°

RD Sharma Class 9 Solution Chapter 15 Areas of Parallelograms and Triangles MCQS

Question 1.
If the length of a chord of a circle is 16 cm and is at a distance of 15 cm from the centre of the circle, then the radius of the circle is
(a) 15 cm
(b) 16 cm
(c) 17 cm
(d) 34 cm
Solution:
Length of chord AB of circle = 16 cm
Distance from the centre OL = 15 cm

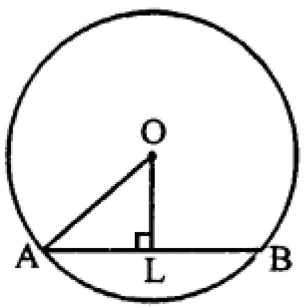


Let OA be the radius, then in right ΔOAL,
$OA^2 = OL^2 + AL^2$
16
$= (15)^2 + 162 = 15^2 + 8^2$
$= 225 + 64 = 289 = (17)^2$
∴ OA = 17 cm
Hence radius of the circle = 17 cm (c)
Question 2.
The radius of a circle Js 6 cm. The perpendicular distance from the centre of the circle to the chord which is 8 cm in length, is

(a) $\sqrt{5}$ cm (b) $2\sqrt{5}$ cm

(c) $2\sqrt{7}$ cm (d) $\sqrt{7}$ cm

Solution:
Radius of the cirlce (r) = 6 cm
Perpendicular distance from centre = ?
Length of chord = 8 cm



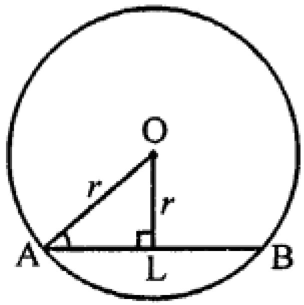
Let AB be chord, OL is the distance
In right $\triangle OAL$
$OA^2 = AL^2 + OL^2$

$$(6)^2 = \frac{8}{2}^{2} + OL^2 \qquad \left\{\because AL = \frac{1}{2}AB\right\}$$

$$36 = 16 + OL^2 \Rightarrow OL^2 = 36 - 16 = 20$$

$$\therefore OL = \sqrt{20} = \sqrt{4\times 5} = 2\sqrt{5} \text{ cm} \qquad \text{(b)}$$

Question 3.
If O is the centre of a circle of radius r and AB is a chord of the circle at a distance r2 from O, then ∠BAO =
(a) 60°
(b) 45°
(c) 30°
(d) 15°
Solution:
r is the radius of the circle with centre O
AB is the chord, at a distance of r2 from the centre



Let θ be the angle $\angle OAB$

$$\sin\theta = \frac{OL}{OA} = \frac{\frac{r}{2}}{r}$$

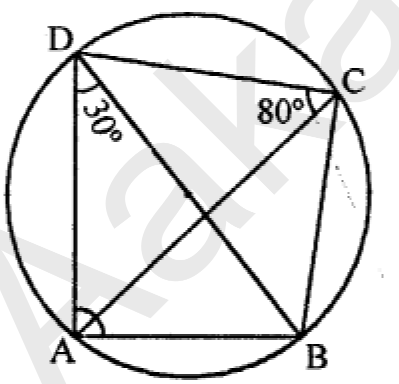
$$= \frac{r}{2\times r} = \frac{1}{2} = \tan 30^\circ$$

$\therefore\ \theta = 30^\circ$

$\therefore\ \angle OAB = 30^\circ$

or $\angle BAO = 30^\circ$ **(c)**

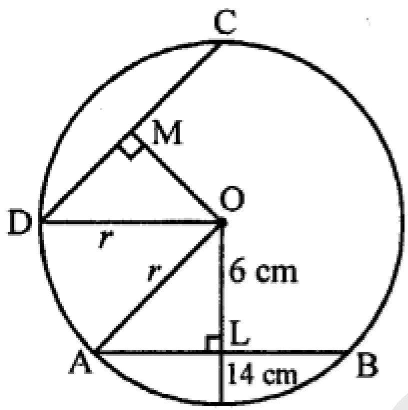
Question 4.
ABCD is a cyclic quadrilateral such that ∠ADB = 30° and ∠DCA = 80°, then ∠DAB=
(a) 70°
(b) 100°
(c) 125°
(d) 150°
Solution:
ABCD is a cyclic quadrilateral ∠DCA = 80° and ∠ADB = 30°



