

## HC VERMA Solutions for Class 11 Physics Chapter 13 Fluid Mechanics

### Question 1

The surface of water in a water tank on the top of a house is 4m above the tap level. Find the pressure of water at the tap when the tap is closed? Is it necessary to specify that the tap is closed? Take  $g = 10\text{m/s}^2$

### Solution 1

Pressure due to water

$$\Rightarrow P = h\rho g = 4 \times 1000 \times 10 \\ = 40000 \text{ N/m}^2$$

Since pressure is proportional to height of water. If tap is open height of water decreases, so pressure decreases.

### Question 2

The height of mercury surfaces in the two arms of the manometer shown in figure (13-E1) are 2 cm and 8 cm.

Atmospheric pressure =  $1.01 \times 10^5 \text{ N/m}^2$ . Find (a) the pressure of the gas in the cylinder and (b) the pressure of mercury at the bottom of the U tube.

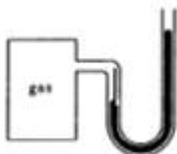
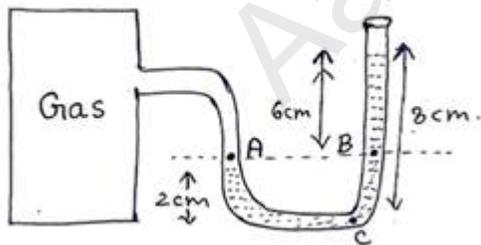


Figure 13-E1

### Solution 2

(a)



$$P_A = P_B$$

$$P_{\text{gas}} = P_0 + \frac{6}{100} \rho g$$

$$= 1.01 \times 10^5 + \frac{6}{100} \times 13.6 \times 1000 \times 9.80$$

$$= 1.09 \times 10^5 \frac{\text{N}}{\text{m}^2}$$

(b) Pressure at bottom of mercury

$$\begin{aligned}\Rightarrow P_c &= P_0 + \left(\frac{8}{100}\right)\rho g \\ &= 1.01 \times 10^5 + \frac{8}{100} \times 13.6 \times 1000 \times 9.8 \\ &= 1.12 \times 10^5 \frac{N}{m^2}\end{aligned}$$

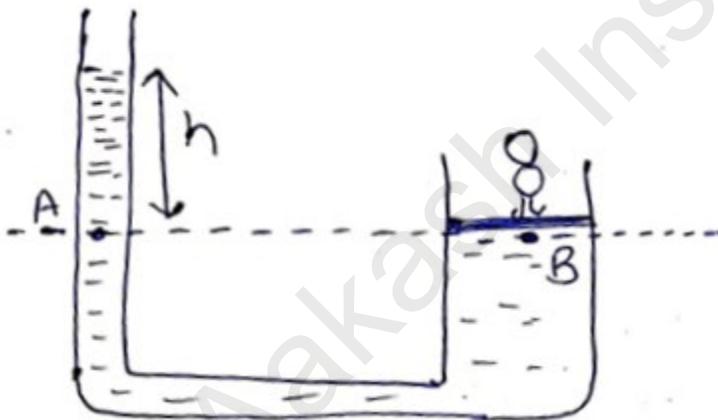
### Question 3

The area of cross-section of the wider tube shown in figure (13-E2) is  $900 \text{ cm}^2$ . If the boy standing on the piston weighs  $45 \text{ kg}$ , find the difference in the levels of water in the two tubes.



Figure 13-E2

### Solution 3



$$P_A = P_B$$

$$P_0 + h\rho g = P_0 + \frac{mg}{A}$$

$$h = \frac{m}{A\rho} = \frac{45}{900 \times 10^{-4} \times 1000}$$

$$h = 50 \text{ cm}$$

### Question 4

A glass of water has a bottom of area  $20 \text{ cm}^2$ , top of area  $20 \text{ cm}^2$ , height  $20 \text{ cm}$  and volume half a litre.

(a) Find the force exerted by the water on the bottom.

(b) Considering the equilibrium of the water, find the resultant force exerted by the sides of the glass on the water. Atmospheric pressure =  $1.01 \times 10^5 \text{ kg/m}^3$  and  $g = 10 \text{ m/s}^2$ . Take all numbers to be exact.

