

Test Date: 29/07/2020



Aakash

Medical | IIT-JEE | Foundations
(Divisions of Aakash Educational Services Limited)

A
CODE

Regd. Office: Aakash Tower, 8, Pusa Road, New Delhi-110005, Ph.011-47623456

Mock Test
for JEE (Advanced) - 2020
Test - 3A (Paper - I)

ANSWERS

PHYSICS

1. (A, D)
2. (B, C)
3. (A, C)
4. (A, C)
5. (A, C)
6. (B, C)
7. (16)
8. (02)
9. (10)
10. (34)
11. (16)
12. (60)
13. (08)
14. (02)
15. (D)
16. (B)
17. (A)
18. (A)

CHEMISTRY

19. (B, C, D)
20. (A, B, C, D)
21. (A, B, C)
22. (A, B, C)
23. (A, B, C, D)
24. (A, B, C, D)
25. (31)
26. (07)
27. (07)
28. (05)
29. (06)
30. (25)
31. (09)
32. (07)
33. (A)
34. (C)
35. (B)
36. (C)

MATHEMATICS

37. (A, B)
38. (A, D)
39. (A, D)
40. (A, C, D)
41. (A, C)
42. (A, B, C, D)
43. (18)
44. (23)
45. (01)
46. (08)
47. (09)
48. (08)
49. (80)
50. (46)
51. (B)
52. (D)
53. (A)
54. (C)

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ANSWERS & SOLUTIONS

PART – I : PHYSICS

1. Answer (A, D)

$$V_1 \cos 45^\circ = V_2$$

$$\tan \theta = \frac{h}{x}$$

$$x = h \cot \theta$$

$$V_1 = h \cosec^2 \theta \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{V_1}{2h}$$

2. Answer (B, C)

$$\vec{V} = \frac{d\vec{r}}{dt} = (-4\omega \sin 2\omega t \hat{i} + 6\omega \sin 2\omega t \hat{j})$$

$$|\vec{V}| = 5\omega \sin 2\omega t = \left(\frac{5}{2}\right)(2\omega) \sin 2\omega t$$

$$\Rightarrow A = \frac{5}{2}$$

3. Answer (A, C)

$$\int d\phi = \alpha t d \int_{d}^{2d} x dx$$

$$\phi = \frac{3}{2} \alpha t d^3$$

4. Answer (A, C)

$$X_{cm} = \frac{2L}{3}$$

$$I_A = \int_0^L \lambda_0 x^3 dx = \frac{\lambda_0 L^4}{4}$$

$$mg \left(\frac{2l}{3} \right) = \frac{\lambda_0 L^4}{4} \alpha$$

$$mg \left(\frac{2l}{3} \right) = \frac{ml^2}{2} \alpha$$

$$\alpha = \frac{4g}{3l}$$

5. Answer (A, C)

$$\frac{5R}{2} \frac{dT}{dt} = \frac{kA}{d} (2T_0 - T)$$

$$\int_{T_0}^T \frac{dT}{2T_0 - T} = \int_0^t \frac{2KA}{5Rd} dt$$

$$\ln \frac{(2T_0 - T)}{T_0} = -\frac{2kA}{5Rd} t$$

$$T = T_0 \left(2 - e^{-\frac{2kA}{5Rd} t} \right)$$

6. Answer (B, C)

$$J = \frac{3mV_0}{2}$$

$$\mu JR = \frac{2}{5} mR^2 \left[\frac{V_0}{R} - \omega \right]$$

$$\frac{3mV_0}{4} = \frac{2}{5} mV_0 - \frac{2}{5} m\omega R$$

$$\frac{2}{5} m\omega R = -\frac{7}{20} mV_0$$

$$\omega = -\frac{7}{8} \frac{V_0}{R}$$

$$V_p = \left[\frac{V_0}{2} - \frac{7}{8} \frac{V_0}{R} \right] = \frac{3V_0}{8R}$$

