HC VERMA Solutions for Class 11 Physics Chapter 15 Waves Motion and Waves on a String

Question 1
A wave pulse passing on a string with a speed of 40 cm/s in the negative x-direction has its maximum at x=0 at t=0. Where will this maximum be located at t= 5 s?

Solution 1

Speed = $\frac{distance}{time}$
X= $\frac{40}{100} \times 5$
X= 2m (in negative direction)

Question 2
The equation of a wave travelling on a string stretched along the X-axis is given by
Y= A.$e^{-(\frac{x}{a}+\frac{t}{T})^2}$

a) Write the dimensions of A, a and T.
b) Find the wave speed
c) In which direction is the wave travelling
d) Where is the maximum of the pulse located at t=T? At t = 2T?

Solution 2
a)
[y]=[A]=[$M^{\circ}L^{1}T^{\circ}$] ;
and
$\frac{[x]}{[a]} = \frac{[t]}{T}$ = [$M^{\circ}L^{\circ}T^{\circ}$]
[a]= [$M^{\circ}L^{1}T^{\circ}$]
[T]= [$M^{\circ}L^{\circ}T^{1}$]

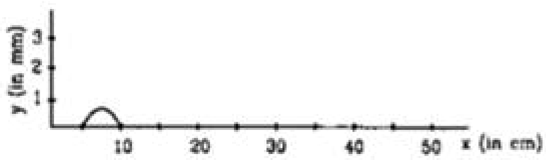
b)
$\text{wave speed} = \frac{|coefficient\, of\, t|}{|coefficient\, of\, x|} = \left(\frac{a}{T}\right)$

c)
$\frac{x}{a}+\frac{t}{T}$ = constant
As time increases, x must decrease
So, wave travels in negative direction

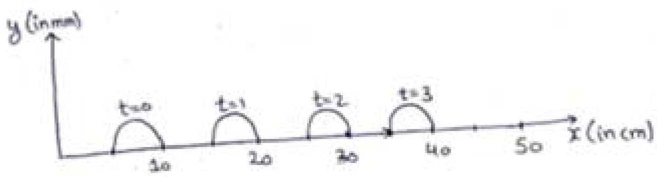
d) For maximum pulse location
y= A, i.e. when $\frac{x}{a}+\frac{t}{T}=0$
$x = -\left(\frac{t}{T}\right)$

at
$t = T;\ x = -a$
$t = 2T;\ x = -2a$

Question 3
The Figure shows a wave pulse at t=0. The pulse moves to the right with a speed at 10 cm/s. Sketch the shape of the string at t=1s, 2s, 3s.



Solution 3



At,
$t = 1\text{sec}\ \ d_1 = \text{vxt} = 10\text{x}1 = 10\text{m}$
$t = 2\text{ sec}\ \ d_2 = \text{vxt} = 10\text{x}2 = 20\text{m}$
$t = 3\text{ sec}\ \ d_3 = \text{vxt} = 10\text{x}3 = 30\text{m}$

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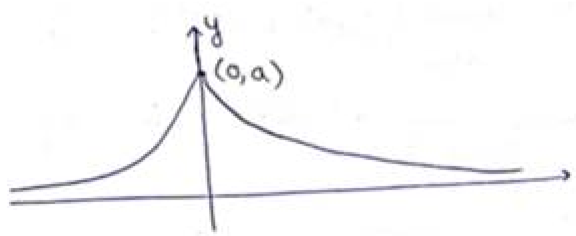
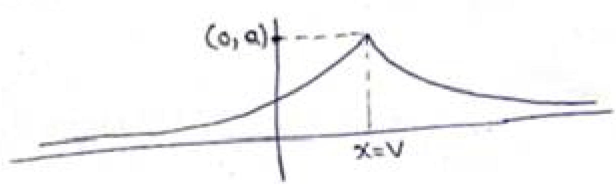
Question 4
A pulse travelling on a string is represented by the function
$$y = \frac{a^3}{(x - vt)^2 + a^2}$$
Where a= 5mm and v=20 cm/s. Sketch the shape of the string at t=0, 1s, 2s. Take x=0 in the middle of the string.

Solution 4
At t=0
y= $\frac{a^3}{x^2+a^2}$
y(x=0)=a
y(x=$\pm\infty$)=0

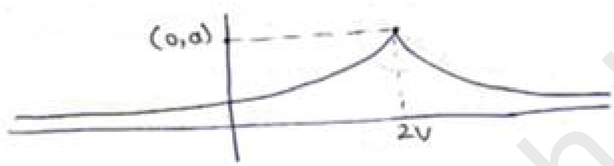


At t=1 sec

y= $\frac{a^3}{(x-v)^2+a^2}$ The wave is moving in positive direction with wave speed of v so in 1 sec it moves by distance v in positive direction.



At t=2 sec wave travels distance of 2v



Question 5

The displacement of the particle at x=0 of a stretched string carrying a wave in the positive x-direction is given by

$f(t) = A\sin(t/T)$

The wave speed is *v. Write the wave equation.*

Solution 5

Since wave is travelling in positive direction

$\text{Replace } t \rightarrow t - \frac{x}{v}$

$\text{Equation of wave} = A\sin\left(\frac{1}{T}\left(t - \frac{x}{vT}\right)\right)$

$y = A\sin\left(\frac{t}{T} - \frac{x}{vT}\right)$

Question 6

A wave pulse is travelling on a string with a speed *v* towards the positive x-axis. The shape of the string at t=0 is given by

$g(x) = A\sin(x/a)$, where A and a are constants.

a) What are the dimensions of A and a?

b) Write the equation of the wave for a general time t, if the wave of the speed is t.

Solution 6

a) $[A] = [M^{\circ}L^{1}T^{\circ}]$

and

$\frac{[x]}{[a]} = [M^{\circ}L^{\circ}T^{\circ}]$ [Since Trignometric ratios are dimensionless]

$[a] = [M^{\circ}L^{1}T^{\circ}]$

b)wave travels in positive direction

replace x by $x - vt$

equation of wave $= A \sin \frac{(x - vt)}{a}$

Question 7

A wave propagates on a string with a speed *v.* The shape of the string at $t = t_0$ is given by $g(x, t_0) = A \sin (x/a)$. Write the wave equation for a general time t.

Solution 7

In time $(t - t_0)$ wave travels distance of $v(t - t_0)$

Since wave is travelling in positive x direction.

Replace x by $x - v(t - t_0)$

Equation of a wave $A \sin \frac{(x - v(t - t_0))}{a}$

$y = A \sin \frac{(x + vt_o - vt)}{a}$

Question 8

The equation of a wave travelling on a string is

$y = (0.10 \text{ mm}) \sin[(31.4 \text{ m}^{-1})x + (314 s^{-1})t]$

a) In which direction does the wave travel

b) Find the wave speed the wavelength and the frequency of the wave.

c) What is the maximum displacement and the maximum speed of a portion of the string?