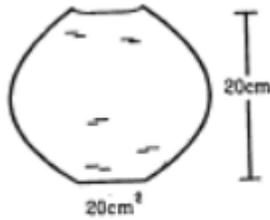


Figure 13-E3

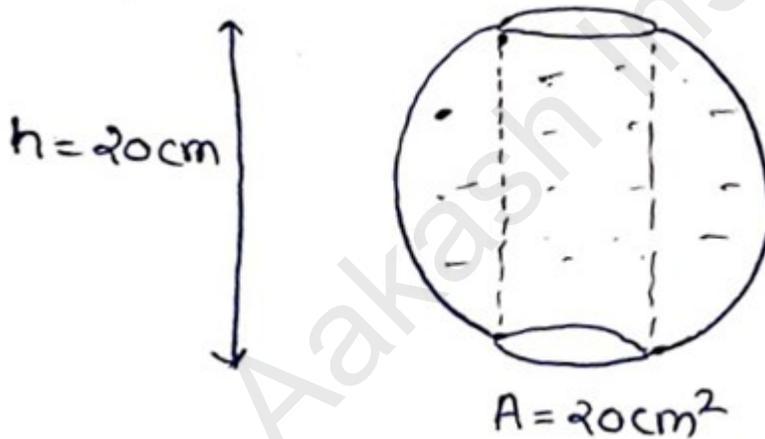
Solution 4

$$(a) P_{bottom} = P_0 + h\rho g$$

$$F_{bottom} = P_{bottom} A_{bottom}$$

$$= \left( 10^5 + \frac{20}{100} \times 1000 \times 10 \right) (20 \times 10^{-4}) = 204 \text{ N}$$

(b)



Cylindrical water column is balanced by bottom of glass and remaining by side wall.

$$\Rightarrow F_{sidewall} = \text{Total Weight} - \text{Wt of cylinder water column}$$

$$= mg - h\rho gA$$

$$\Rightarrow F_s = V\rho g - h\rho gA \text{ (upward direction)}$$

$$\Rightarrow F_s = \frac{1}{2} \times 10^{-3} \times 1000 \times 10 - \frac{20}{100} \times 1000 \times 10 \times 20 \times 10^{-4}$$

$$\Rightarrow F_s = 1 \text{ N}$$

Question 5

Suppose the glass of the previous problem is covered by a jar and the air inside the jar is completely pumped out.

(a) What will be the answers to the problem?

(b) Show that the answers do not change if a glass of different shape is used provided the height, the bottom area and the volume are unchanged.

Solution 5

Now atmospheric pressure will not act.

(a)  $F_{bottom} = P_{bottom} A_{bottom}$

$$F_b = h\rho gA = \left( \frac{20}{100} \times 1000 \times 10 \times 20 \times 10^{-4} \right) = 4N$$

(b)  $F_s = mg - h\rho gA$  (No change)

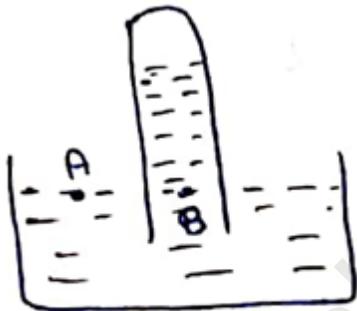
$$F_s = 1N$$

No change will occur in answer if height, area and volume is kept same irrespective of shape of container.

Question 6

If water be used to construct a barometer, what would be the height of water column at standard atmospheric pressure (76 cm of mercury)?

Solution 6



$$P_A = P_B$$

$$P_0 = h_{Hg}\rho_{Hg}g = h_w\rho_w g$$

$$h_w = \frac{h_{Hg}\rho_{Hg}}{\rho_w} = \frac{76 \times 13.6}{1}$$

$$h_w = 1033.6cm$$

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Question 7

Find the force exerted by the water on a  $2m^2$  plane surfaces of a large stone placed at the bottom of a sea 500m deep. Does the force depend on the orientation of the surface?

Solution 7

$$F = PA$$

$$F = (\rho_w gh)A = (1000 \times 10 \times 500) \times 2 = 10^7 \text{ N/m}^2$$

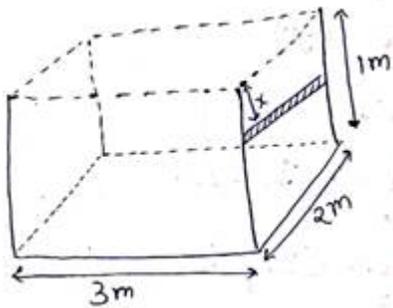
Force depends upon area not on orientation.

### Question 8

Water is filled in a rectangular tank of size 3m × 2m × 1m.