∵∠ADB = ∠ACB (Angles in the same segment)
∴ ∠ACB = 30°
∴ ∠BCD = 80° + 30° = 110°
∵ ABCD is a cyclic quadrilateral
∴∠BAD + ∠BCD = 180°

$\Rightarrow \angle BAD + 110° = 180°$
$\Rightarrow \angle BAD = 180° - 110° = 70°$
or $\angle DAB = 70°$ (a)

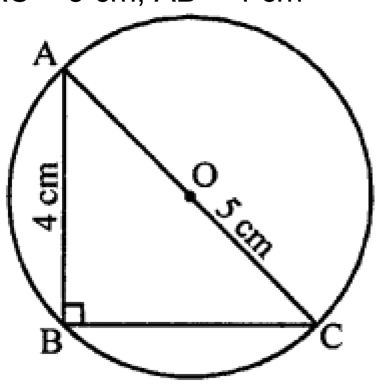
Question 5.
A chord of length 14 cm is at a distance of 6 cm from the centre of a circle. The length of another chord at a distance of 2 cm from the centre of the circle is
(a) 12 cm
(b) 14 cm
(c) 16 cm
(d) 18 cm
Solution:
In a circle AB chord = 14 cm
and distance from centre OL = 6 cm
Let r be the radius of the circle, then $OA^2 = AL^2 + OL^2$
$\Rightarrow r^2 = (7)^2 + (6)^2 = 49 + 36 = 85$



In the same circle length of another chord CD = ?
Distance from centre = 2 cm
$\therefore r^2 = OM^2 + MD^2$
$\Rightarrow 85 = (2)^2 + DM^2$
$\Rightarrow 85 = 4 + DM^2$
$\Rightarrow DM^2 = 85-4 = 81 = (9)^2$
$\therefore DM = 9$
$\therefore CD = 2 \times DM = 2 \times 9 = 18$ cm
$\therefore$Length of another chord = 18 cm (d)

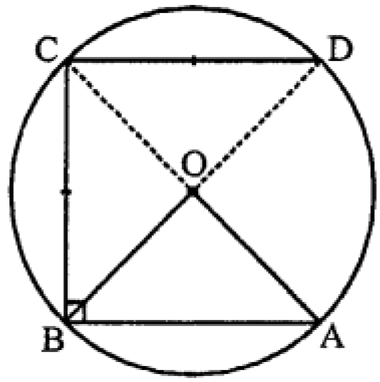
Question 6.
One chord of a circle is known to be 10 cm. The radius of this circle must be
(a) 5 cm
(b) greater than 5 cm
(c) greater than or equal to 5 cm
(d) less than 5 cm
Solution:
Length of chord of a circle = 10 cm

Length of radius of the circle greater than half of the chord
More than 102 = 5 cm (b)
Question 7.
ABC is a triangle with B as right angle, AC = 5 cm and AB = 4 cm. A circle is drawn with O as centre and OC as radius. The length of the chord of this circle passing through C and B is
(a) 3 cm
(b) 4 cm
(c) 5 cm
(d) 6 cm
Solution:
In right ΔABC, $\angle B = 90°$
AC = 5 cm, AB = 4 cm



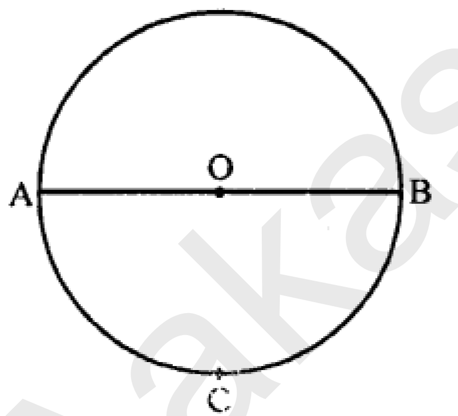
$\therefore BC^2 = AC^2 - AB^2$
$= 5^2 - 4^2 = 25 - 16$
$= 9 = (3)^2$
$\therefore$ BC = 3 cm
$\therefore$ Length of chord BC = 3 cm (a)

