Test Date: 05/08/2020

A
CODE

Aakash
Medical | IIT-JEE | Foundations
(Divisions of Aakash Educational Services Limited)

Regd. Office: Aakash Tower, 8, Pusa Road, New Delhi-110005, Ph.011-47623456

Mock Test
for JEE (Advanced) - 2020
Test – 4A (Paper - I)

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(B, C)	19.	(A, C, D)	37.	(A, D)
2.	(C, D)	20.	(A, B, C)	38.	(A, C, D)
3.	(A, B, C, D)	21.	(A, B, C, D)	39.	(A, B)
4.	(B, C)	22.	(A, B, C)	40.	(B, D)
5.	(A, B, C)	23.	(A, D)	41.	(A, B)
6.	(B, D)	24.	(A, B, C, D)	42.	(A, C, D)
7.	(60)	25.	(04)	43.	(06)
8.	(20)	26.	(04)	44.	(06)
9.	(22)	27.	(64)	45.	(02)
10.	(15)	28.	(12)	46.	(00)
11.	(20)	29.	(13)	47.	(07)
12.	(15)	30.	(25)	48.	(38)
13.	(14)	31.	(64)	49.	(03)
14.	(08)	32.	(35)	50.	(03)
15.	(B)	33.	(D)	51.	(C)
16.	(D)	34.	(B)	52.	(C)
17.	(A)	35.	(A)	53.	(C)
18.	(B)	36.	(B)	54.	(C)

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ANSWERS & SOLUTIONS

PART – I : PHYSICS

1. Answer (B, C)

$$dU = Q - \frac{Q}{2} = \frac{Q}{2} = dW$$

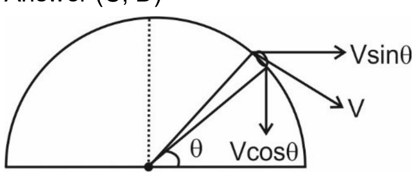
$$\Rightarrow n\left(\frac{3R}{2}\right)dT = PdV$$

$$\Rightarrow \frac{3}{2}(PdV + VdP) = PdV$$

$$\Rightarrow PV^{\frac{1}{3}} = \text{const} \Rightarrow P\sqrt{T} = \text{ constant}$$

$$\therefore\ C = \frac{3R}{2} - \frac{R}{\frac{1}{3}-1} = 3R$$

2. Answer (C, D)

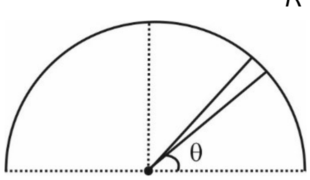


$$\therefore \int \overrightarrow{dP} = \int_{\theta=0}^{\pi} (dm)\left(V\sin\theta\hat{i} - V\cos\theta\hat{j}\right)$$

$$\Rightarrow \vec{P} = \int_0^{\pi} \frac{m}{\pi R} \times Rd\theta \times V\sin\theta\hat{i} + 0$$

$$\vec{P} = \frac{2mV}{\pi}\hat{i} = m\vec{V}_{cm}$$

Centripetal force = $(dm)\dfrac{V^2}{R}$



$$\therefore F_y = \frac{m}{\pi R}(Rd\theta) \times \frac{V^2}{R}\sin\theta$$

$$= \frac{mV^2}{\pi R} \times \int_0^{\pi} \sin\theta d\theta$$

$$= \frac{2mV^2}{\pi R}$$

3. Answer (A, B, C, D)

$$Q_B(t=0) = 4\pi\epsilon_0(3a) \times V = 12\pi\epsilon_0 aV$$

$$Q_A(t \to \infty) = \frac{4\pi\epsilon_0 a}{4\pi\epsilon_0 a + 12\pi\epsilon_0 a} \times (12\pi\epsilon_0 aV) = 3\pi\epsilon_0 aV$$

$$\Delta H = \frac{1}{2} \times \left(\frac{3}{4} \times 4\pi\epsilon_0 a\right) \times V^2 = \frac{3\pi\epsilon_0 aV^2}{2}$$

$$\left(\because \Delta H = \frac{1}{2}\frac{C_1C_2}{C_1 + C^2} \times V^2\right)$$

$$\sigma_B = \frac{Q_B}{4\pi(3a)^2} = \frac{9\pi\epsilon_0 aV}{4\pi 9a^2} = \frac{\epsilon_0 V}{4a}$$

4. Answer (B, C)

$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \left(\frac{9+7}{9-7}\right)^2 = 64 \quad \text{and}$$

$$\nu_1 = \frac{440}{2\pi} = 70\text{ Hz}$$

$$\nu_2 = \frac{484}{2\pi} = 77\text{ Hz}$$

$\therefore$ Beats = 77 – 70 = 7 Hz

5. Answer (A, B, C)

i = 0.5 × 10^{-3} c/s = n × 1.6 × 10^{-19}

$\Rightarrow$ n = 3.1 × 10^{15}

$\therefore$ P = 3.1 × 10^{15} × 1.6 × 10^{-19} × 10^4 = 5 J/s

$$\lambda_{\min} = \frac{1242}{10^4}\text{nM} = 1.24\text{Å}$$

6. Answer (B, D)

$$\frac{100-T_c}{R/2}=\frac{T_c-0}{R/2}+\frac{T_c-25}{R}$$

$\Rightarrow T_c = 45°C$

$$i_{CB}=\frac{45}{5}=9W;\ i_{CD}=\frac{45-25}{5}=4W$$

7. Answer (60)

One-fourth of total flux originated from A terminates at B

$$\therefore 2\pi(1-\cos\theta)=\frac{1}{4}\times(4\pi)$$

$$\Rightarrow 1-\cos\theta=\frac{1}{2}$$

$$\Rightarrow \cos\theta=\frac{1}{2}$$

$\Rightarrow \theta = 60°$

8. Answer (20)

$$V=\frac{40\times 20}{(40-20)}=40\text{ cm}$$

$\therefore$ object distance for mirror = 40 – 20 = 20 cm

$\therefore$ Image distance from mirror = 20 cm

$\because$ mirror is rotated by 30° clockwise image would rotate by 60°

$\therefore$ Distance OP = 20 cm

9. Answer (22)

$$\phi_1=\frac{\mu_0 Ia}{2\pi}\ln 2$$

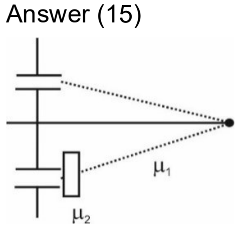
$$\phi_2=-\frac{\mu_0 Ia}{2\pi}\left(\ln\frac{3}{2}\right)$$