- Find the total force exerted by the water on the bottom surface of the tank.
- Consider a vertical side of area 2m × 1m. Take a horizontal strip of width  $\partial x$  metre in this side, situated at a depth of  $x$  metre from the surface of water. Find the force by the water on strip.
- Find the torque of the force calculated in part (b) about the bottom edge of this side.
- Find the total force by the water on this side.
- Find the total torque by the water on this side about the bottom edge. Neglect the atmospheric pressure and take  $g = 10 \text{ m/s}^2$ .

### Solution 8



$$(a) F_{\text{bottom}} = P_{\text{bottom}} A_{\text{bottom}}$$

$$F_b = (\rho gh)(A_b) = (1000 \times 10 \times 1)(3 \times 2) = 60000 \text{ N}$$

(b) Force of strip of width 'dx'

$$dF = (P)(A)$$

$$= (x\rho g)(A)$$

$$= (x\rho g)(2 \cdot dx) = x \times 1000 \times 10 \times 2 dx$$

$$= 20,000 x dx$$

(c) Torque = force × distance

$$d\tau = dF(1 - x) = 20000x(1 - x)$$

(d) Total force by water on side wall

$$F = \int_0^1 dF = \int_0^1 20000x(dx) = 20000 \left[ \frac{x^2}{2} \right]_0^1$$

$$= 10000 \text{ N}$$

(e) Total torque on side wall

$$\tau = \int_0^1 d\tau = \int_0^1 20000x(1 - x) = \frac{10000}{3}$$

### Question 9

An ornament weighing 36g in air, weighs only 34 g in water. Assuming that some copper is mixed with gold to prepare the ornament, find the amount of copper in it. Specific gravity of gold is 19.3 and that of copper is 8.9

Solution 9

$$m_{\text{air}} = 36\text{gm}$$

$$m_{\text{Au}} + m_{\text{Cu}} = 36 - (1)$$

Weight difference in air and water is due to buoyancy force

$$(V_{\text{Au}} + V_{\text{Cu}})\rho_w g = 2g$$

$$\left(\frac{m_{\text{Au}}}{\rho_{\text{Au}}} + \frac{m_{\text{Cu}}}{\rho_{\text{Cu}}}\right)\rho_w = 2$$

$$\frac{m_{\text{Au}}}{19.3} + \frac{m_{\text{Cu}}}{8.9} = 2 - (2)$$

Solving, equation (1) and (2)

$$m_{\text{Cu}} = 2.2\text{gm}$$

Question 10

Refer to the previous problem. Suppose, the goldsmith argues that he has not mixed copper or any other material with gold, rather some cavities might have been left inside the ornament. Calculate the volume of the cavities left that will allow the weights given that problem.

Solution 10

$$m_{\text{air}} = m_{\text{Au}} = 36\text{gm}$$

Now,

$$(V_{\text{Au}} + V_c)\rho_w g = 2g$$

$$\left(\frac{m_{\text{Au}}}{\rho_{\text{Au}}} + V_c\right)\rho_w = 2$$

$$\left(\frac{m_{\text{Au}}}{19.3} + V_c \rho_w\right) = 2$$

$$V_c = 0.112 \text{ cm}^3$$

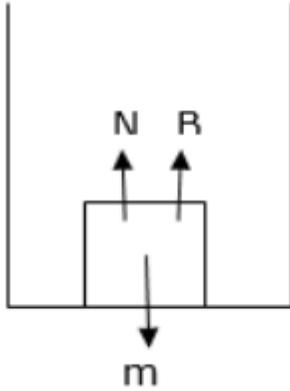
Question 11

A metal piece of mass 160 g lies in equilibrium inside a glass of water (figure 13-E4). The piece touches the bottom of the glass at a small number of points. If the density of the metal is  $8000 \text{ kg/m}^3$



Figure 13-E4

Solution 11



$$\begin{aligned}
 N + B &= mg \\
 \Rightarrow N &= mg - B \\
 &= 160 \times 10^{-3} \times 10 - V \rho_w g \\
 &= 1.6 - \left(\frac{m}{\rho}\right) \rho_w g \\
 &= 1.6 - \left(\frac{160 \times 10^{-3}}{8000}\right) (1000)(10) \\
 \Rightarrow N &= 1.4
 \end{aligned}$$

### Question 12

A ferry boat has internal volume  $1 \text{ m}^3$  and weight  $50 \text{ kg}$ .

- (a) Neglecting the thickness of the wood, find the fraction of the volume of the boat immersed in water.
- (b) If a leak develops in the bottom and water starts coming in, what fraction of the boat's volume will be filled with water before water starts coming in from the sides?