$$K = \frac{1}{2} m \frac{V_0^2}{4} + \frac{1}{5} m \times \frac{49V_0^2}{64}$$

$$= \frac{49}{320} + \frac{40}{320} = \frac{89}{320}$$

7. Answer (16)

$$i = \frac{\frac{dV}{dx}}{\sigma 4\pi x^2}$$

$$i = \frac{Edx}{\frac{1}{KE} \frac{dx}{4\pi x^2}} = 4\pi x^2 KE^2$$

$$\sqrt{\frac{i}{4\pi k}} \int_a^b \frac{dx}{x} = \int_0^v dv$$

$$i = 16A$$

8. Answer (02)

$$\vec{V} = V_x \hat{i} + V_y \hat{j}$$

$$\vec{B} = B_0 x \hat{k}$$

$$F = q \vec{V} \times \vec{B}$$

$$mV_x \frac{dV_x}{dx} = -qV_y B_0 x$$

$$\text{At } X = X_{\max}, V_x = 0, V_y = \sqrt{V^2 - V_x^2}$$

9. Answer (10)

$$1 \times \frac{r_1}{\frac{Rr_2}{R+r} + R + r_1} = 10 \text{ mA}$$

$$\frac{10 + r_2}{R + r_2} = 1 \text{ mA}$$

10. Answer (34)

$$n = 4$$

$$\Delta E = 12.75 \text{ eV}$$

$$\frac{1}{2} mv^2 = \frac{1}{2} mv^2 + \frac{1}{2} mv_2^2 + \Delta E$$

$$\frac{1}{2} mv^2 = \frac{16}{6} \Delta E$$

$$K = 34 \text{ eV}$$

11. Answer (16)

$$|\vec{V}| = 4\sqrt{\sin^2 2t + (1 - \cos 2t)^2}$$

$$= 4\sqrt{\sin^2 2t + 1 + \cos^2 2t - 2\cos 2t}$$

$$= 4\sqrt{2 - 2\cos 2t}$$

$$|\vec{V}| = 8 \sin t$$

$$d = \int \vec{V} dt = 8 \int_0^\pi \sin t dt = 16$$

12. Answer (60)

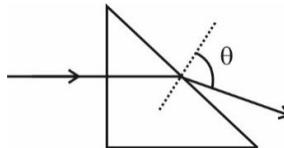
$$V_0 = 6I + 1.2 I^2$$

$$V_0 = 60 \text{ V}$$

13. Answer (08)

$$h = f \theta$$

14. Answer (02)



$$\sin \theta = \frac{\mu}{\sqrt{2}}$$

$$\cos \theta \frac{d\theta}{dt} = \frac{1}{\sqrt{2}} \frac{d\mu}{dt}$$

15. Answer (D)

16. Answer (B)

Solution of Q 15 and Q 16

$$C_{eq} = \frac{\epsilon_0 A}{3d} \text{ at } t = t_0$$

$$C_{eq} = \frac{\epsilon_0 A}{x+d}$$

$$i = V_0 \frac{d}{dt} C_{eq} = \frac{V_0 \epsilon_0 A}{(x+d)^2} \times V$$

$$U = \frac{Q^2}{2C} = \frac{Q}{C} \frac{dQ}{dt}$$

$$= \frac{2V_0}{3} \times \frac{\epsilon_0 A V_0 V}{9d^2}$$

17. Answer (A)

18. Answer (A)

Solution Q 17 and Q18

$$Y_1 = \sqrt{3} \sin \pi \left(t - \frac{6}{4} \right)$$

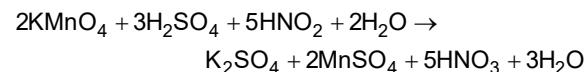
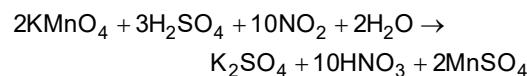
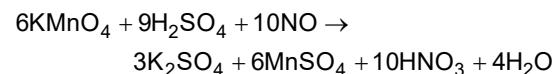
$$Y_2 = \sin \pi \left(t - \frac{4}{4} \right)$$

$$Y = Y_1 + Y_2$$

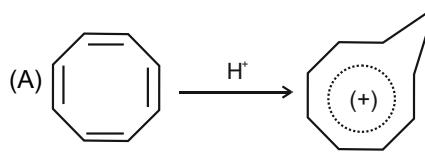
$$= \sqrt{3} \sin \pi \left(t - \frac{3}{2} \right) + \sin \pi(t-1)$$