Solution 8

$31.4x + 314t = \text{constant}$

As time increases, x decreases

So, wave travels in negative direction.

b) $\text{wave speed} = \frac{|\text{coefficient of t}|}{|\text{coefficient of y}|}$

$= \frac{314}{31.4} = 10$ m/s

Comparing with $y = A \sin (\omega t + Kx)$

$K = \frac{2\pi}{\lambda} = 3.14$

$\lambda = 0.2m = 20cm$

ω= 2⊓f= 314
f=50 Hz.

c) maximum displacement = A= 0.1mm

Maximum speed = A$^{\omega}$
=(0.1 $\times 10^{-3}$) (314)
=3.14 × 10^{-2} m/s
=3.14 cm/sec

Question 9

A wave travels along the positive x-direction with a speed of 20 m/s. The amplitude of the wave is 0.20cm and the wavelength is 2.0cm
a) Write suitable wave equation which describes this wave
b) What is the displacement and velocity of the particle at x=2.0cm at time t=0 according to the wave equation written?

Can you get different values of this quantity if the wave equation is written in a different fashion?

Solution 9

$K = \frac{2\pi}{\lambda} = \frac{2\pi}{2} = \pi\,cm^{-1}$

$\upsilon = f \times \lambda$

20 = f $\times 2 \times 10^{-2}$ sec^{-1}
f = 1000 Hz
ω = 2π×$10^3 sec^{-1}$
Equation of wave travelling in positive direction
$y = A \sin(Kx - \omega t)$
$= (0.2cm)\sin((\pi cm^{-1})x - (2\pi \times 10^3)t$

b)
At $x = 2$ cm and $t = 0$
$y = 0.2 \sin(2\pi - 0)$
y=0
Particle velocity
$v_p = \omega\sqrt{(A^2 - y^2)}$
= (2 π x 10^3) $\sqrt{((0.2 \times 10)^2 - 0^2)}$
$v_p = 4\pi$ m/s

It depends upon displacement not on fashion of wave equation

Question 10

A wave is described by the equation
$y = (1.0mm)\sin\pi\left[\frac{x}{2.0cm} - \frac{t}{0.01s}\right]$
a. Fin the time period and the wave length.
b. Write the equation for the velocity of the particles. Find the speed of the particle at x= 1.p cm at the time t=0.01 s
c. What are the speeds of the particles at x=3.o cm, 5.0 cm and 7.0 cm at t=0.01s?
d. What are the speeds of the particle at x=0.011, 0.012 and 0.013 s?

Solution 10

Comparing equation with $y = A\sin(Kx - \omega t)$

a)

$K = \frac{2\pi}{\lambda} = \frac{\pi}{2}$

$\lambda = 4cm$

and $\omega = \frac{2\pi}{T} = \frac{\pi}{0.01}$

T=0.02= 20ms

b)

Differentiate the displacement equation

$v_p = \frac{dy}{dt}$

$v_p = (1mm)\left[\cos\left(\frac{\pi x}{2} - \frac{\pi t}{0.01}\right)\right]\left(\frac{-\pi}{0.01}\right)$

At $x = 1$ cm & $t = 0.01$sec

$v_p = \left(\frac{-\pi}{0.01}\right)\left[\cos\left(\frac{\pi x}{2} - \frac{\pi t}{0.01}\right)\right]$

$v_p = 0$

C) At $t = 0.01$ sec

$v_p = \left(\frac{-\pi}{0.01}\right)\cos\left(\frac{\pi x}{2} - \pi\right)$

At $x = 3cm;\ v_p = 0$

$X = 5cm;\ v_p = 0$

$X = 7cm;\ v_p = 0$

d)at $x = 1cm$

$v_p = \left(\frac{-\pi}{0.01}\right)\left[\cos\left(\frac{\pi}{2} - \frac{\pi t}{0.01}\right)\right]$

At $t = 0.011$

$v_p = \left(\frac{-\pi}{0.01}\right)\cos\left(\frac{\pi}{2} - \frac{\pi * 0.011}{0.01}\right)$

v_p=9.7 cm/s

At t=0.012

$v_p = \left(\frac{-\pi}{0.01}\right)\cos\left(\frac{\pi}{2} - \frac{\pi * 0.012}{0.01}\right)$

v_p=18 cm/s

At t=0.013

$v_p = \left(\frac{-\pi}{0.01}\right)\cos\left(\frac{\pi}{2} - \frac{\pi * 0.013}{0.01}\right)$

v_p=25 cm/s

Question 11

A particle on a stretched string supporting a travelling wave takes 5.0 ms to move its mean position to the extreme position. The distance between two consecutive particles which are at their mean position is 2.0 cm. Find the frequency, the wavelength and the wave speed.

Solution 11

$\frac{T}{4}$ = 5ms

T = 20ms = $\frac{1}{f}$

f= 50 Hz

Wavelength:

$\frac{\lambda}{2}$ = 2cm

λ = 4cm

Wave speed:

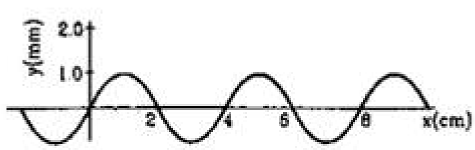
υ= f × λ

= 50 × 4 × 10^{-2}

υ= 2 m/s

Question 12

Figure shows a plot of the transverse displacements of the particles of a string at t=0 through which a travelling wave is passing in the positive x-direction. The wave speed is 20 cm/s. Find a) the amplitude b) The wavelength v) The wave number and d) The frequency of the wave.



Solution 12

a) Amplitude = y_{max}= 1mm

b) wavelength = 4 cm

c) wave matter K = $\frac{2\pi}{\lambda}$ = $\frac{2\pi}{4}$

k= 1.57 ≅1.6 cm^{-1}

d) frequency

υ = f × λ

20 = f × 4

f = 5 Hz

Question 13

A wave travelling on a string at a speed of 10 m/s causes each particle of the string to oscillate with a time period of 20 ms.

a. What is the wavelength of the wave

b. If the displacement of a particle is 1.5mm at a certain instant, what will be the displacement of a particle 10 cm away from it at the same instant?

Solution 13

υ= f × λ

$\upsilon = \frac{1}{t} \times \lambda$

$\lambda = 10 \times 20 \times 10^{-3}$

$\lambda = 0.2m = 20$ cm

b) Displacement equation of particle be

$y = A\sin(\omega t - Kx)$

At any instant 1.5 = A sin (ωt- Kx)

Phase difference for particle at a distance x= 10 cm

$\emptyset = \frac{2\pi}{20} \times 10$

$= \pi$

The displacement is given by

y'= A sin ($\omega t - Kx + \pi$)

$= -A\sin((\omega t - Kx)$

$y' = -1.5mm$

Question 14

A steel wire of length 64 cm weighs 5 g. If it is stretched by a force of 8N, what would be the speed of transverse wave passing on it?

Solution 14

$\upsilon = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Tl}{m}}$

$= \sqrt{\frac{8 \times 64 X 10^{-2}}{5 \times 10^{-3}}}$

$\upsilon = 32$ m/s

Question 15

A string of length 20 cm and linear mass density 0.40 g/cm is fixed at both ends and is kept under a tension of 16N. A wave pulse is produced at t=0 near an end as shown in figure. Which travels towards the other end.

a) When will the string have the shape shown in the figure again?

b) Sketch the shape of the string at a time half that found in part (a)

Solution 15

a) Velocity of the wave = $\sqrt{\frac{T}{\mu}}$

$= \sqrt{16 \ X 10^{-2}} / \sqrt{(0.4 X 10^{-3})}$

= 20 m/s

Distance travelled by wave = 20cm + 20 cm= 40 cm

T=0.02 sec

In transverse wave, Phase difference of π occurs after reflection from denser medium.