Question 8.
If AB, BC and CD are equal chords of a circle with O as centre, and AD diameter then $\angle AOB$ =
(a) 60°
(b) 90°
(c) 120°
(d) none of these
Solution:
In a circle chords AB = BC = CD



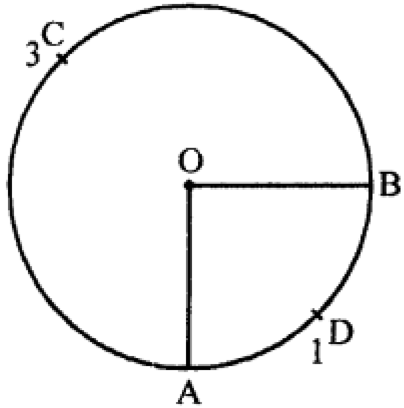
O is the centre of the circle
∴ ∠AOB = cannot be found (d)

Question 9.
Let C be the mid-point of an arc AB of a circle such that $m\overset{\frown}{AB} = 183^\circ$. If the region bounded by the arc ACB and line segment AB is denoted by S, then the centre O of the circle lies
(a) in the interior of S
(b) in the exterior of S
(c) on the segment AB
(d) on AB and bisects AB
Solution:
$\overset{\frown}{AB} = 183^\circ$
∴ AB is the diameter of the circle with centre O and C is the mid point of arc AB

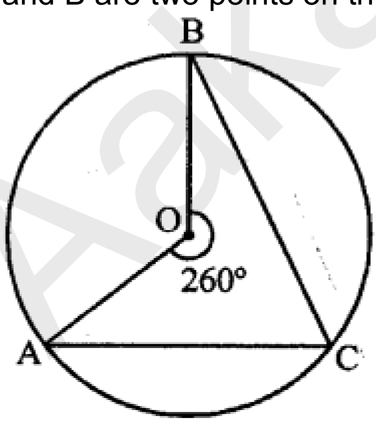


Line segment AB = S
∴ Centre will lie on AB (c)
Question 10.
In a circle, the major arc is 3 ti.nes the minor arc. The corresponding central angles and the degree measures of two arcs are
(a) 90° and 270°
(b) 90° and 90°
(c) 270° adn 90°
(d) 60° and 210°

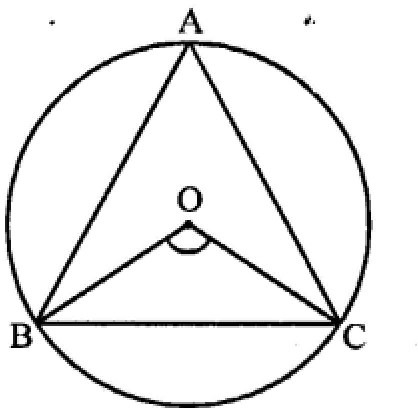
Solution:
In a circle, major arc is 3 times the minor arc i.e. arc ACB = 3 arc ADB
∴ Reflex ∠AOB = 3∠AOB
But angle at O = 360°



and let ∠AOB = x
Then reflex ∠ADB = x
x + 3x – 360°
⇒ 4x = 360°
⇒ x = 62∘2 = 90°
∴ 3x = 90° x 3 = 270°
Here angles are 270° and 90° (c)
Question 11.
If A and B are two points on a circle such that $m(\breve{AB})$ = 260°. A possible value for the angle subtended by arc BA at a point on the circle is
(a) 100°
(b) 75°
(c) 50°
(d) 25°
Solution:
A and B are two points on the circle such that reflex ∠AOB = 260°

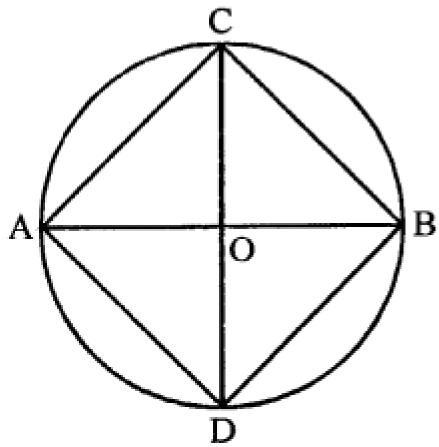


∴ ∠AOB = 360° – 260° = 100°
C is a point on the circle

∴ By joining AC and BC,
∠ACB = 12∠AOB = 12 x 100° = 50° (c)
Question 12.
An equilateral triangle ABC is inscribed in a circle with centre O. The measures of ∠BOC is
(a) 30°
(b) 60°
(c) 90°
(d) 120°
Solution:
∆ABC is an equilateral triangle inscribed in a circle with centre O