### Solution 12

(a) In equilibrium,  
Weight of boat = Buoyancy force

$$mg = V_{im} \rho_w g$$

$$50 = V_{im} (1000)$$

$$V_{im} = 0.05 \text{ m}^3$$

(b) Let  $V_f$  be the volume of boat filled with water before water starts coming in from the sides. So,

$$mg + V_f \rho_w g = V_{boat} \rho_w g$$

$$50 + V_f \times 1000 = 1 \times 1000$$

$$V_f = 0.95 \text{ m}^3$$

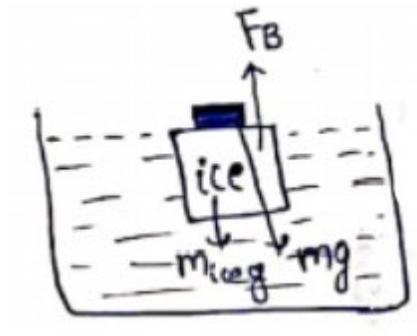
$$\text{Fraction of boat's volume filled} = \frac{0.95}{1} = \frac{19}{20}$$

### Question 13

A cubical block of ice floating in water has to support a metal piece weighing  $0.5 \text{ kg}$ . What can be the minimum edge of the block so that it does not sink in water?

Specific gravity of ice = 0.9

Solution 13



Let side of ice cube be  $x$ .

In equilibrium,

$$M_{ice}g + mg = V_{imm}\rho_w g$$

$$(\rho_{ice}x^3) + 0.5 = x^3\rho_w$$

$$(0.9 \times 10^3)x^3 + 0.5 = x^3(10^3)$$

$$X = 17 \text{ cm}$$

Question 14

A cube of ice floats partly in water and partly in K.oil. Find the ratio of the volume of ice immersed in water to that in K.oil.

Specific gravity of K.oil is 0.8 and that of ice is 0.9.

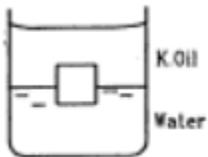


Figure 13-E5

Solution 14

Volume of ice inside kerosene oil and water be  $V_k$  and  $V_w$  respectively

$$V_{ice} = V_k + V_w \text{ ---- (1)}$$

Since, Ice is in equilibrium,

$$M_{ice}g = F_B$$

$$V_{ice}\rho_{ice}g = V_k\rho_k g + V_w\rho_w g$$

$$V_{ice}(0.9) = V_k(0.9) + V_w \text{ ---- (2)}$$

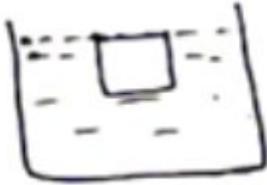
Solving equation (1) and (2)

$$\frac{V_w}{V_k} = 1$$

Question 15

A cubical box is to be constructed with iron sheets 1 mm in thickness. What can be the minimum value of the external edge so that the cube does not sink in water?  
Density of iron =  $8000 \text{ kg/m}^3$  and density of water =  $1000 \text{ kg/m}^3$

Solution 15

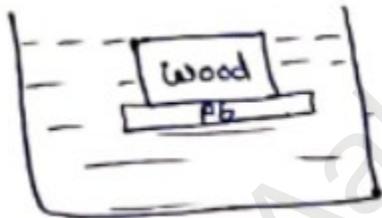


Let external edge of iron cube be  $x$   
In equilibrium,  
Weight of iron cube = Buoyancy force  
 $V_i \rho_i g = V_{im} \rho_w g$   
 $6x[(0.1)x^2(8000)] = x^3(1000)$   
 $X = 4.8 \text{ cm}$

Question 16

A cubical block of wood weighing 200 g has a lead piece fastened underneath. Find the mass of the lead piece which will just allow the block to float in water.  
Specific gravity of wood is 0.8 and that of lead is 11.3.

Solution 16



In equilibrium

$$(m_w + m_{pb})g = (V_w + V_{pb})\rho_w g$$

$$(0.2 + m_{pb}) = (m_{wood}/\rho_{wood} + m_{pb}/\rho_{pb})\rho_{water}$$

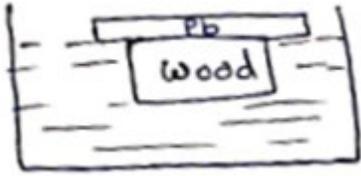
$$(0.2 + m_{pb}) = (0.2/0.8 + m_{pb}/11.3)$$

$$m_{pb} = 0.0548 \text{ kg}$$

Question 17

Solve the previous problem if the lead piece is fastened on the top surface of the block and the block is to float with its upper surface just dipping into water.