Question 16

A string of linear mass density 0.5 g/cm and a total length 30 cm is tied to a fixed wall at one end and to a frictionless ring at the other end. The ring can move on a vertical rod. A wave pulse is produced on the string which moves towards the ring at a speed of 20 cm/s. The pulse is symmetric about its maximum which is located at a distance of 20 cm from the end joined to the ring.

a. Assuming that the wave is reflected from the ends without loss of energy, find the time taken by the string to regain its shape.
b. The shape of the string changes periodically with time. Find this time period.
c. What is the tension in the string?

Solution 16

When transverse wave travels from dense medium to a rarer medium no phase change occurs.

a) Distance to travel by wave = 20 + 20
= 40 cm

$$Time = \frac{distance}{speed} = \frac{40}{20} = 2\ sec$$

b) Phase change occurs at fixed end and not at free end.
Distance travelled to regain shape=
30 + 30 = 60 cm

$$Time = \frac{d}{v} = \frac{60}{20} = 3\ sec$$

c) v = $\sqrt{\frac{T}{\mu}}$

$2 \times 10^{-2} = \sqrt{\frac{T}{0.5 \times 10^{-1}}}$ = T = 2×10^{-3} N

Question 17

The wires of different densities but same area of cross section are soldered together at one end and are stretched to a tension T. The velocity of a transverse wave in the first wire is double of that in the second wire. Find the ratio of the density of the first wire to that of the second wire.

Solution 17

$$\upsilon = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T \times L}{m}} = \sqrt{\frac{TL}{\rho AL}} = \sqrt{\frac{T}{\rho A}}$$

$\rho \propto \frac{1}{\upsilon^2}$

$\frac{\rho_1}{\rho_2} = \frac{\upsilon_2^2}{\upsilon_1^2} = \frac{\upsilon_2^2}{(2\upsilon_2^2)}$

$\rho_1 : \rho_2$ = *1 : 4*

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Question 18

A transverse wave described by

y = (0.02m) sin [(1.0 m^{-1})x + (30 s^{-1})t]

Propagates on a stretched string having a linear mass density of 1.2 $\times 10m^{-4}$kg/m. Find the tension in the string.

Solution 18

$\text{Velocity of wave} = \frac{|\text{coefficient of t}|}{|\text{coefficient of x}|}$

υ= 30 m/s

$\upsilon = \sqrt{\left(\frac{T}{\mu}\right)}$

$30 = \sqrt{\left(\frac{T}{1.2\times10^{-4}}\right)}$

T = 0.108 N

Question 19

A travelling wave is produced on a long horizontal string by vibrating an end up and down sinusoidally. The amplitude of vibration is 1.0cm and the displacement becomes zero 200 times per second. The linear mass density of the string is o.10 kg/m and it is kept under a tension of 90 N.

a. Find the speed and the wavelength of the wave

b. Assume that the wave moves in the positive x direction and at t=0 the end x=0 is at its positive extreme position. Write the wave equation.

c. Find the velocity and acceleration of the particle at x = 50 cm at time t = 10 ms.

Solution 19

A= 1 cm;

μ = 0.1 kg/m;

T= 90 N

In one direction, displacement becomes

$f = \frac{200}{2}$

= 100 Hz

a) Speed of wave

$\upsilon = \sqrt{\frac{T}{\mu}}$

$\upsilon = \sqrt{\frac{90}{0.01}}$ = 30 m/s

wavelength:

$\upsilon = f \times \lambda$

$30 = 100 \times \lambda$

λ = 0.3m = 30cm

b)

At $x = 0$, displacement is maximum wave equation.

$y = (1cm)\cos 2\pi\left(\frac{t}{0.01} - \frac{x}{30}\right)$

Where x is in cm and t is in sec

c) $\upsilon_p = \frac{dy}{dt} = -\frac{2\pi}{0.01}\sin 2\pi\left(\frac{t}{0.01} - \frac{x}{30}\right)$

$\text{when } x = 50cm \ \& \ \text{time} = 10ms = 0.01sec$

υ_p = -5.4 m/s

$\text{Acceleration of particle} = \frac{d}{dt}\upsilon_p$

$A = -\left(\frac{2\pi}{0.01}\right)^2 \cos 2\pi\left(\frac{t}{0.01} - \frac{x}{30}\right)$

$\text{When } x = 50 \text{ cm and } t = 0.01sec$

a= 2km/sec^2

Question 20

A string of length 40 cm and weighing 10 g is attached to a spring at one end and to a fixed wall at the other end. The spring has a spring constant of 160 N/m and is stretched by 1.0cm. If a wave pulse is produced on the string near the wall, how much time will it take to reach the spring?

Solution 20

$\upsilon = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{(Kx)}{mass} \times length}$

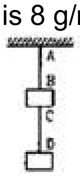
$v^2 = \frac{160 \times 0.01 \times 0.4}{10 \times 10^{-3}}$

$\upsilon = 8m/s$

$time = \frac{distance}{speed} = \frac{0.4}{8}$ = 0.05 sec.

Question 21

Two blocks each having a mass of 3.2kg are connected by a wire CD and the system is suspended from the ceiling by another wire AB. The linear mass density of the wire AB is 10 g/m and that of CD is 8 g/m. Find the speed of transverse wave pulse produced in AB and in CD



Solution 21

Tension in string AB $= (3.2 + 3.2)g = 64N$

Tension in string CD $= 3.2g = 32\,N$

$\upsilon = \sqrt{\left(\frac{T}{\mu}\right)}$

For wire AB

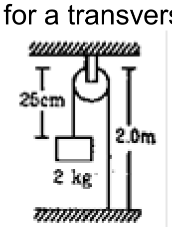
$$\upsilon = \sqrt{\frac{64}{10 \times 10^{-3}}} \simeq 80\frac{m}{s}$$

For wire CD

$\upsilon = \sqrt{\frac{32}{8 \times 10^{-3}}} \simeq 63\,\frac{m}{s}$

Question 22

In the arrangement shown in the given figure, the string has a mass of 4.5g. How much time will it take for a transverse disturbance produced at the floor to reach the pulley? take g= 10 m/s^2



Solution 22

$\upsilon = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{2g \times 2.25}{4.5 \times 10^{-3}}}$

υ= 100m/s

$$\text{Time to reach pulley} = \frac{\text{distance}}{\text{speed}}$$

$t = \frac{2}{100}$

= 0.02sec

Question 23

A 4.0 kg block is suspended from the ceiling of an elevator through a string having a linear mass density of 19.2 × 10^{-3} kg/m. Find the speed (with respect to the string) with which a wave pulse can proceed on the string if the elevator accelerates up at the rate of 2.0 m/s^2.