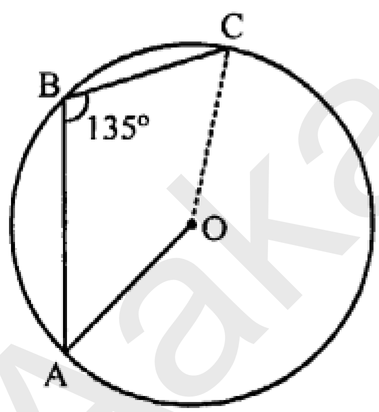
∴ Measure of ∠BOC = 2∠BAC
= 2 x 60° = 120° (d)

Question 13.
If two diameters of a circle intersect each other at right angles, then quadrilateral formed by joining their end points is a
(a) rhombus
(b) rectangle
(c) parallelogram
(d) square
Solution:
Two diameter of a circle AB and CD intersect each other at right angles



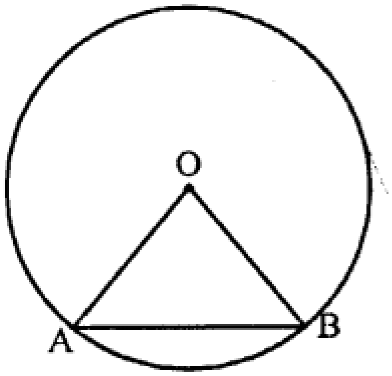
AD, DB, BC and CA are joined forming a quad. ABCD.
∵ The diagonals are equal and bisect each other at right angles
∴ ACBD is a square (d)

Question 14.
In ABC is an arc of a circle and ∠ABC = 135°, then the ratio of arc $\breve{AB}$ to the circumference is
(a) 1 : 4
(b) 3 : 4
(c) 3 : 8
(d) 1 : 2
Solution:
Arc ABC of a circle and ∠ABC = 135°
Join OA and OC



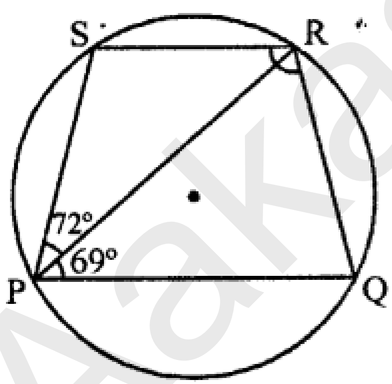
∴ Angle subtended by arc ABC at the centre = 2 x ∠ABC = 2 x 135° = 270°
Angle at the centre of the circle = 360°
∴ Ratio with circumference = 270° : 360° = 3:4 (b)
Question 15.
The chord of a circle is equal to its radius. The angle subtended by this chord at the minor arc of the circle is
(a) 60°
(b) 75°

(c) 120°
(d) 150°
Solution:
The chord of a circle = radius of the circle In the figure OA = OB = AB



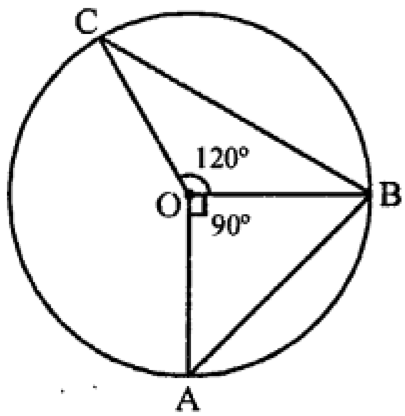
∴ ∠AOB = 60°
(Each angle of an equilateral = 60°) (a)

Question 16.
PQRS is a cyclic quadrilateral such that PR is a diameter of the circle. If ∠QPR = 67° and ∠SPR = 72°, then ∠QRS =
(a) 41°
(b) 23°
(c) 67°
(d) 18°
Solution:
PQRS is a cyclic quadrilateral with centre O and ∠QPR = 67°
∠SPR = 72°