Solution 17



In equilibrium,

$$m_{\text{Pb}} + m_{\text{Wg}} = V_{\text{wood}} \rho_{\text{water}} g$$

$$m_{\text{Pb}} + 200 = 200/0.8$$

$$m_{\text{Pb}} = 50\text{g}$$

### Question 18

A cubical metal block of edge 12 cm floats in mercury with one fifth of the height inside the mercury. Water is poured till the surface of the block is just immersed in it. Find the height of the water column to be poured. Specific gravity of mercury = 13.6.

### Solution 18

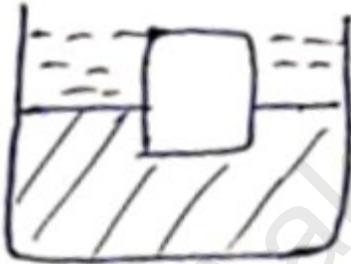
Initially,

$$mg = F_B$$

$$\rho_b (12)^3 g = (12)^2 (12/5) (\rho_{\text{Hg}}) g$$

$$\rho_b = 13.6/5 \text{ gm/cc}$$

Let x be the height of water column after the water is poured



$$mg = x(12)^2 \rho_{\text{wg}} + (12-x)(12)^2 \rho_{\text{Hg}} g$$

$$(13.6/5)(12)^3 = x(12)^2 + (12-x)(12)^2(13.6)$$

$$X = 10.4\text{cm}$$

### Question 19

A hollow spherical body of inner and outer radii 6 cm and 8 cm respectively floats half submerged in water. Find the density of the material of the sphere.

### Solution 19

In equilibrium,

$$mg = F_B$$

$$\rho \left[ \frac{4\pi}{3} (8)^3 - \frac{4\pi}{3} (6)^3 \right] = \frac{2\pi}{3} (8)^3 \rho_w g$$

$$\rho = 0.865 \rho_w$$

$$\rho = 865 \text{ kg/m}^3$$

### Question 20

A solid sphere of radius 5 cm floats in water. If a maximum load of 0.1 kg can be put on it without wetting the load, find the specific gravity of the material of the sphere.

### Solution 20

$$\begin{aligned} (m_s + m_L)g &= V \rho_w g \\ \frac{4\pi}{3} (5)^3 \rho_s + 0.1 \times 1000 &= \frac{4\pi}{3} (5)^3 \\ \rho_s &= 0.8 \text{ gm/cc} \end{aligned}$$

### Question 21

Find the ratio of the weights, as measured by a spring balance, of a 1 kg block of iron and a 1 kg block of wood. Density of iron = 7800 kg/m<sup>3</sup>, density of wood = 800 kg/m<sup>3</sup> and density of air = 1.293 kg/m<sup>3</sup>.

### Solution 21

$$\begin{aligned} \text{Reading by spring balance} &= mg - \text{buoyancy force} \\ &= mg - V \rho_{\text{air}} g \\ &= mg - \frac{m}{\rho} \rho_{\text{air}} g \\ W_{\text{iron}} &= [1 - (1/7800)(1.293)]g \\ W_{\text{wood}} &= [1 - (1/800)(1.293)]g \\ W_{\text{iron}} / W_{\text{wood}} &= 1.0015 \end{aligned}$$

### Question 22

A cylindrical object of outer diameter 20 cm and mass 2 kg floats in water with its axis vertical. If it is slightly depressed and then released, find the time period of the resulting simple harmonic motion of the object.

### Solution 22

$$\begin{aligned} T &= 2\pi \sqrt{\frac{m}{k}} \\ \text{Water behaves as spring having spring constant } K &= A \rho g \\ T &= 2\pi \sqrt{\frac{m}{A \rho g}} \\ &= 2\pi \sqrt{\frac{2 \times 10^3}{\pi (10)^2 (1) (980)}} \\ T &= 0.5 \text{ sec} \end{aligned}$$

### Question 23

A cylindrical object of outer diameter 10 cm, height 20 cm and density 8000 kg/m<sup>3</sup> is supported by a vertical spring and is half dipped in water as shown in figure(13-E6). (a) Find the elongation of the spring in equilibrium condition. (b) If the object is slightly depressed and released, find the time period of resulting oscillations of the object. The spring constant = 500 N/m.