$$\therefore \Delta\phi=\frac{\mu_0 Ia}{2\pi}\ln 3$$

$$=\frac{2\times 10^{-7}\times 10\times 0.1}{10^{-2}}\times 1.1$$

$= 22 \times 10^{-6}$C

10. Answer (15)



$$\Delta x=(\mu_2-\mu_1)\times t$$

$$\Rightarrow 5\times\left(\frac{\lambda_0}{2}\right)=\left(\frac{3}{2}-\frac{4}{3}\right)\times t$$

$$\Rightarrow 5\frac{\lambda_0}{2}=\frac{1}{6}\times t$$

$\Rightarrow t = 15\lambda_0$

11. Answer (20)

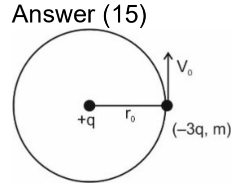
$$V=\frac{15\times 10}{(15-10)}=30\text{ cm}\Rightarrow m=\frac{30}{15}=2$$

$$\therefore V_x=-2^2\times 4\hat{i}+4\hat{i}=-12\hat{i}$$

$$V_y=-2\times(11-2)\hat{j}+2\hat{j}=-16\hat{j}$$

$$\therefore V=\sqrt{V_x^2+V_y^2}=20\text{ m/s}$$

12. Answer (15)



$$\therefore \frac{mV_0^2}{r_0}=\frac{3q^2}{4\pi\epsilon_0 r_0^2}\text{ and } mv_0r_0=n\frac{h}{2\pi}=\frac{h}{2\pi}$$

$$\Rightarrow mV_0^2\times\frac{h}{2\pi mV_0}=\frac{3q^2}{4\pi\epsilon_0}$$

$$\Rightarrow V_0=\frac{3q^2}{2\epsilon_0 h}$$

$\Rightarrow n = 15$

13. Answer (14)

Using projection concept

$$F=\int_2^4\frac{\mu_0 I_1}{2\pi r}\times I_2 dr=\frac{\mu_0}{2\pi}\times I_1 I_2\ln(2)$$

$= 2 \times 10^{-7} \times 5 \times 20 \times (0.7)$

$= 14 \times 10^{-6}$ N

14. Answer (08)

$$W_1=32P_0\times(2V_0-V_0)=32P_0V_0$$

$$W_2=\frac{(32P_0\times 2V_0)-P_0\times 16V_0}{\left(\frac{5}{3}-1\right)}=72P_0V_0$$

$$W_3=P_0\times(32V_0-16V_0)=16P_0V_0$$

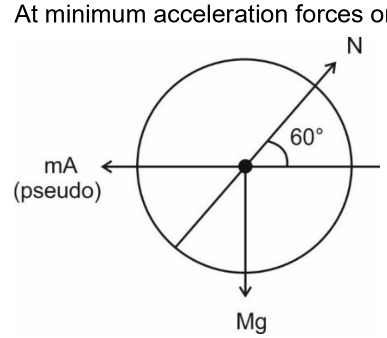
$$W_4=32P_0V_0\ln\left(\frac{1}{3^2}\right)=-160P_0V_0\ln 2$$

$= –112\ P_0V_0$

$$\therefore \Delta Q=\sum W=(32+72+16-112)P_0V_0=8P_0V_0$$

15. Answer (B)

At minimum acceleration forces on top cylinder



$\therefore N \times \frac{\sqrt{3}}{2} = \text{Mg and N} \times \frac{1}{2} = \text{MA}$

$\Rightarrow \sqrt{3} = \frac{g}{A} \Rightarrow A = \frac{g}{\sqrt{3}}$

$\therefore F_{\text{min}} = (3\text{M}) \times \left(\frac{g}{\sqrt{3}}\right) = \sqrt{3}\text{Mg}$

16. Answer (D)

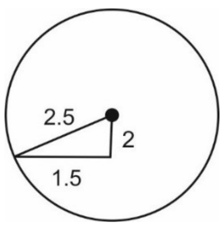
$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + A^2}}}$

$\because A_{\text{min}} = \frac{g}{\sqrt{3}}$

$\therefore T_{\text{max}} = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{g^2}{3}}}}$

$= 2\pi\sqrt{\frac{\sqrt{3}\,l}{2g}}$

17. Answer (A)



$\omega = \frac{3}{\left(\frac{2}{100}\right)} = 150$

After collision

$\therefore L = I\omega - mV_c r_\perp$

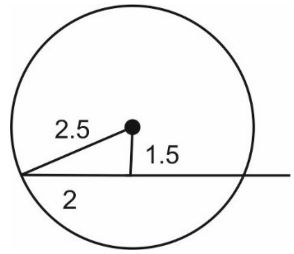
$= \frac{2}{5} m \times \left(\frac{2.5}{100}\right)^2 \times 150 - m \times 3 \times \left(\frac{2}{100}\right)$