Solution 23

Tension in string

$T = m(g + a)$

$= 4(10 + 2)$

T= 48N

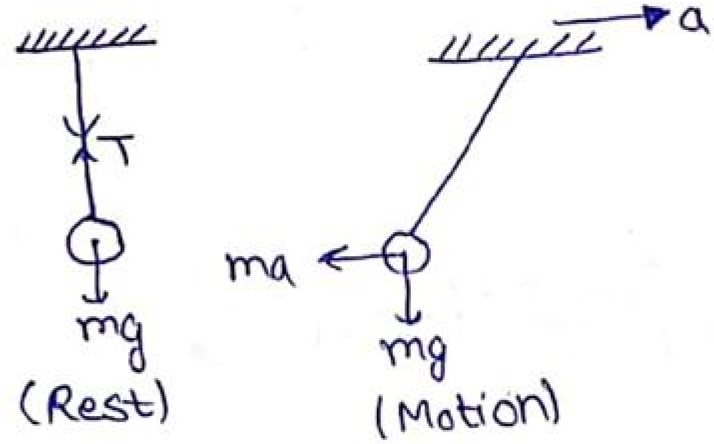
$\upsilon = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{48}{19.2 \times 10^{-3}}}$

υ= 50 m/s

Question 24

A heavy ball is suspended from the ceiling of a motor car through a light string. A transverse pulse travels at a speed of 60 cm/s on the string when the car is at rest and 62 cm/s when the car accelerates on a horizontal road. Find the acceleration of the car. Tale g = 10 m/s^2.

Solution 24



$$T = mg$$

$$\upsilon = \sqrt{\frac{T}{\mu}}$$

$$0.6 = \sqrt{\frac{mg}{\mu}} \;\ldots\ldots(1)$$

$$T' = \sqrt{((mg)^2 + (ma^2))}$$

$$\upsilon = \sqrt{\frac{T}{\mu}}$$

$$0.62 = \sqrt{\frac{m(g^2 + a^2)}{\mu}} \;\ldots..(2)$$

Diving (2) by (1)

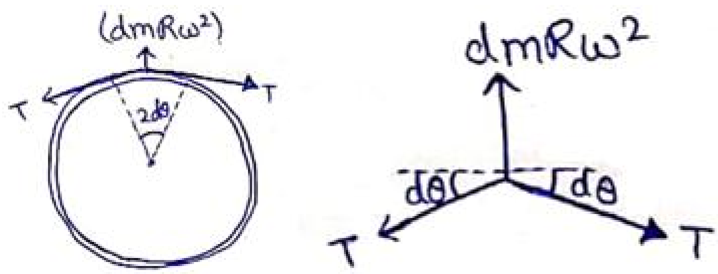
$$\frac{0.62}{0.6} = \frac{\sqrt{g^2+a^2}}{\sqrt{g}}$$

a = 3.7 m/s^2

Question 25

A circular loop of string rotates about its axis on a frictionless horizontal plane at a uniform rate so that the tangential speed of any particle of the string is υ. If a small transverse disturbance is produced at a point of the loop, with what speed (relative to the string) will this disturbance travel on the string?

Solution 25



Let mass per unit length be μ

Mass of small section on 2dΘ; $dm = \mu(2Rd\theta)$

Now,

$2T\sin d\theta = dmR\omega^2$

$2Td\theta = \mu(2R)d\theta\, R\omega^2$

$T = \mu R^2\omega^2$

$\text{Velocity wave} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\mu R^2\omega^2}{\mu}}$

$= R\omega = \upsilon$

Question 28

A transverse wave of 0.50mm and frequency of 100 Hz is produced on a wire stretched to a tension of 100 N. If the wave speed is 100 m/s, what average power is the source transmitting to the wire?

Solution 28

$P = 2\pi^2 f^2 A^2 \tau\rho s$

Since $\upsilon = \sqrt{\frac{T}{\rho s}}$

$\rho s = \frac{1}{100}$

Now,

P=2$\pi^2(100)^2(0.5\times10^{-3})\frac{1}{100}$(100)

= 49mW

Question 30

A tuning fork of frequency 400 Hz is attached to a long string of linear mass density 0.01 kg/m kept under a tension 49N. The fork produces transverse waves of amplitude 0-50 mm on the string.

a) Find the wave speed and wavelength of the waves.

b) Find the maximum speed and acceleration of a particle of the string.

c) At what average rate is the tuning fork transmitting energy to the string?

Solution 30

a) $\upsilon_A = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{49}{0.01}}$

υ=70 m/s

wavelength

υ = f × λ

70 = 440 × λ

λ =16cm

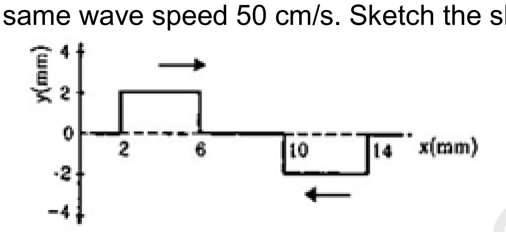
b) Maximum speed of the particle = A$^{\omega}$
$= (0.5 \times 10^{-3})(2 \times \pi \times 440) = 1.4 m/s$

Maximum acceleration of particle =A$^{\omega^2}$
$= (0.5 \times 10^{-3})(2 \times \pi \times 400)^2$
= 3.8 Km/s^2
c) $P = 2\pi^2 f^2 A^2 \upsilon \mu$
$= 2\pi^2(400).(440).(0.54 \times 10^{-3})).(0.54 \times 10^{-3)})(70)(0.01)$
p =0.67W

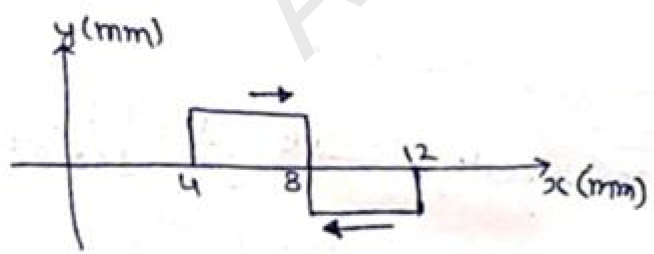
Question 31
Two waves, travelling in the same direction through the same region, have equal frequencies, wavelengths and amplitudes. If the amplitude pf each wave is 4mm and the phase difference between the waves is 90°, what is the resultant amplitude?

Solution 31
$A_{R=}\sqrt{(A_1^2 + A_2^2 + 2A_1A_2\cos\emptyset)}$
$=\sqrt{(4)^2 + (4)^2 + 2(4)(4)\cos 0} = 4\sqrt{2}$ mm

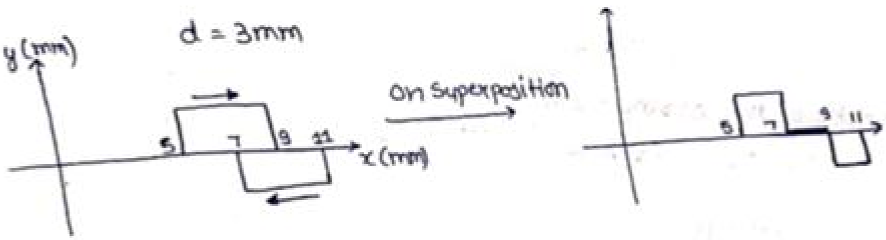
Question 32
The given figure shows two wave pulses at t=0 travelling on a string in opposite direction with the same wave speed 50 cm/s. Sketch the shape of the string at t-4 ms, 6 ms, 8 ms and 12 ms.