∴ ∠QPS = 67° + 72° = 139°
∵ ∠QPS + ∠QRS = 180° (Sum of opposite angles of a cyclic quad.)
⇒ 139° + ∠QRS = 180°
⇒ ∠QRS = 180° – 139° = 41° (a)

Question 17.
If A, B, C are three points on a circle with centre O such that ∠AOB = 90° and ∠BOC =

120°, then ∠ABC =
(a) 60°
(b) 75°
(c) 90°
(d) 135°
Solution:
A, B and C are three points on a circle with centre O
∠AOB = 90° and ∠BOC = 120°


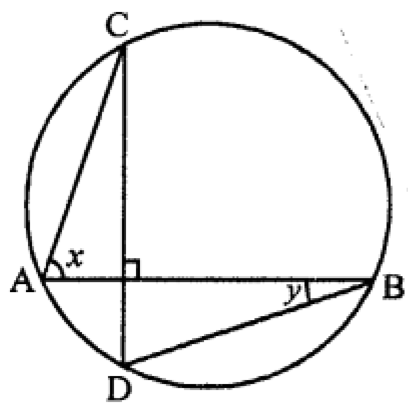
∴ ∠AOC = 360° – (120° + 90°)
= 360° -210°= 150°
But ∠AOC is at the centre made by arc AC and ∠ABC at the remaining part of the circle
∴ ∠ABC = 12 ∠AOC
= 12 x 150° = 75° (b)
Question 18.
The greatest chord of a circle is called its
(a) radius
(b) secant
(c) diameter
(d) none of these
Solution:
The greatest chord of a circle is called its diameter. (c)

Question 19.
Angle formed in minor segment of a circle is
(a) acute
(b) obtuse
(c) right angle
(d) none of these
Solution:
The angle formed in minor segment of a circle is obtuse angle. (b)

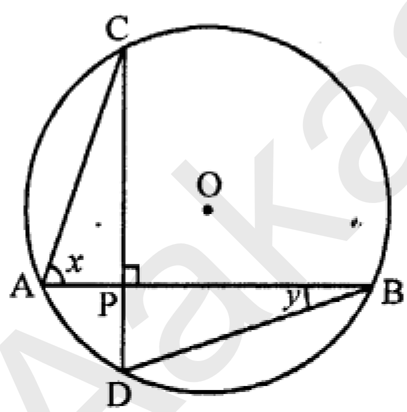
Question 20.
Number of circles that can be drawn through three non-collinear points is
(a) 1

(b) 0
(c) 2
(d) 3
Solution:
The number of circles that can pass through three non-collinear points is only one. (a)

Question 21.
In the figure, if chords AB and CD of the circle intersect each other at right angles, then x + y =
(a) 45°
(b) 60°
(c) 75°
(d) 90°

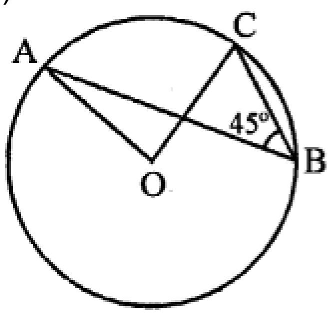


Solution:
In the circle, AB and CD are two chords which intersect each other at P at right angle
i.e. ∠CPB = 90°

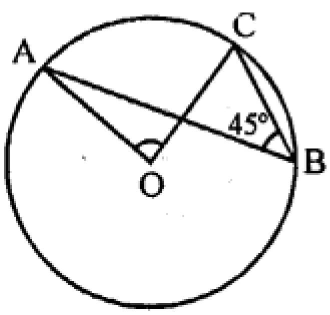


∠CAB and ∠CDB are in the same segment
∴ ∠CDB = ∠CAB = x
Now in ∆PDB,
Ext. ∠CPB = ∠D + ∠DBP
⇒ 90° = x + y (∵ CD ⊥ AB)
Hence x + y = 90° (d)

Question 22.
In the figure, if ∠ABC = 45°, then ∠AOC=
(a) 45°
(b) 60°
(c) 75°
(d) 90°

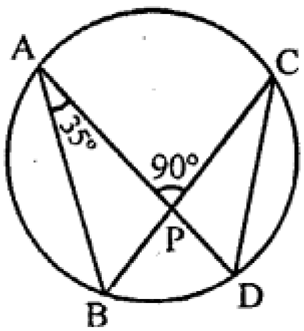


Solution:
∵ arc AC subtends
∠AOC at the centre of the circle and ∠ABC
at the remaining part of the circle