= – 0.0225 × m

$\because$ L is negative, the ball does not return after collision

$\therefore$ No. of collision = 1

18. Answer (B)



$\omega = \frac{3}{\left(\frac{1.5}{100}\right)} = 200$ rad/s

After collision

$L = I\omega - mvr_\perp$

$= \left(\frac{2}{5}mr^2\right)\omega - mVcr_\perp$

$= \frac{2}{5} m \times \left(\frac{2.5}{100}\right)^2 \times 200 - m \times 3 \times \frac{1.5}{100}$

= 0.005 × m

$\because$It is positive, there would be second collision

$L_i = 0.005 \times m$

$L_f = \left[\frac{2}{5} m \times \left(\frac{2.5}{100}\right)^2 + m \times \left(\frac{1.5}{100}\right)^2\right] \times \frac{V_0^1}{\left(\frac{1.5}{100}\right)}$

$\because L_i = L_f$

$\Rightarrow V_c^1 = 0.16$ m/s

$\therefore$ Velocity of approach = 0.32 m/s

PART – II : CHEMISTRY

19. Answer (A, C, D)

In case of Phenolphthalein, partial neutralisation of Na_2CO_3 occurs.

Number of moles of HCl = no. of moles of Na_2CO_3

0.05 M × 'x' × 10^{-3} (L) = 0.05 × 40 × 10^{-3}

$\therefore$ x = 40 ml

In case of Methyl orange, end point is at the complete neutralisation of Na_2CO_3 & $NaHCO_3$ occurs

$\therefore$ No. of equivalent of HCl = (No. of equivalent of Na_2Co_3) + (No. of equivalent of $NaHCO_3$)

1 × 0.05 × y = (2 × 0.05 × 40 × 10^{-3}) + (1 × 0.05 × 40 × 10^{-3})

x = 40 ml & y = 120 ml

in option (A) : y – x = 80 ml $\Rightarrow$ 120 – 40 = 80

$\therefore$ option A is correct

In option (B) : x + y = 80 but x = 40 & y = 120

$\therefore$ x + y = 160

therefore option B is incorrect

If the titration is started with phenolphthalein indicator then in solution

no of moles of $NaHCO_3$ = 2 × n_{NaHCO_3} (due to partial neutralisation of Na_2CO_3)

$\therefore$ no. of equivalent of HCl = no. of eq. of $NaHCO_3$

$1 \times V \times 0.05 = 1 \times 2 \times 0.05 \times 40 \times 10^{-3}$

$\therefore$ V_{HCl} = 80 ml

$V_{HCl} = 2x \Rightarrow 2 \times 40 = 80$ ml

$\therefore$ Option C is correct

When some solution is titrated with 0.1 M NaOH no. of eq. of NaOH = no. of eq. of $NaHCO_3$

$1 \times 0.1 \times V = 1 \times 0.05 \times 40 \times 10^{-3}$

$V = 20 \times 10^{-3}$ L $\Rightarrow$ 20 ml

We know that x = 40 ml

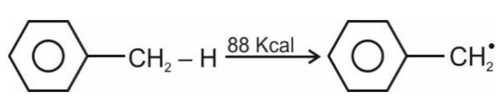
$\therefore \frac{x}{2} =$ 20 ml . Thus option D is correct

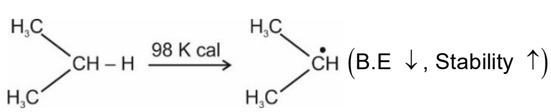
20. Answer (A, B, C)

* The greater the difference between activation energies, the larger the selectivity. On going from activation energy difference of 1 Kcal (chlorination) to about 3 Kcal (bromination) can mean the difference between a reaction with a selectivity of 3.5 : 1 for chlorination & 97 : 1 for bromination

Reactivity order: $F_2 > Cl_2 > Br_2 > I_2$

E_a order: $F_2 < Cl_2 < Br_2 < I_2$

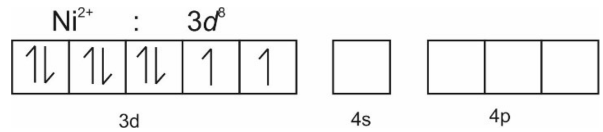




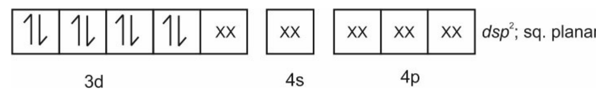
$H_2C = CH - \dot{C}H_2 > H_2C = \dot{C}H$

(Due to resonance stabilisation)

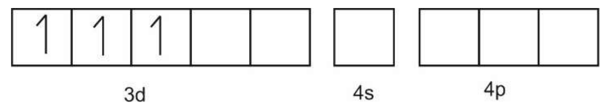
21. Answer (A, B, C, D)



$[Ni(CN)_4]^{2-}$



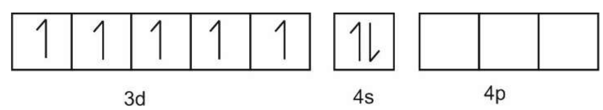
Cr(24): [Ar] $3d^5$ $4s^1$: Cr^{+3}: $3d^3$



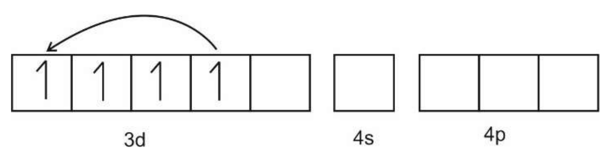
Already two 3*d* orbitals are vacant to form inner orbital d-complex & always have unpaired e^- so octahedral complexes are always paramagnetic.