Solution 32
Distance travelled by pulse in 4ms
$d = \upsilon \times t$
= (50 $\times 10$)(4$\times 10^{-3}$)
d = 2mm



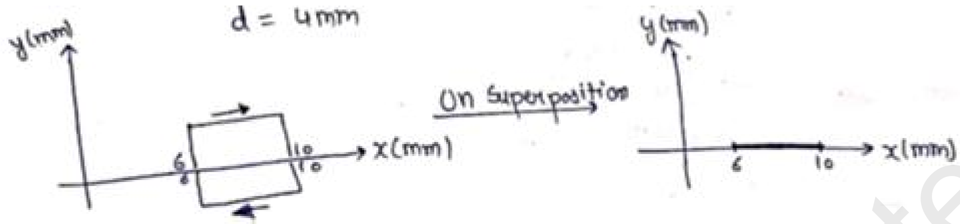
* Distance travelled by pulse in 6ms
d = (50 $\times 10$)(6$\times 10^{-3}$)
d = 3mm



* Distance travelled by pulse in 8ms

d = (50 $\times 10$)(8$\times 10^{-3}$)

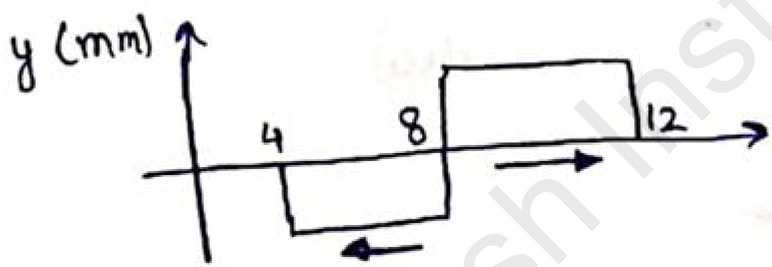
d= 4mm



* Distance travelled by a pulse in 12ms

d = (50 $\times 10$)(12$\times 10^{-3}$)

d=6mm



Chapter 15 - Waves Motion and Waves on a String
Exercise 326

Question 33

Two waves each having a frequency of 100 Hz and a wavelength of 2.0 cm, are travelling in the same direction on a string. What is the phase difference between the waves

a) If the second wave was produced 0.015s later than the first one at the same place

b) If the two waves were produced at the same instant but the first one was produced at the same instant but the first one was produced at a distance 4.0 cm behind the second one?

c) If each of the waves has an amplitude of 2.0mm, what would be the amplitude of the resultant waves in part (a) and (b)?

Solution 33

a) $\emptyset = \frac{2\pi}{T} \times t = 2\pi f \times t$

=2$\pi(100)(0.05)$

$\emptyset = 3\pi$

b) $\emptyset = \frac{2\pi}{\lambda} .(x)$

$= \frac{2\pi}{4}.(4)$

$\emptyset = 4\pi$

c) $A_R = 2A\cos(^{\emptyset}/_2)$

when $\emptyset = 3\pi$ $A_R = 2A\cos(^{3\pi}/_2)$

$A_R = 0$

when $\emptyset = 4\pi$ $A_R = 2A\cos(^{4\pi}/_2)$

$= 2\,(2mm)\cos 2\pi$

$A_R = 4mm$

Question 34

If the speed of a transverse wave on a stretched string of length 1 m is 60 m/s, what is the fundamental frequency of vibration?

Solution 34

Fundamental frequency

f= $\frac{\upsilon}{2l}$

f= $\frac{60}{2(1)}$

=30 Hz

Question 35

A wire of 2oom is stretched in a tension of 160 N. If the fundamental frequency of vibration is 100 Hz, find its linear mass density.

Solution 35

f= $\frac{\upsilon}{2l} = \sqrt{\frac{T}{\mu}} \times \frac{1}{2l}$

100= $\sqrt{\frac{160}{\mu}} \times \frac{1}{2X2}$

$\mu = 1 \times 10^{-3} kg/m$

$\mu = 1gm/m$

Question 36

A steel wire of mass 40g and length 80 cm is fixed at the two ends. The tension in the wire is 50 N. Find the frequency and wavelength of the fourth harmonic of the fundamental.

Solution 36

Wave speed

$\upsilon = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{50 \times 0.8}{4 \times 10^{-3}}}$

$\upsilon = 100 \frac{m}{s}$

Fundamental frequency $f_0 = \frac{\upsilon}{2l} = \frac{100}{2(0.8)}$

= 62.5 Hz

Fourth harmonic frequency = 4 f_0

=4 (62.5)
= 250 Hz
Wavelength of the fourth harmonic
$\upsilon = f \times \lambda$
100 = 250 × λ
λ = 0.4m = 40cm

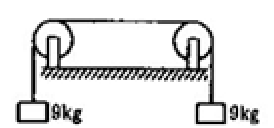
Question 37
A piano wire weighing 6.00g and having a length of 90.0cm emits a fundamental frequency of vibration is 100 Hz, find its linear mass density.

Solution 37
Fundamental frequency
$$f_0 = \frac{\upsilon}{2l}\sqrt{\frac{T}{\mu}}$$
$261.63 = \frac{1}{2(0.9)}\sqrt{\frac{T\times 0.9}{6\times 10^{-3}}}$
T ≃1480N

Question 38
A sonometer wire having a length of 1.50m between the bridges vibrates in its second harmonic resonance with a tuning fork of frequency 256 Hz. What is the speed of the transverse wave on the wire?

Solution 38
Frequency in IInd harmonic = $\frac{2}{2l}\sqrt{\frac{T}{\mu}}$
$$256 = \frac{1}{1.5}\times \upsilon$$
υ= 384 m/s

Question 39
The length of the wire shown in figure between the pulleys is 1.5m and its mass is 120g. Find the frequency of vibration with which the wire vibrates in two loops leaving the middle point of the wire between the pulleys at rest.



Solution 39
Wire is in IInd harmonic as two loops are formed
Frequency of IInd harmonic = $\frac{2}{2l}\sqrt{\frac{T}{\mu}}$
$= \frac{1}{1.5}\sqrt{\frac{9g\times 1.5}{12\times 10^{-3}}} \simeq 70\text{ Hz}$.

Question 40

A one meter long stretched string having a mass of 40g is attached to a tuning fork. The fork vibrates 128 Hz in a direction perpendicular to the string. What should be the tension in the string if it is to vibrate in four loops?

Solution 40

If string forms 4 loops, it will be in 4^{th} harmonic.

$f=\frac{4}{2l}\sqrt{\frac{T}{\mu}}$

$128=\frac{2}{1}\sqrt{\frac{T}{40\times10^{-3}}}$

T = 164 N

Question 41

A wire, fixed at both ends is seen to vibrate at a resonant frequency of 240Hz and also at 300 Hz

a. What could be the maximum value of the fundamental frequency?

b. If transverse waves can travel on this string at a speed 0f 40 m/s, what is its length?

Solution 41

a) Maximum fundamental frequency will be highest common factor of 240 hz and 320Hz

So, 80 Hz is the fundamental frequency.

b) Fundamental frequency = $\frac{\upsilon}{2l}$

$80=\frac{40}{2\times l}$

$l = 0.25\text{m or } 25\text{ cm}$

Question 42

A string, fixed at both ends, vibrates in a resonant mode with a separation of 20 cm between the consecutive nodes. For the next higher resonant frequency, this separation is reduced by 16cm. Find the length of the string.