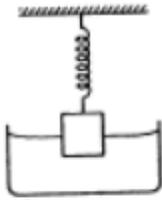
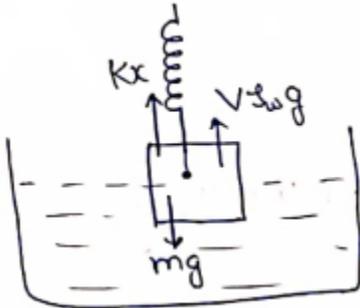


Figure 13-E6

Solution 23



In equilibrium

$$Kx + V\rho g = mg$$

$$(500 \times 10^3)x + \pi(5)^2\left(\frac{20}{2}\right)(1)(980) = \pi(5)^2(20)(8)(980)$$

$$x = 23.5 \text{ cm}$$

$$T = 2\pi \sqrt{\frac{m}{K_{\text{eq}}}}$$

Water behaves as spring of spring constant  $K_1 = A\rho g$

Now, spring and water spring are in parallel combination

$$K_{\text{eq}} = K_1 + K_2$$

$$T = 2\pi \sqrt{\frac{\pi(5)^2(20)(8)}{(500 \times 10^3 + \pi(5)^2(1)(980)}}$$

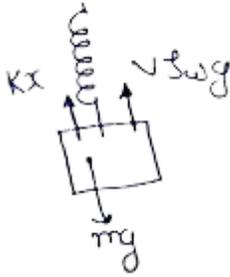
$$T = 0.935 \text{ sec}$$

Question 24

A wooden block of mass 0.5 kg and density 800 kg/m<sup>3</sup> is fastened to the free end of a vertical spring of spring constant 50 N/m fixed at the bottom. If the entire system is completely immersed in water, find (a) the elongation (or compression) of the spring in equilibrium and (b) the time-period of vertical oscillations of the block when it is slightly depressed and released.

Solution 24

(a)



In equilibrium,

$$Kx + V\rho_w g = mg$$

$$(50)x + (0.5/800)(1000)(10) = (0.5)(10)$$

$$X = -0.025 \text{ m}$$

$$X = -2.5 \text{ cm}$$

So, spring will be in compression

(b) Since, system is completely inside water so unbalanced force will be due to spring only. Buoyancy force doesn't change

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$= 2\pi \sqrt{\frac{0.5}{50}}$$

$$T = \pi / 5 \text{ Type equation here.}$$

### Question 25

A cube of ice of edge 4 cm is placed in an empty cylindrical glass of inner diameter 6 cm. Assume that the ice melts uniformly from each side so that it always retains its cubical shape. Remembering that ice is lighter than water, find the length of the edge of the ice cube at the instant it just contact with the bottom of the glass.

### Solution 25

Let the length of the edge of the ice block when it just leaves contact with the bottom of glass be  $x$  and height of water after melting be  $h$ .

Weight = Buoyancy force

$$(x)^3 \rho_{ice} g = (x^2 h) \rho_w g$$

$$0.9x = h \text{ ---- (1)}$$

Volume of water formed by melting of ice

$$(4)^3 - (x)^3 = \pi r^2 h - x^2 h \text{ ---- (2)}$$

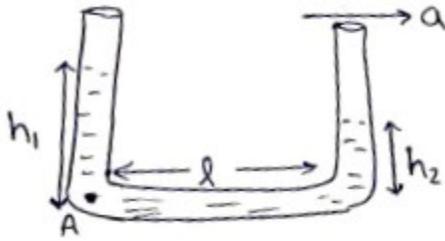
Solving equation (1) and (2)

$$X = 2.26 \text{ cm}$$

### Question 26

A U-tube containing a liquid is accelerated horizontally with a constant acceleration  $a_0$ . If the separation between the vertical limbs is  $l$ , find the difference in the heights of the liquid in the two arms.

### Solution 26



$$P_A = P_0 + h_1 \rho g = P_0 + h_2 \rho g + \rho l a$$

$$h_1 - h_2 = l a / g$$

### Question 27

At Deoprayag (Garhwal, UP) river Alaknanda mixes with the river Bhagirathi and becomes river Ganga. Suppose Alaknanda has a width of 12m, Bhagirathi has a width of 8 m and Ganga has a width of 16 m. Assume that the depth of water is same in the three rivers. Let the average speed of water in Alaknanda be 20 km/h and in Bhagirathi be 16 km/h. Find the average speed of water in the river Ganga.