In Tetrahedral complexes of Zn^{+2}, d-Orbital is completely filled so all complexes are diamagnetic

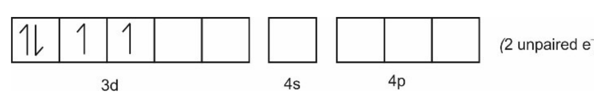
Mn(25):



Mn^{+3} :



In d^2sp^3 hybridisation:



22. Answer (A, B, C)

e^- density is more on N atom in $(C_2H_5)_2NH$ than $C_2H_5NH_2$ because more +I effect. So $(C_2H_5)_2NH$ more basic than $C_2H_5NH_2$. $C_6H_5NH_2$ & $C_6H_5NHCH_3$ less basic than $(C_2H_5)_2NH$ & $C_2H_5NH_2$ due to delocalisation of e^- pair

Hence the order of basicity of given compounds are:

$C_6H_5NH_2 < C_6H_5NHCH_3 > C_2H_5NH_2 > (C_2H_5)_2NH$ and higher the basic strength, the lower is the Pkb value therefore: for Pkb

$C_6H_5NH_2 > C_6H_5NHCH_3 > C_2H_5NH_2 > (C_2H_5)_2NH$

In the gaseous phase there is no effect of solvation so basic strength maily depends upon the +I effect, Higher the +I effect, Stronger is the base. Therefore:

$(C_2H_5)_3N$ > $(C_2H_5)_2NH$ > $C_2H_5NH_2$ > NH_3 so option B is correct

More H-bonding, more B. Pt

$\therefore$ B.Pt of $C_2H_5NH_2$ > $(C_2H_5)_2NH$, further oxygen is more E.N than Nitrogen thus C_2H_5OH forms stronger H-bond than $C_2H_5NH_2$ therefore correct of B.Pt is

$(CH_3)_2NH$ < $C_2H_5NH_2$ < C_2H_5OH

so option C is correct

More H-bonding, more solubility $\therefore$ Solubility of $C_2H_5NH_2$ > $(C_2H_5)_2NH$ further solubility of amine decreases with inverse in molecular mass because hydrophobic part increases thus statement D is correct

23. Answer (A, D)

The order with t-alkyl the best migrating sec. alkyl closely followed by phenyl then ethyl then methyl, very roughly follows the order in which the group are able to stabilise a positive charge. Primary groups are much more reluctant to undergo migration than sec. ones or alkyl groups and this make regioselective Baeyer-Villiger reaction possible

24. Answer (A, B, C, D)

At eqm. moles of CO_3^{-2} ions formed

= 0.1520 – 0.0358 $\Rightarrow$ 0.1162

After Reaction conc. of $K_2C_2O_4$ = $\dfrac{0.1162}{0.5} = 0.2324$ M

Option (D) correct

at eqm $\left[CO_3^{-2}\right] = \dfrac{0.0358}{0.5} \Rightarrow 0.0716$ M

$\therefore$ Option (B) correct

For $Ag_2C_2O_4$, $K_{SP} = [Ag^+]^2\left[C_2O_4^{-2}\right]$

$\Rightarrow 1.29 \times 10^{-11} = [Ag^+]^2 \times 0.2324$

$$\therefore \left[Ag^+\right] = \sqrt{\frac{1.29\times10^{-11}}{0.2324}} \Rightarrow 7.45\times10^{-6}$$

Option (C) correct

$$K_{SP}\left(Ag_2CO_3\right) = \left[Ag^+\right]^2\left[CO_3^{-2}\right]$$

$$\Rightarrow \frac{1.29\times10^{-11}}{0.2324}\times0.0716$$

$$\Rightarrow 3.97\times10^{-12}\ \text{mol}^3\,L^{-3}$$

$\therefore$ Option (A) correct

25. Answer (04)

Cations soluble in excess NH_3 solution are :

$Ni^{+2}(aq.), Cd^{+2}(aq.),\ Zn^{2+}(aq.),\ Ag^+(aq.)$

26. Answer (04)

E_{inc} = work function (ϕ) + K.E

4.9 = 3.4 + K.E $\Rightarrow$ K.E = 1.5 eV

$$\& \ \lambda = \left(\frac{150}{K.E}\right)^{\frac{1}{2}} \ \therefore \lambda = \sqrt{\frac{150}{1.5}} \Rightarrow 10\text{Å}$$

We know that $\Delta P.\Delta x = \dfrac{h}{4\pi}$

$$\frac{h}{\lambda^2}.\Delta\lambda\times\Delta x = \frac{h}{4\pi}$$

$$\text{or} \ \frac{\Delta\lambda}{\left(10\times10^{-10}\right)^2}\times\frac{25}{4\pi}\times10^{-10} = \frac{1}{4\pi}$$

$$\therefore \Delta\lambda = 4\times10^{-10}\ \text{m} = 4\text{Å}$$

27. Answer (64)

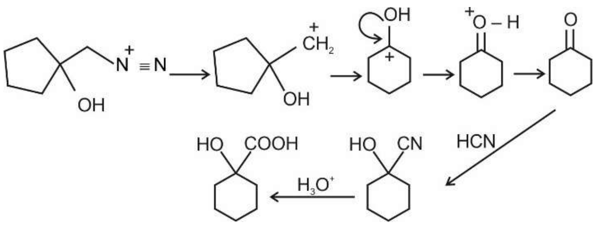
Process AC is polytropic : (P = KV) & molar heat capacity (Cm) = C_v + $\dfrac{R}{2} \Rightarrow 2R$

Process AB is isobaric process $\therefore$ $c_m = c_p = \dfrac{5}{2}$ R

$$\frac{q_{(AC)}}{q_{(AB)}} = \frac{\int_{T_A}^{T_C} n.C_m.\Delta T}{\int_{T_A}^{T_B} n.C_{p.\,m}.\Delta T} \Rightarrow \frac{2R}{\frac{5}{2}R} \Rightarrow 0.8$$

$$\therefore \left(\frac{q_{(AC)}}{q_{(AB)}}\right)^2\times100 = 64$$

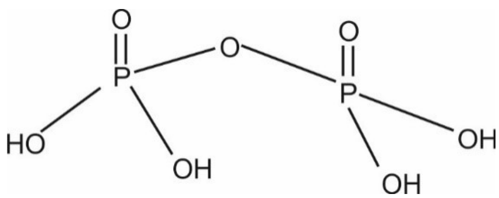
28. Answer (12)