Solution 42

If n loops are formed initially then (n+1) loops will be formed in next resonance frequency

So, length of string

$n\times 2 = (n+1)(1.6)$

0.4n=1.6

n=4

∴ Length of string = 4 × 2

= 8 cm

Question 43

A 660 Hz tuning fork sets up vibration in a string clamped at both ends. The wave speed for a transverse wave on this string is 220 m/s and the string vibrates in here loops

a. Find the length of the string

b. If the maximum amplitude of a particle is o.5 cm, write a suitable equation describing the motion.

Solution 43

f = 660 Hz; υ = 220 m/s

$\upsilon = f\times\lambda$

$220= 660\times\lambda$

$\lambda = \frac{1}{3}$ m

a) 3 loops are formed, so length of wire = $\frac{3\lambda}{2}$

$=\frac{3}{2}\left(\frac{1}{3}\right)$

$=\frac{1}{2}$m

=50cm

b) Standing wave equation if Node is at x=0

$y = (\mathrm{Amp})\sin kx.\cos\omega t$

=0.5cm sin($\frac{2\pi}{\lambda}$).× cos (ωt)

= 0.5cm sin ($\frac{2\pi}{1} \times \frac{3}{200}\mathrm{cm}^{-1}$)x cos (2$\pi \times 660$)t

$y = 0.5\mathrm{cm}\ [\sin(0.06\pi \mathrm{cm}^{-1})]\mathrm{x}\cos(1320\pi \mathrm{s}^{-1})\mathrm{t}]$

Question 44

A particular guitar wire is 30.0cm long and vibrates at a frequency of 196 Hz when no finger is placed on it. The next higher notes on the scale are 220 Hz, 247Hz, 262 Hz and 294 Hz. How far from the end of the string must the finger be placed to play these notes?

Solution 44

F $\propto \frac{1}{l}$ [As velocity remains constant for given medium]

here, $f_1 = 196\ \mathrm{Hz};\ f_2 = 220\ \mathrm{Hz}, f_3 = 247\mathrm{Hz};\ f_4 = 262\ \mathrm{Hz}, f_5 = 294\ \mathrm{Hz}$

$\frac{f_1}{f_2} = \frac{l_2}{l_1}$

$\frac{196}{220} = \frac{l_2}{0.3}$

$l_2 = 26.7\ \mathrm{cm}$

Now,

$\frac{f_1}{f_3} = \frac{l_3}{l_1}$

$\frac{196}{247} = \frac{l_3}{(0.3)}$

l_3=23.8cm

Similarly

$\frac{196}{262} = \frac{l_4}{(0.3)}$ and $\frac{196}{294} = \frac{l_5}{(0.3)}$

$l_4 = 22.4\mathrm{cm}\ \&\ l_5 = 20\mathrm{cm}$

Question 45

A steel wire fixed at both ends has a fundamental frequency of 200 Hz. Aa person can hear sound of maximum frequency 14Khz. What is the highest harmonic that can be played on this string which is audible to the person?

Solution 45

Let highest harmonic be n^{th}

So. $f_{n^{th}}$ = nf_0

$14{,}000\ =\ n(200)$

n = 70

Question 46

Three resonant frequencies of a string are 90, 150, 210 Hz.

a. Find the highest possible fundamental frequency of vibration of this string.
b. Which harmonics of the fundamental are the given frequencies?
c. Which overtones are these frequencies
d. If the length of the string is 80 cm, what would be the speed of the transverse wave on this string?

Solution 46

a) Maximum fundamental frequency will be highest common factor of 90Hz, 150 Hz and 210 Hz.

f_0= 30 Hz

b) and c)

when f_1 = 90 Hz

f_1 = 3 f_0 (3rd harmonic and 2nd overtone)

When f_2 = 150 Hz

f_1 = 5 f_0 (5th harmonic and 4th overtone)

When f_3 = 210 Hz

f_3 = 7 f_0 (7th harmonic and 6th overtone)

d) $f_0 = \frac{\upsilon}{2l}$

$30 = \frac{\upsilon}{2(0.8)}$

υ= 48 m/s

Question 47

Two wires are kept tight between the same pair of supports. The tensions in the wires are in the ratio 2: 1, the radii are in the ratio 3: 1 and the densities are in the ratio 1: 2. Find the ratio of their fundamental frequencies.

Solution 47

$f_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2l}\sqrt{\frac{T}{m} \times l}$

$f_0 \propto \sqrt{\left(\frac{T}{ml}\right)}$; $f_0 \propto \sqrt{\frac{T}{(\rho\pi r^2)l l}}$

$f_0 \propto \sqrt{\frac{T}{(\rho r^2)}}$ [since l is same for both wires]

$\frac{f_{01}}{f_{02}} = \sqrt{\frac{T_1}{T_2} \times \frac{\rho_2}{\rho_1} \times \frac{r_2^2}{r_1^2}}$

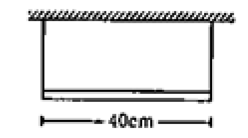
$= \sqrt{\left(\frac{2}{1}\right)\left(\frac{2}{1}\right)\left(\frac{1}{3}\right)^2}$

$\frac{f_{01}}{f_{02}} = \frac{2}{3}$

Question 48

A uniform horizontal rod of length 40 cm and mass 1.2kg is supported by two identical wires as shown in figure. Where should a mass of 4.8 kg be placed on rod so that the same tuning fork may excite the wire on left into its fundamental vibrations and that on right into its first overtone?

Take g=10 m/s^2

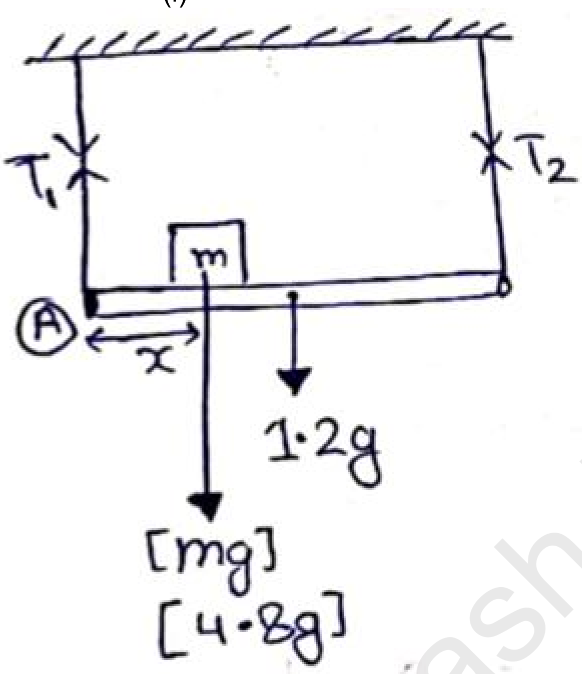


Solution 48

Fundamental frequency of 1st wire = first overtone of frequency of 2nd wire

$\frac{1}{2l}\sqrt{\frac{T_1}{\mu}} = \frac{2}{2l}\sqrt{\frac{T_2}{\mu}}$

$T_1 = 4T_2$ -------(i)



Let mass m=4.8kg be placed at distance x from left end.