### Solution 27

From continuity equation

$$A_1 V_1 + A_2 V_2 = A_3 V_3$$

$$(12 \times d)(20) + (8 \times d)(16) = (16 \times d)(V_g)$$

$$V_g = 23 \text{ km/hr}$$

### Question 28

Water flows through a horizontal tube of variable cross-section (figure 13-E7). The area of cross-section at A and B are  $4 \text{ mm}^2$  and  $2 \text{ mm}^2$  respectively. If 1 cc of water enters per second through A, find (a) the speed of water at A, (b) the speed of water at B and (c) the pressure difference  $P_A - P_B$ .



Figure 13-E7

### Solution 28

A) Discharge = Area  $\times$  velocity

$$(1 \times 10^{-6}) = (4 \times 10^{-6}) V_A$$

$$V_A = 0.25 \text{ m/s}$$

B)  $A_1 V_1 = A_2 V_2$

$$(4)(0.25) = (2) V_B$$

$$V_B = 0.5 \text{ m/s}$$

C) From Bernoulli's equation

$$P_A + \frac{\rho V_A^2}{2} = P_B + \frac{\rho V_B^2}{2}$$

$$P_A + \frac{(1000)(0.25)^2}{2} = P_B + \frac{(1000)(0.5)^2}{2}$$

$$P_A - P_B = 94 \text{ N/m}^2$$

### Question 29

Supposed the tube in the previous problem is kept vertical with A upward but the other conditions remain the same. The separation between the cross-section at A and B is 15/16 cm. Repeat parts (a), (b) and (c) of the previous problem. Take  $g = 10 \text{ m/s}^2$ .

### Solution 29

(a)  $V_A = 0.25 \text{ m/s}$

(b)  $V_B = 0.5 \text{ m/s}$

(c)  $P_A + \frac{\rho V_A^2}{2} + \rho gh_A = P_B + \frac{\rho V_B^2}{2} + \rho gh_B$

$$P_A + \frac{(1000)(0.25)^2}{2} = P_B + \frac{(1000)(0.5)^2}{2} + 1000 \times 10 \times (h_B - h_A)$$

[here,  $h_B - h_A = \frac{15}{16} \text{ cm}$ ]

$$P_A - P_B = 0 \text{ N/m}^2$$

### Question 30

Suppose the tube in the previous problem is kept vertical with B upward. Water enters through B at the rate of  $1 \text{ cm}^3/\text{s}$ . Repeat parts (a), (b) and (c). Note that the speed decreases as the water falls down.

### Solution 30

(a)  $V_A = 0.25 \text{ m/s}$

(b)  $V_B = 0.5 \text{ m/s}$

(c)  $P_A + \frac{\rho V_A^2}{2} + \rho gh_A = P_B + \frac{\rho V_B^2}{2} + \rho gh_B$

$$P_A + \frac{(1000)(0.25)^2}{2} = P_B + \frac{(1000)(0.5)^2}{2} + 1000 \times 10 \times (h_B - h_A)$$

[here,  $h_B - h_A = \frac{16}{16} \text{ cm}$ ]

$$P_A - P_B = 188 \text{ N/m}^2$$

### Question 31

Water flows through a tube shown in figure (13-E8). The areas of cross-section at A and B are  $1 \text{ cm}^2$  and  $0.5 \text{ cm}^2$  respectively. The height difference between A and B is 5 cm. If the speed of water at A is  $10 \text{ cm/s}$  find (a) the speed at B (b) the difference in pressures at A and B.



Figure 13-E8

### Solution 31

(a)  $A_a V_a = A_b V_b$

$(1)(10) = (0.5)(V_b)$

$V_b = 20 \text{ cm/s}$

$$(b) P_A + \frac{\rho V_A^2}{(1)(10)^2} + \rho gh_A = P_B + \frac{\rho V_B^2}{(1)(0.2)^2} + \rho gh_B$$

$$P_A + \frac{\rho V_A^2}{100} + 1 \times 1000 \times h_A = P_B + \frac{\rho V_B^2}{0.04} + 1 \times 1000 \times h_B$$

[here,  $h_B - h_A = 5 \text{ cm}$ ]  
 $P_B - P_A = 4850 \text{ dyne/cm}^2$   
 $P_B - P_A = 485 \text{ N/m}^2$

### Question 32

Water flows through a horizontal tube as shown in figure (13-E9). If the difference of heights of water column in the vertical tubes is 2 cm, and the areas of cross-section at A and B are  $4 \text{ cm}^2$  and  $2 \text{ cm}^2$  respectively, find the rate of flow of water across any section.