% of carbon = $\frac{84}{144}\times 100 = 58.33$

$\therefore \frac{700}{x} = 58.33$ thus x = $\frac{700}{58.33} \Rightarrow 12$

29. Answer (13)



A = 1

B = 4

C = 2

$\therefore B^2 - (A + C)$

$= (4)^2 - (1 + 2)$

= 13

30. Answer (25)

$\Delta T_b = m \times K_b$

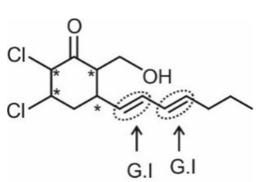
$\Rightarrow \frac{\text{effective no. of moles of } (Na_3PO_4 + MgSO_4)\times 1000}{\text{mass of solvent (in gm)}} \times K_b$

$$= \frac{\left(\frac{8.2}{164}\times 2.5\right)+\left(\frac{12}{120}\times 1.6\right)}{79.8}\times 1000 \times 0.50$$

$\Delta T_b = 1.785°C$

14 × ΔTb = 14 × 1.785 = 25

31. Answer (64)



$2^n \Rightarrow 2^6 \Rightarrow 64$

32. Answer (35)

$$A(aq) \longrightarrow B(aq) + C(aq)$$

Initial con. (M) a

Conc. At time t a – x x x

Specific rotations of A, B and C are given as

R_A = +40°, R_B = – 60° and Rc = +50°

Angle of rotations of reaction mix at (t = 0)

$R_0 = aR_A = +40° \Rightarrow a = 1$ M

At time t = 46.06 min

$R_t = (a – x) R_A + xR_B + xR_C = 0°$ (given)

$40° + (R_B + R_C – R_A)x = 0$

(–60 + 50 – 40)x = –40; x = 0.8

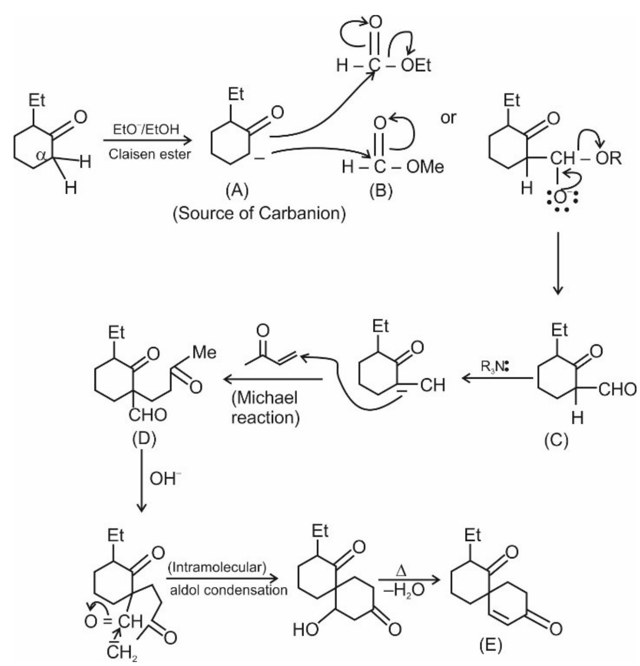
Rate constant, K = $\frac{2.303}{46.06}\log\frac{1}{1-0.8}$

$= \frac{2.303\times 0.7}{46.06} = 0.035$

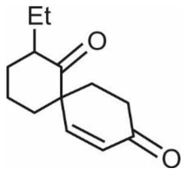
$= 35 \times 10^{-3}\ min^{-1} = Y \times 10^{-3}\ min^{-1}$

$\therefore$ Y = 35

33. Answer (D)



34. Answer (B)



35. Answer (A)

Z_{eff} = corners + body centre $\Rightarrow \left(8\times\frac{1}{8}+1\right)\Rightarrow 2$ atoms/unit cell

Atomic weight of Mg = 24.3 g

$\therefore$ Mass = $\frac{2\times 24.3}{6\times 10^{23}}\Rightarrow 8.07\times 10^{-23}$ gm

Volume = $\frac{m}{d}=\frac{8.07\times 10^{-23}}{1.74\text{ g cm}^{-3}}\times\left(10^{8}\text{ Å}\right)^{3}=46.4\text{Å}^{3}$

* Base Area from figure (B)

$\sin 60^\circ=\frac{x}{a}\Rightarrow x=a\sin 60^\circ$

$\Rightarrow$ 0.866 a $\therefore$ Area = ax $\Rightarrow$ 0.866 a^2

Volume = Area × height $\Rightarrow$ 0.866 a^2 × (1.633 a)

= 1.41 a^3 = 46.4 $Å^3$

$\Rightarrow$ a = 3.2 Å

36. Answer (B)

* nearest neighbour's are along the base edge at 3.2 Å

* 12 neighbour atoms, 6 atoms in the same plane, 3 above and 3 atoms below

PART – III : MATHEMATICS

37. Answer (A, D)

Circles with points $P\left(2t_1,\frac{2}{t_1}\right)$ and $Q\left(2t_2,\frac{2}{t_2}\right)$ as diameter given by

$(x-2t_1)(x-2t_2)+\left(y-\frac{2}{t_1}\right)\left(y-\frac{2}{t_2}\right)=1$ …(i)

Given that slope of PQ = 1

$-\frac{1}{t_1t_2}=1\rightarrow t_1t_2=-1$

(i) $\rightarrow(x^2+y^2-8)+(t_1+t_2)(x-y)=0$

This is in the form 5 + λL = 0. The circles passes through points of intersection of