Now,

$T_1 + T_2$ = 1.2g + 4.8g (Translatory equilibrium)

4$T_1 + T_2$ = 6g

T_2 = 12 N

Apply rotational equilibrium at point (A)

Clockwise torque = anticlockwise torque

$(4.8g)x + (1.2)g(0.2) = T_2 \times (0.4)$

48x + 2.4 = 4.8

48x = 2.4

$x = \frac{2.4}{48} \times 100$

x=5cm

Question 49

The given figure shows an aluminum wire of length 60cm joined to a steel wire of length 80cm and stretched between two fixed supports. The tension produced is 40 N. The cross-sectional area of the steel wire is 1.0mm^2 and that of the aluminum wire is 3.001.0mm^2. What could be the minimum frequency of a tuning fork which can produce standing waves in the system with the joint as a node?

The density of aluminum is 2.6 $\frac{g}{cm}^3$ and that of steel is 7.8 $\frac{g}{cm}^3$

Solution 49

Velocity of wave in wire $V=\sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}}$

For steel wire

$V_{steel} = \sqrt{\frac{40}{(7.8\times10^3)(1\times10^{-6})}}$ = 71.6 m/s

For Aluminum wire

$V_{AL}=\sqrt{\frac{40}{(2.6\times10^3)(3\times10^{-6})}}$ = 71.6 m/s

Let p^{th} harmonic frequency of steel wire matches with q^{th} harmonic of Aluminum wire

$P \times \frac{V_{steel}}{2(l_{steel})} = q \times \frac{V_{Al}}{2(l_{Al})}$

$\frac{p}{80} = \frac{q}{60}$

3p=4q

P	4	8	12
q	3	6	9

For minimum, p = 4 loops and q= 3 loops

So, minimum frequency = $P \times \frac{V_{steel}}{2(l_{steel})}$

$= 4 \times \frac{71.6}{2(0.8)}$

$\simeq$ 180 Hz

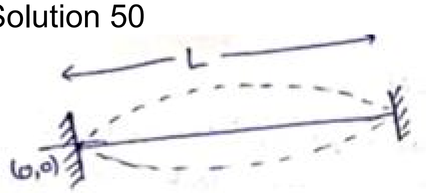
Question 50

A string of length L fixed at both ends vibrates in its fundamental mode at a frequency v and a maximum amplitude A.

a. Find the wavelength and the wave number k.

b. Take the origin at one end of the string and the X-axis along the string. Take the Y-axis along the direction of the displacement. Take t=0 at the instant when the middle point of the string passes through its mean position and is going towards the positive y-direction. Write the equation describing the standing wave.

Solution 50



a) Distance between two consecutive nodes is $\frac{\lambda}{2}$=L
λ = 2L
wave number k=$\frac{2\pi}{\lambda}$ =$\frac{2\pi}{2L}$
k=$\frac{\pi}{L}$
b) Equation of standing wave having having node at origin and antinode of mean position
$y = A\sin kx \sin \omega t$
y=A sin ($\frac{\pi x}{L}$)sin(2$\pi\omega t$)

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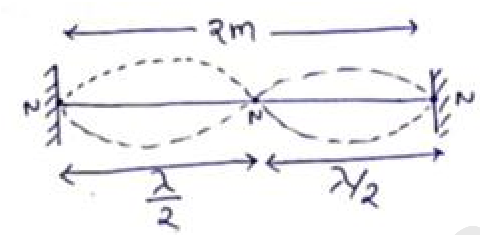
Question 51

A 2m long string fixed at both ends is set into vibrations in its first overtone. The wave speed on the string is 200 m/s and the amplitude is 0.5cm
a. Find the wavelength and frequency
b. Write the equation giving the displacement of different points as a function of time. Choose the X-axis along the string with the origin at one end and t=0 at the instant when the point x=50cm has reached its maximum displacement.

Solution 51



a) wavelength λ = 2m
frequency=$\vartheta \times \lambda$
200=$\vartheta \times 2$
$\vartheta = 100$ Hz
b) Equation of a standing wave having node at origin and antinode at maximum displacement at t=0
$y = A\sin kx \cos \omega t$
$y = (0.5\text{cm})\sin\left(\frac{2\pi}{2}x\right)\cos(2\pi \times 100t)$
y= $0.5\text{cm} \sin \pi x \cos 200\pi t$
Here x is in meter and t is in second

Question 52

The equation for the vibration of a string, fixed at both ends vibrating in its third harmonic is given by $y = (0.4\text{cm})\sin[(0.314\ \text{cm}^{-1})x]\cos[(600\pi s^{-1})t]$.
a. At what frequency of vibration?
b. What are the position of the nodes?
c. What is the length of the string?
d. What is the wavelength and the speed of two travelling waves that can interfere to give this vibration?

Solution 52

Comparing equation with y= A.sinkx.cosωt

a) $\omega = 600\pi$

2$\pi f = 600\pi$

f=600Hz

b) K=0.314

$\frac{2\pi}{\lambda}$ = 0.314

λ = 20cm

Nodes are at distance of 0, $\frac{\lambda}{2}\frac{2\lambda}{2}\frac{3\lambda}{2}$ from one end i.e., *x=0,* 10cm, 20cm, 30cm.

c) Length of string $= \frac{3\lambda}{2} = \frac{3}{2}(20) = 30cm$

d) $\upsilon = f \times \lambda$

$= 300 \times \frac{20}{100}$

υ = 60 m/s

Question 53

The equation of a standing wave, produced on a string fixed at both ends, is

$y = (0.4\text{cm})\sin[(0.314\text{cm}^{-1})x]$cos[600$\pi s^{-1}$)t]

What could be the smallest length of the string?

Solution 53

Here K=0.314

$\frac{2\pi}{\lambda} = 0.314$

λ = 20 cm

minimum length of string = $\frac{\lambda}{2} = \frac{20}{2} =$ 10cm

Question 54

A 40 cm wire having a mass of 3.2g is stretched between two fixed supports 40.05cm apart. In its fundamental mode, the wire vibrates at 220 Hz. If the area of cross-section of the wire is 1.0mm^2, find the Young's modulus.

Solution 54

Fundamental frequency

$f_0 = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

220= $\frac{1}{2(0.4005)}\sqrt{\frac{T\times40\times10^{-2}}{3.2\times10^{-3}}}$

T $\simeq$248.2 N

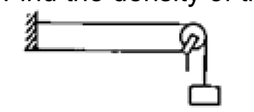
$Y = \frac{Tl}{A\Delta l}$

$= \frac{248.2 \times (0.4)}{(1 \times 10^{-6}) \times (0.4005 - 0.40)}$

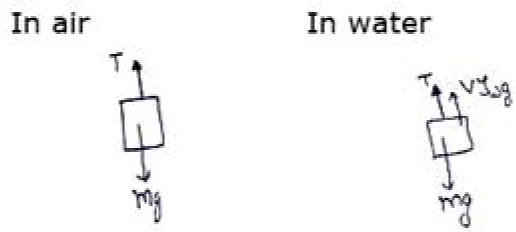
Y= 1.98$\times 10^{11}\ N/m^2$

Question 55

The figure shows a string stretched by a block going over a pulley. The string vibrates in its tenth harmonic in unison with a particular tuning fork. When a beaker containing water is brought under the block so that the block is completely dipped into the beaker, the string vibrates in its eleventh harmonic. Find the density of the material of the block.