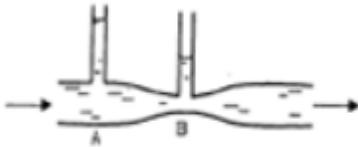


Figure 13-E9

### Solution 32

$$A_a V_a = A_b V_b$$

$$4V_a = 2V_b$$

$$2V_a = V_b \text{ ---- (1)}$$

$$P_A + \frac{\rho V_A^2}{(1)V_A^2} + \rho gh_A = P_B + \frac{\rho V_B^2}{(1)(2V_A)^2} + \rho gh_B$$

$$P_A + \frac{\rho V_A^2}{3V_A^2} = P_B + \frac{\rho V_A^2}{4} \text{ [here, } h_A = h_B = 0 \text{ cm]}$$

$$P_A - P_B = \frac{3\rho V_A^2}{4}$$

$$2 \times 1 \times 1000 = \frac{3\rho V_A^2}{4}$$

$$V_A = 36.5 \text{ cm/sec}$$

$$\therefore \text{Rate of discharge} = V_A A_a$$

$$= (36.5)(4)$$

$$Q = 146 \text{ cm}^3/\text{sec}$$

### Question 33

Water flows through the tube shown in figure(13-E10). The areas of cross-section of the wide and the narrow portions of the tube are  $5 \text{ cm}^2$  and  $2 \text{ cm}^2$  respectively. The rate of flow of water through the tube is  $500 \text{ cm}^3/\text{s}$ . Find the difference of mercury levels in the U-tube.



Figure 13-E10

### Solution 33

$$Q = A_a V_a = A_b V_b$$

$$500 = 5(V_a) = 2(V_b)$$

$$V_a = 100 \text{ cm/sec}$$

$$V_b = 250 \text{ cm/sec}$$

$$P_A + \frac{\rho V_A^2}{2} = P_B + \frac{\rho V_B^2}{2}$$

$$P_A - P_B = \frac{\rho}{2}(V_B^2 - V_A^2)$$

$$980 \times 13.6 \times h = \frac{1}{2} [(250)^2 - (100)^2]$$

$$h = 1.97 \text{ cm}$$

### Question 34

Water leaks out from an open tank through a hole of area  $2 \text{ mm}^2$  in the bottom. Suppose water is filled up to a height of  $80 \text{ cm}$  and the area of cross-section of the tank is  $0.4 \text{ m}^2$ . The pressure at the open surface and at the hole are equal to the atmospheric pressure. Neglect the small velocity of the water near the open surface in the tank. (a) Find the initial speed of water coming out of the hole. (b) Find the speed of water coming out when half of water has leaked out. (c) Find the volume of water leaked out during a time interval  $dt$  after the height remained is  $h$ . Thus find the decrease in height  $dh$  in terms of  $h$  and  $dt$ . (d) From the result of part (c) find the time required for half of the water to leak out.

### Solution 34

$$(a) V = \sqrt{2gh}$$

$$= \sqrt{2 \times 10 \times 0.8}$$

$$V = 4 \text{ m/s}$$

$$(b) V = \sqrt{2gh'}$$

$$= \sqrt{2 \times 10 \times 0.4}$$

$$V = \sqrt{8} \text{ m/s}$$

$$(c) Q = \frac{dV}{dt} = av$$

$$dV = (2 \text{ mm}^2)(\sqrt{2gh})dt$$

$$(d) dV = -Adh = a(\sqrt{2gh})dt$$

$$\int_{0.8}^{0.4} \frac{dh}{\sqrt{2gh}} = \frac{a}{A} \int_0^t dt$$

$$t = 6.5 \text{ hrs}$$

### Question 35

Water level is maintained in a cylindrical vessel upto a fixed height  $H$ . The vessel is kept on a horizontal plane. At what height above the bottom should a hole be made in the vessel so that the water stream coming out of the hole strikes the horizontal plane at the greatest distance from the vessel. (figure 13-11)

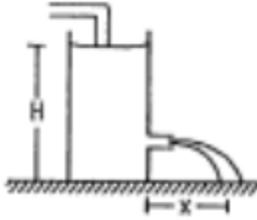


Figure 13.11

Solution 35

Let  $h$  be the height of hole from bottom of tank

$$v = \sqrt{2g(H-h)}$$

$$T = \sqrt{\frac{2h}{g}}$$

$$\therefore R = vT$$

$$= \sqrt{4(Hh-h^2)}$$

To maximize Range,

$$\frac{d}{dh}(Hh-h^2) = 0$$

$$H-2h = 0$$

$$h = \frac{H}{2}$$

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