$x^2+y^2=8$, $x-y=0$

i.e. (2, 2), (–2, – 2)

38. Answer (A, C, D)

$\cos A+\cos C=4\sin^2\frac{B}{2}$

$2\cos\frac{A+C}{2}\cos\frac{A-C}{2}=4\sin^2\frac{B}{2}$

$\cos\frac{A-C}{2}=2\sin\frac{B}{2}$

$2\cos\frac{B}{2}\cos\frac{A-C}{2}=2\sin B$

$\sin A+\sin C=2\sin B\Rightarrow$ Option (D)

a, *b*, *c* are in AP

we know $\cos A+\cos B+\cos C=1+\frac{r}{R}$

$\therefore 4\sin^2\frac{B}{2}+\cos B=1+\frac{r}{R}$

$2(1-\cos B)+\cos B=1+\frac{r}{R}$

$\cos B=1-\frac{r}{R}\Rightarrow$ Option (C)

$\cos A+1-\frac{r}{R}+\cos C=1+\frac{r}{R}$

$\cos A+\cos C=\frac{2r}{R}\Rightarrow$ Option (A)

39. Answer (A, B)

minimum % of combatant who lost one eye and one leg = 75 + 80 – 100 = 55%

minimum % of combatant who lost one eye, one leg and one arm = 55 + 90 – 100 = 45%

min % of comparts lost all organs

= 45 + 85 – 100 = 30%

40. Answer (B, D)

$\because g(f(x))=x\Rightarrow g'(f(x))=\frac{1}{f'(x)}$

$$\Rightarrow g''(f(x)).f'(x) = -\frac{1}{(f'(x))^2}.f''(x)$$

$$\Rightarrow g''(f(x)) = -\frac{f''(x)}{(f'(x))^3}$$

$$\Rightarrow g'''(f(x)).f'(x) = \frac{-(f'(x))^3.f'''(x) + f''(x).3(f'(x))^2.f''(x)}{(f'(x))^6}$$

$$\Rightarrow g'''(f(x)) = \frac{3(f''(x))^2 - f'(x).f'''(x)}{(f'(x))^5}$$

41. Answer A, B()

Given equation is $x^3 - y^2 = 0$ …(i)

$P = (4m^2, 8m^3)$

(i) $\Rightarrow 3x^2 - 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{3x^2}{2y}$

at $P(4m^2, 8m^3), \frac{dy}{dx} = \frac{3(16m^4)}{2(8m^3)} = 3m$

equation of tangent at P is

$y - 8m^3 = 3m(x - 4m^2)$

$y = 3mx - 4m^3$ …(ii)

By solving (i) , (ii) we get x = $4m^2$, m^2

Let $x = m^2$

(ii) $\Rightarrow y = 3m^3 - 4m^3 = -m^3$

Let Q = $(m^2, -m^3)$

Slope of the tangent at Q is

$$\frac{dy}{dx} = \frac{3m^4}{2(-m^3)} = \frac{-3}{2}m.$$

Slope of the normal at Q is $\frac{2}{3m}$

Given that slope of the tangent at P is equal to slope of the normal at Q

$$\frac{2}{3m} = 3m \Rightarrow m^2 = \frac{2}{9} \Rightarrow m = \pm\frac{\sqrt{2}}{3}$$

42. Answer (A, C, D)

Let $I = \int \frac{\sin^2 x}{(x - \sin x\cos x)^2}dx$

$$I = \int \frac{\sin^2 x}{\cos^4 x(x\sec^2 x - \tan)}dx = \int \frac{\sec^2 x\tan^2 x}{(x\sec^2 x - \tan x)^2}dx$$

$$\left[\frac{d}{dx}(x\sec^2 x - \tan x) = x(2\sec x)(\sec x\tan x) + \sec^2 x - \sec^2 x\right]$$

$$= 2x\sec^2 x\tan x$$

$$I = \frac{1}{2}\int \frac{2x\sec^2 x\tan x}{(x\sec^2 x - \tan x)^2} \times \frac{\tan x}{x}dx$$

$$I = \frac{1}{2}\left[\frac{\tan x}{x}\int \frac{2x\sec^2 x\tan x}{(x\sec^2 x - \tan x)^2}dx - \int\left[\frac{x\sec^2 x - \tan x}{x^2}\int \frac{2x\sec^2 x - \tan x}{(x\sec^2 x - \tan x)^2}dx\right]dx\right]$$

$$I = \frac{1}{2}\left[\frac{\tan x}{x} \times \frac{-1}{x\sec^2 x - \tan x} - \int\left[\frac{x\sec^2 x - \tan x}{x^2} \times \frac{-1}{x\sec^2 x - \tan x}\right]dx\right]$$

$$I = \frac{1}{2}\left[\frac{\tan x}{x(\tan x - x\sec^2 x)} - \frac{1}{x}\right] + C$$

43. Answer (06)

sinx $\to 0$ as $x \to 0$

Let t = sin$x$

$$L = \lim_{t\to 0}\frac{t + \log\left(\sqrt{1+t^2} - t\right)}{t^3}$$

Apply L-Hospital rule

$$L = \lim_{t\to 0}\frac{1 + \frac{1}{\sqrt{1+t^2} - t}\left(\frac{1}{2\sqrt{1+t^2}}(2t) - 1\right)}{3t^2}$$

$$L = \lim_{t\to 0}\frac{1 + \frac{1}{\sqrt{1+t^2} - t}\left(\frac{t - \sqrt{1+t^2}}{\sqrt{1+t^2}}\right)}{3t^2}$$

$$L = \lim_{t\to 0}\frac{1 - \frac{1}{\sqrt{1+t^2}}}{3t^2} = \lim_{t\to 0}\frac{\sqrt{1+t^2} - 1}{3t^2\sqrt{1+t^2}}$$