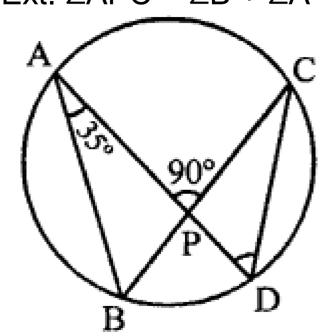


∴ ∠AOC = 2∠ABC
= 2 x 45° = 90°
Hence ∠AOC = 90° (d)

Question 23.
In the figure, chords AD and BC intersect each other at right angles at a point P. If ∠D AB = 35°, then ∠ADC =
(a) 35°
(b) 45°
(c) 55°
(d) 65°

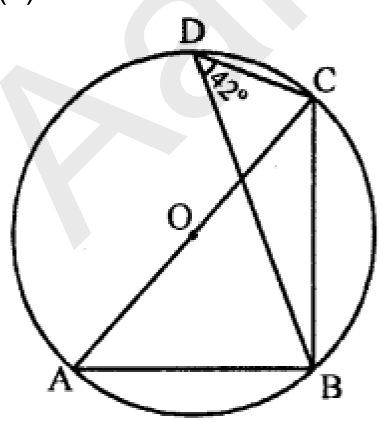


Solution:
Two chords AD and BC intersect each other at right angles at P, ∠DAB = 35°
AB and CD are joined
In ΔABP,
Ext. ∠APC = ∠B + ∠A

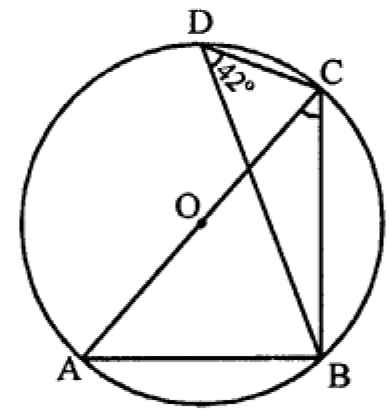


⇒ 90° = ∠B + 35°
∠B = 90° – 35° = 55°
∵ ∠ABC and ∠ADC are in the same segment
∴ ∠ADC = ∠ABC = 55° (c)

Question 24.
In the figure, O is the centre of the circle and ∠BDC = 42°. The measure of ∠ACB is
(a) 42°
(b) 48°
(c) 58°
(d) 52°



Solution:
In the figure, O is the centre of the circle
$\angle BDC = 42°$
$\angle ABC = 90°$ (Angle in a semicircle)
and $\angle BAC$ and $\angle BDC$ are in the same segment of the circle.
$\therefore \angle BAC = \angle BDC = 42°$
Now in ΔABC,
$\angle A + \angle ABC + \angle ACB = 180°$ (Sum of angles of a triangle)



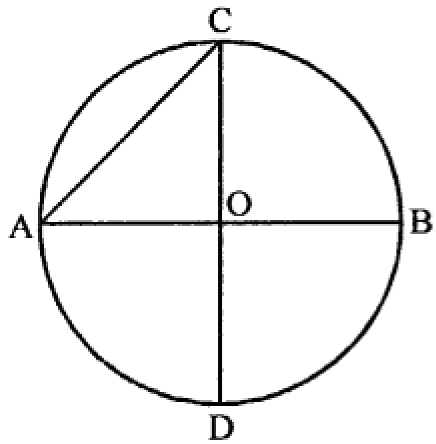
$\Rightarrow 42° + 90° + \angle ACB = 180°$
$\Rightarrow 132° + \angle ACB – 180°$
$\Rightarrow \angle ACB = 180° – 132° = 48°$ (b)

Question 25.
In a circle with centre O, AB and CD are two diameters perpendicular to each other. The length of chord AC is

(a) 2AB
(b) $\sqrt{2}$
(c) $\frac{1}{2}$AB
(d) $\frac{1}{\sqrt{2}}$AB

Solution:

AB and CD are two diameters of a circle with centre O



Length of chord AC = $\sqrt{OA^2 + OC^2}$

$= \sqrt{r^2 + r^2} = \sqrt{2r^2} = \sqrt{2}\,r$

$= \sqrt{2} \times \frac{1}{2} AB = \frac{1}{\sqrt{2}} AB$ (d)