Solution 55



$T = mg = dVg$ $T' = mg — V\rho_w g$

$= d\upsilon g — V\rho_w g$

$T' = (d — \rho_w)Vg$

Since, $f_{fork} = f_{10th\ harm} f_{11th\ harm}$

$= 10 \times \frac{1}{2l}\sqrt{\frac{T}{\mu}} = 11 \times \frac{1}{2l}\sqrt{\frac{T}{\mu}}$

$100(dVg) = 121(d — \rho_w)\upsilon_g$

$d = 5.8 \times 10^3\ kg/m^3$

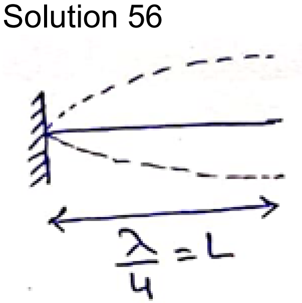
Question 56

A 2.00m long rope, having a mass of 80g, is fixed at one end and is tied to a light string at the other end. The tension in the string is 256 N

a. Find the frequencies of the fundamental and the first two overtones.

b. Find the wavelength in the fundamental and the first overtones.

Solution 56



a) Fundamental frequency

$\frac{\lambda}{\mu} = 2m$; υ $= \sqrt{\frac{T}{\mu}} = \sqrt{\frac{256\times2}{80\times10^{-3}}}$

λ= 8m υ = 80 m/s

$f_0 = \frac{1}{4l}\sqrt{\frac{T}{\mu}}$

$= \frac{1}{4(2)} \frac{256\ \times2}{80\times10^{-3}}$

f_0 = 10 Hz.

b) 1st overtone frequency $f_1 = 3f_0 = 3(10) = 30\ Hz$

υ= f × λ

80=30 × λ_1

λ_1 = 2.67m

2nd overtone frequency $f_2 = 3f_0 = 5(10) = 50\ Hz$

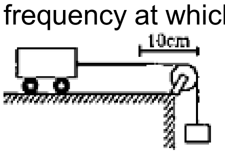
υ= f × λ

80=50 × λ_2

λ_2= 1.6m

Question 57

A heavy string is tied at one end to a movable support and to a light thread at the other end as shown in the below figure. The thread goes over a fixed pulley and supports a weight to produce a tension. The lowest frequency with which the heavy string resonates is 120 Hz. If the movable support is pushed to the right by 10cm so that the joint is placed on the pulley, what will be the minimum frequency at which the heavy string can resonate?



Solution 57

Initially, heavy string is fixed at one end only

So, lowest frequency means fundamental frequency = 120 Hz

$\therefore f_0 = \frac{\upsilon}{4l}$ = 120

Now, when moved by 10cm, heavy string is fixed at both ends

So, minimum frequency = $\frac{\upsilon}{2l} = 2f_0$

= 2(120)

= 240Hz

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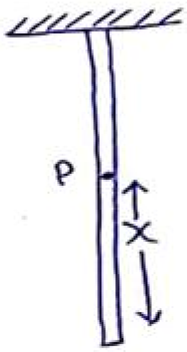
Question 26

A heavy but uniform rope of length L is suspended from a ceiling a) write the velocity of a transverse wave travelling on the string as a function of the distance from the lower end. b) If the rope is given a sudden sideways jerk at the bottom, how long will it take for the pulse to reach the ceiling? c) A particle is dropped from the ceiling at the instant the bottom end is given the jerk, where will the particle meet the pulse?

Solution 26



a) Let mass per unit length be μ

Tension at point P; $T = \text{mass} \times g$

$T = \mu x g$

$\upsilon = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{uxg}{\mu}}$

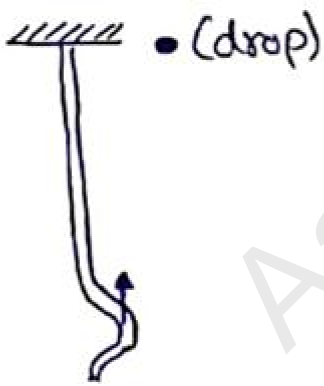
$\upsilon = \sqrt{xg}$

b) $\upsilon = \frac{dx}{dt} = \sqrt{xg}$

$\int_0^L \frac{dx}{\sqrt{x}} = \sqrt{g}\int_0^t dt$

$T = \sqrt{\frac{4L}{g}}$

c)



Acceleration of pulse wave

$a = \upsilon\frac{dv}{dx} = \sqrt{xg}).\frac{d}{dx}(\sqrt{xg}$

$a = \frac{g}{2}$

Now

$\mu_{rel}=0$

$a_{rel}=3g/2$

$s_{rel}=L$

$s_{rel} = \mu_{rel} + \frac{1}{2}a_{rel}t^2$

$L = 0 + \frac{1}{2}(3g/2)\,t^2$

$t = \sqrt{\frac{4L}{3g}}$

For pulse,

$u=0,\ a=\left(\frac{g}{2}\right),\ t=\sqrt{\frac{4L}{3g}}$

$s = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2}\left(\frac{g}{2}\right)\left(\frac{4L}{3g}\right)$

$s = \frac{L}{3}$ from bottom

Question 27

Two long strings A and B, each having linear mass density 1.2×10^{-1} kg/m are stretched by different tensions 4.8N and 7.5N respectively and are kept parallel to each other with their left ends at x=0. Wave pulses are produced on the strings at the left ends at t=0 on string A and t = 20 ms on string B. When and where will the pulse on B overtake that on A?

Solution 27

$\upsilon_A = \sqrt{\frac{T}{\mu}} = \sqrt{\left(\frac{4.8}{1.2\times10^{-2}}\right)}$ = 20 m/s

$\upsilon_B = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{7.5}{1.2\times10^{-2}}}$ = 25 m/s

In 20ms, pulse in string A travels = speed $\times$ time

$= 20 \times 20 \times 10^{-3}$

= 0.4m

Now, U_{rel} = 25-20 = 5 m/s

s_{rel} = 0.4m

$a_{rel} = 0$

$t = \frac{s_{rel}}{u_{rel}}$ = 0.08 sec = 80 ms

Total time = 20 + 80

= 100ms

In 100ms, string A travels a distance of $= \upsilon_A \times t$

$= 20 \times 100 \times 10^{-3}$

$X = 2m$

Question 29

A 200 Hz wave with amplitude 1 mm travels on a long string of linear mass density 6 g/m kept under a tension of 60 N.

a) Find the average power transmitted across a given point on the string.

b) Find the total energy associated with the wave In a 20 m long portion of the string.

Solution 29

a) $\text{Velocity of wave} = \upsilon_A = \sqrt{\frac{T}{\mu}} = \sqrt{\left(\frac{60}{6\times10^{-3}}\right)}$

υ = 100 m/s

Power = P= $2\pi^2 f^2 A^2 \rho \upsilon_s$

P= $2\pi^2(100)^2(10^{-3})^2(100)(6 \text{ X } 10^{-3})$

P=0.47W

b)

Length of the string = 2m

Time to cover the distance t=$\frac{2}{100}$ = 0.02 sec

$$P = \frac{\text{Energy}}{\text{Time}}$$

$\text{Energy} = (0.47)(0.02)$

E= 9.4mJ