$$L = \lim_{t\to 0} \frac{\left(\sqrt{1+t^2}-1\right)}{3t^2\sqrt{1+t^2}\left(\sqrt{1+t^2}+1\right)}$$

$$= \lim_{t\to 0} \frac{1}{3\sqrt{1+t^2}\left(\sqrt{1+t^2}+1\right)}$$

$$L = \frac{1}{3(1)(1+1)} = \frac{1}{6}$$

44. Answer (06)

Given equation is $6\int_1^x f(t)\,dt = 3xf(x) - x^3$

differentiate with respect to x on both sides

$6[f(x) - 0] = 3(x f'(x) + f(x)) - 3x^2$

$2f(x) = xf'(x) + f(x) - x^2$

$xf'(x) - f(x) = x^2$

$\cot y = f(x) \rightarrow \frac{dy}{dx} = f'(x)$

$x\frac{dy}{dx} - y = x^2$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = x,\ P = -\frac{1}{x},\ Q = x$

$I.F. = e^{\int P dx} = e^{-\int \frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$

Solution is y(I.F.) = $\int Q(I.F.)dx + C$

$y\left(\frac{1}{x} = \int x\left(\frac{1}{x}\right)dx + C\right)$

$\frac{y}{x} = x + C$

$y = x^2 + Cx$

$f(x) = x^2 + Cx$

$f(1) = 2 \Rightarrow 2 = 1 + C \Rightarrow C = 1$

$f(x) = x^2 + x$

$f(2) = 4 + 2 = 6$

45. Answer (02)

$$A = \begin{bmatrix} 4 & -1 \\ 15 & -4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & -1 \\ 15 & -4 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 15 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\Rightarrow tr\,(A^{2n}) = 2$ …(i)

$$B = \begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix} = B$$

$\Rightarrow tr\,(B^n) = 1$ …(ii)

$$C = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix}$$

$$C^3 = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow tr\,(C^{3n}) = 0$ …(iii)

Now from (i), (ii), (iii)

$$P_n = k^n \begin{bmatrix} 2 & 1 & tr\left(C^{2n}\right) \\ tr\left(A^n\right) & 1 & tr\left(C^n\right) \\ tr\left(A^{3n}\right) & 1 & 0 \end{bmatrix}$$

$\Rightarrow tr\,(P_n) = k^n \times 3$

$$\sum_{n=1}^{\infty} tr\left(P_n\right) = 1$$

$\Rightarrow 3(k + k^2 + k^3 + \ldots..\infty) = 1$

$\Rightarrow \frac{3k}{1-k} = 1$

$\Rightarrow 3k = 1 - k$

$\Rightarrow 4k = 1$

$\Rightarrow k = \frac{1}{4}$

Now $\left(\frac{1}{k}\right)^{\sqrt{k}} = (4)^{1/2} = 2$

46. Answer (00)

Consider $a = x + 1$

$b = x - 1$

$c = (-2x)$

$a + b + c = x + 1 + x - 1 - 2x = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

Now from above

$(x + 1)^3 + (x - 1)^3 - 8x^3$

$= 3((x + 1)(x - 1)(-2x))$

$= 6(x(1 + x)(1 - x))$

$= 6(x(1 - x^2))$

$$\Rightarrow I(x) = \int \frac{x^2(1-x^2)^2(1-3x^2)dx}{1+\left(6x(1-x^2)\right)^3}$$

$$\Rightarrow I(x) = \int \frac{x^2(1-x^2)^2(1-3x^2)dx}{1+6^3(x-x^3)^3}$$

Let $6^3(x - x^3)^3 = t$

$\Rightarrow 6^3 \times 3(x - x^3)^2(1 - 3x^2)dx = dt$

$\Rightarrow 3 \times 6^3 x^2(1 - x^2)^2(1 - 3x^2)\,dx = dt$

$$\Rightarrow I(x) = \frac{1}{3\times 6^3}\int \frac{dt}{1+t}$$

$$= \frac{1}{3\times 6^3}\ln\left(1+6^3(x-x^3)^3\right)+C$$

Now $I(0) = 0$

$$\frac{1}{3\times 6^3}\ln 1 + C = 0$$

$\Rightarrow C = 0$

Now $I(1) = \frac{1}{3\times 6^3}\ln 1 + 0 \quad [\because C = 0]$

$\Rightarrow I(1) = 0$

47. Answer (07)

$$\frac{dy}{dx} = y\frac{(1-x)}{e^x} - \frac{y}{x}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1-x}{e^x} - \frac{1}{x}$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = e^{-x} - xe^{-x}$$

$$\Rightarrow \frac{dx}{x} + \frac{1}{y}dy = \left(e^{-x} - xe^{-x}\right)dx$$

On integrating

$$\Rightarrow \ln x + \ln y = \int e^{-x}dx - \int xe^{-x}dx$$

$$\Rightarrow \ln(xy) = xe^{-x} + \int xe^{-x}dx - \int xe^{-x}dx$$

$$\Rightarrow \ln(xy) = xe^{-x} + C \qquad \ldots(i)$$

$\left(1, e^{1/e}\right) \rightarrow (i)$

$$\Rightarrow \ln\left(1\times e^{1/e}\right) = 1\times e^{-1} + C$$

$$\Rightarrow \frac{1}{e}\ln e = \frac{1}{e} + C$$

$\Rightarrow C = 0$

$\Rightarrow \ln(xy) = xe^{-x}$

Now for $x = 2$

$\ln 2y = 2e^{-2}$

$$\Rightarrow \ln(2y) = \frac{2}{e^2}$$

$$\Rightarrow y = \frac{1}{2}\left(e^{2/e^2}\right)$$

On comparing with

$y = ae^b$

$$\Rightarrow a = \frac{1}{2}, b = \frac{2}{e^2}$$

48. Answer (38)

$p = {}^{22}C_{18} \times 2 \times 2 \times 2 \times 2$

$q = {}^{22}C_{19} \times 2 \times 2 \times 2$

$$\frac{p}{q} = \frac{\frac{22\times 21\times 20\times 19}{4\times 3\times 2\times 1}}{\frac{22\times 21\times 20}{3\times 2}}\times 2$$

$$= \frac{19}{2} = 9.5$$

49. Answer (03)

On expanding highest power of x is 7 and its coefficient $a_7 = -1$

$f(0) = a_0$

$$\Rightarrow a_0 = \begin{vmatrix} 1 & -1 & -1 \\ 1 & 0 & 3 \\ 3 & 0 & 2 \end{vmatrix}$$