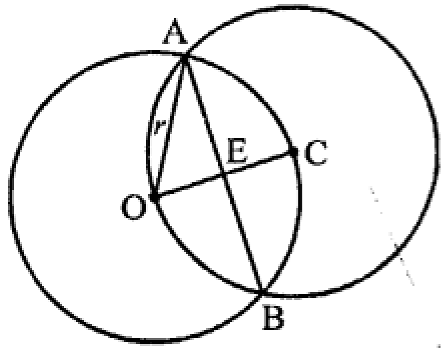
Question 26.
Two equal circles of radius r intersect such that each passes through the centre of the other. The length of the common chord of the circles is

(a) $\sqrt{r}$ (b) $\sqrt{2}\,r AB$

(c) $\sqrt{3}\,r$ (d) $\frac{\sqrt{3}}{2} r$

Solution:
Two equal circles pass through the centre of the other and intersect each other at A and B

Let r be the radius of each circle



$\therefore$ OA = r, OE = $\frac{1}{2}$OC = $\frac{1}{2}r$

$\therefore$ $EA^2 = OA^2 - OE^2$

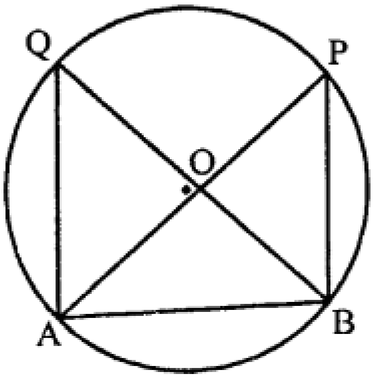
$= (r)^2 - \left(\frac{1}{2}r\right)^2 = r^2 - \frac{1}{4}r^2$

$= \frac{3}{4}r^2 = \frac{\sqrt{3}}{2}r$

$\therefore$ EA = $\frac{\sqrt{3}}{2}r$

and AB = 2 × EA = 2 × $\frac{\sqrt{3}}{2}r = \sqrt{3}\,r$ (c)

Question 27.
If AB is a chord of a circle, P and Q are the two points on the circle different from A and B,then
(a) ∠APB = ∠AQB
(b) ∠APB + ∠AQB = 180° or ∠APB = ∠AQB
(c) ∠APB + ∠AQB = 90°
(d) ∠APB + ∠AQB = 180°
Solution:
AB is chord of a circle,
P and Q are two points other than from points A and B

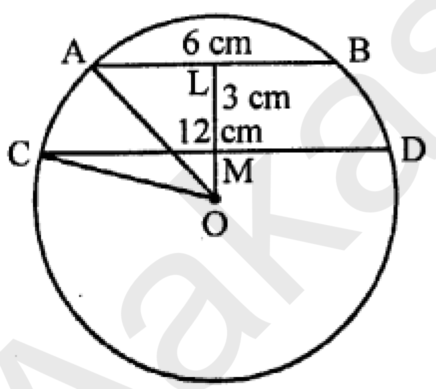


∵ ∠APB and ∠AQB are in the same segment of the circle
∴ ∠APB = ∠AQB (a)

Question 28.
AB and CD are two parallel chords of a circle with centre O such that AB = 6 cm and CD = 12 cm. The chords are on the same side of the centre and the distance between them is 3 cm. The radius of the circle is

(a) 6 cm (b) $5\sqrt{2}$ cm
(c) 7 cm (d) $3\sqrt{5}$ cm

Solution:
AB and CD are two parallel chords of a circle with centre O
Let r be the radius of the circle AB = 6 cm, CD = 12 cm
and distance between them = 3 cm
Join OC and OA, LM = 3 cm