$\Rightarrow a_0 = -7$

And $f'(0) = a_1$

$$\Rightarrow \text{ and } f'(x) = \begin{vmatrix} 2x & 2x-1 & x^3-1 \\ 2 & x^2 & x+3 \\ 2x & x^2+x & x^2+2 \end{vmatrix} + \begin{vmatrix} x^2+1 & 2 & x^3-1 \\ 2x+1 & 2x & x+3 \\ x^2+3 & 2x+1 & x^2+2 \end{vmatrix} + \begin{vmatrix} x^2+1 & 2x-1 & 3x^2 \\ 2x+1 & x^2 & 1 \\ x^2+3 & x^2+x & 2x \end{vmatrix}$$

$$f'(0) = \begin{vmatrix} 0 & -1 & -1 \\ 2 & 0 & 3 \\ 0 & 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 2 & -1 \\ 1 & 0 & 3 \\ 3 & 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 3 & 0 & 0 \end{vmatrix}$$

$a_1 = 4 + 10 + (-3)$

$= 11$

$a_0 = -7$, $a_1 = 11$, $a_7 = -1$

$(a_0 + a_1 + a_7) = 3$

50. Answer (03)

Case 1 (x4 – 16 – 12 = 0)

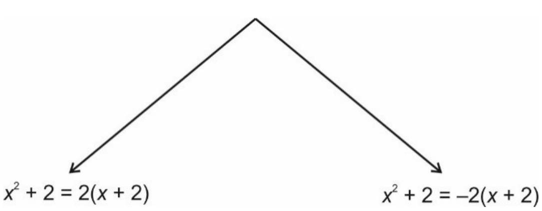
$x^4 = 16x + 12$

$\Rightarrow x^4 + 4x^2 + 4 = 4x^2 + 4 + 16x + 12$

$\Rightarrow (x^2 + 2)- = 4(x^2 + 4x + 4)$

$\Rightarrow (x^2 + 2)^2 = 2^2\,(x + 2)^2$

$x^2 + 2 = \pm\, 2(x + 2)$



$\Rightarrow x^2 - 2x - 2 = 0$ | $\Rightarrow x^2 + 2x + 6 = 0$

$\Rightarrow \because D > 0$ | $\Rightarrow \because D < 0$

$x = \dfrac{2 \pm \sqrt{4+8}}{2}$ | $\because$ No real roots

$x = 1 \pm \sqrt{3}$

$\because a_1 > a_2$

$\Rightarrow a_1 = 1 + \sqrt{3}$ …(i)

$\Rightarrow a_2 = 1 - \sqrt{3}$ …(ii)

Also $a_1a_2 = -2$

Case 2

$x^4 + 16x - 12 = 0$

$\Rightarrow x^4 = -16x + 12$

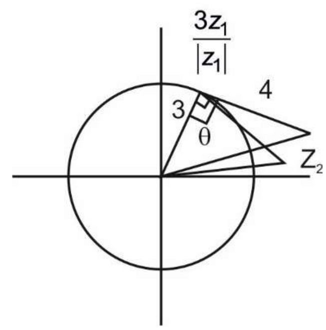
$\Rightarrow x^4 + 4x^2 + 4 = 4x^2 + 4 - 16x + 12$

$\Rightarrow (x^2 + 2)^2 = 4(x^2 - 4x + 4)$

51. Answer (C)

52. Answer (C)

Solution for Q51 and Q52



$\left|\dfrac{3z_1}{|z_1|}\right| = 3 \Rightarrow \dfrac{3z_1}{|z_1|}$ is the point on the circle of radius 3

When $\theta = 60°$

$$\cos 60° = \frac{25 - |z_2|^2}{2 \times 3 \times 4} \Rightarrow 12 = 25 - |z_2|^2$$

$\Rightarrow |z_2| = \sqrt{13}$

When $\theta = \dfrac{\pi}{2}$,

$|z_2| = 5 \Rightarrow \therefore \sqrt{13} \le |z_2| \le 5, \text{when } \dfrac{\pi}{3} \le \theta \le \dfrac{\pi}{2}$

if $\theta = \cos^{-1}\left(\dfrac{2}{3}\right) \Rightarrow \left(\dfrac{2}{3}\right) \Rightarrow |z_2| = 3$

$$\therefore \arg\left(\frac{3z_1}{|z_1|.z_2}\right) = \pi - 2\cos^{-1}\left(\frac{2}{3}\right)$$

$$= 2\left(\frac{\pi}{2} - \cos^{-1}\left(\frac{2}{3}\right)\right) = 2.\sin^{-1}\left(\frac{2}{3}\right)$$

53. Answer (C)

54. Answer (C)

Solution for Q53 and Q54

If exactly one element is zero then number of invertible matrices = 4 × ${}^{4-3+2}C_2$ = 12 and if exactly two elements are zero, then number of invertible matrices = 2 × ${}^{4-2+1}C_1$ = 6

$\therefore$ the number of invertible matrices

M are = 12 + 6= 18

$\because$ $|M|_{max}$ = 4 and $|M|_{min}$ = – 4

$\therefore$ | 4 + 4| = 8

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