Let OM = x, then OL = x + 3

Now in right ΔOCM,

$$OC^2 = OM^2 + CM^2 = (x)^2 + \left(\frac{CD}{2}\right)^2$$

$$\Rightarrow r^2 = x^2 + \left(\frac{12}{2}\right)^2 = x^2 + 36 \quad ...(i)$$

On in right $\angle OAL$,

$OA^2 = OL^2 + AL^2$

$$r^2 = (x+3)^2 + \left(\frac{6}{2}\right)^2 = (x+3)^2 + (3)^2$$

$$= (x+3)^2 + 9 \quad ...(ii)$$

From (*i*) and (*ii*),

$(x+3)^2 + 9 = x^2 + 36$

$\Rightarrow x^2 + 6x + 9 + 9 = x^2 + 36$

$$\Rightarrow 6x = 36 - 18 = 18 \Rightarrow x = \frac{18}{6} = 3$$

$\therefore$ (Radius)$^2 = r^2 = x^2 + 36$

$= (3)^2 + 36 = 9 + 36 = 45$

$= 9 \times 5$

$\therefore r = \sqrt{9 \times 5} = 3\sqrt{5}$ cm (d)

Question 29.

In a circle of radius 17 cm, two parallel chords are drawn on opposite side of a diameter. This distance between the chords is 23 cm. If the length of one chord is 16 cm then the length of the other is

(a) 34 cm

(b) 15 cm

(c) 23 cm

(d) 30 cm

Solution:

Radius of a circle = 17 cm

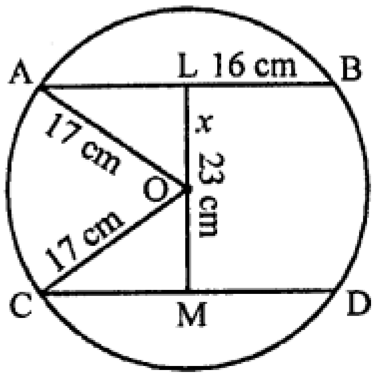
The distance between two parallel chords = 23 cm

AB || CD and LM = 23 cm

Join OA and OC,

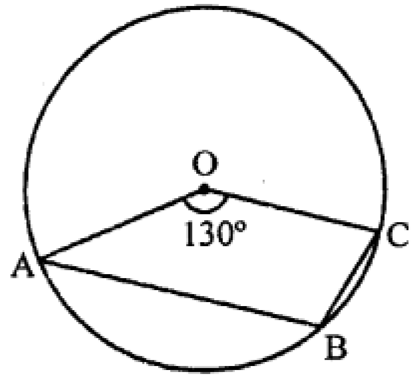
∴ OA = OC = 17 cm

Let OL = x, then OM = (23 – x) cm
AB = 16 cm
Now in right ∆OAL,
$OA^2 = OL2^2 + AL^2$
$\Rightarrow (17)^2 = x^2 + AL^2$
$\Rightarrow 289 = x^2 + AL^2$

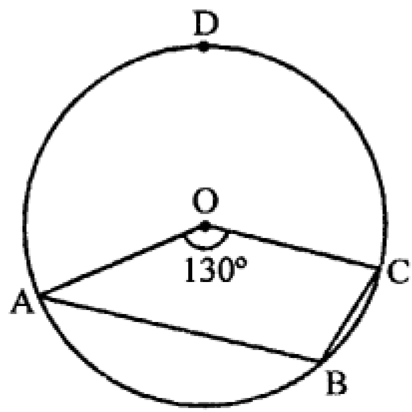


$$\Rightarrow x^2 = 289 - AL^2 = 289 - \left(\frac{16}{2}\right)^2$$
$$= 289 - 64 = 225 = (15)^2$$
$\therefore$ $x = 15$ cm
$\because$ OM $= 23 - x = 23 - 15 = 8$ cm
Now in right ∆OCM
$OC^2 = OM^2 + CM^2$
$(17)^2 = (8)^2 + CM^2 \Rightarrow 289 = 64 + CM^2$
$\Rightarrow CM^2 = 289 - 64 = 225 = (15)^2$
$\therefore$ CM = 15 cm
CD = 2 × CM = 2 × 15 = 30 cm (d)

Question 30.
In the figure, O is the centre of the circle such that ∠AOC = 130°, then ∠ABC =
(a) 130°
(b) 115°
(c) 65°
(d) 165°



Solution:
O is the centre of the circle and ∠AOC = 130°
Reflex ∠AOC = 360° – 130° = 230°



Now arc ADB subtends ∠AOC at the centre and ∠ABC at the remaining part of the circle
∴ ∠ABC = $\frac{1}{2}$reflex ∠AOC
= $\frac{1}{2}$ x 230°= 115° (b)