PART – II : CHEMISTRY

19. Answer (B, C, D)

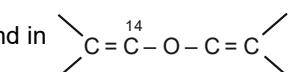


20. Answer (A, B, C, D)



Aromatic

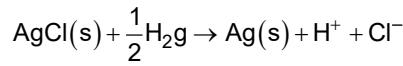
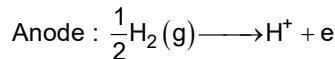
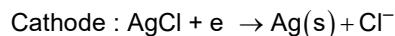
(B) o-Methoxyaniline is more basic than o-methylaniline due to +R effect of methoxy group

(C) $\text{C}^{14}\text{-O}$ bond in  is

longer due to partial availability of lone pair of electrons on O-atom

(D) Phenol and alcohol are different functional groups

21. Answer (A, B, C)



$$E_{\text{Cl}^-}^\circ | \text{AgCl} | \text{Ag} = 0.80 + 0.06 \log 10^{-10} = 0.2 \text{ V}$$

$$E_{\text{cell}} = E_{\text{cell}}^\circ - 0.06 \log \frac{[\text{H}^+] [\text{Cl}^-]}{[\text{P}_{\text{H}_2}]^{1/2}}$$

$$0.44 = (0.2 - 0) - 0.06 \log [\text{H}^+] \times 0.1 ; [\text{H}^+] = 10^{-3} \text{ M}$$

$$[\text{H}^+] = \sqrt{\text{cK}_a} = \sqrt{\text{c} \times 2 \times 10^{-5}} ; \text{c} = 0.05 \text{ M}$$

22. Answer (A, B, C)

$$\text{At pH} = 3, K_{a_1} = \frac{[\text{H}^+] [\text{HA}^-]}{[\text{H}_2\text{A}]} ; \frac{[\text{HA}^-]}{[\text{H}_2\text{A}]} = \frac{10^{-6}}{10^{-3}} = 10^{-3}$$

$$\therefore [\text{H}_2\text{A}] = 1000 [\text{HA}^-]$$

$$\text{At pH} = 6.0, [\text{HA}^-] = [\text{H}_2\text{A}]$$

At pH = 6.5, $[\text{HA}^-] > [\text{H}_2\text{A}]$, So $[\text{H}_2\text{A}]$ is very small

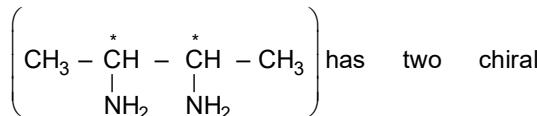
At pH = 8 = $\frac{\text{pK}_{a_1} + \text{pK}_{a_2}}{2}$; $[\text{HA}^-]$ is appreciable so $[\text{A}^{2-}]$ cannot be maximum

23. Answer (A, B, C, D)

By each of the four methods given benzaldehyde can be prepared.

24. Answer (A, B, C, D)

$[\text{Pt}(\text{bn})_2]^{2+}$ is a square planar optically active complex as ligand bn



centres. $[\text{Zn}(\text{gly})_2]$ is tetrahedral and optically active $[\text{Co}(\text{EDTA})]^-$ and $[\text{Cr}(\text{C}_2\text{O}_4)_3]^{3-}$ are octahedral. They also show optical isomerism as they are unsymmetrical.

25. Answer (31)

$$\log \frac{K_2}{K_1} = \frac{\text{Ea}}{2.3 \text{ R}} \left[\frac{1}{T_1} - \frac{1}{T_2} \right]$$

$$= \frac{11454}{2.3 \times 8.3} \left[\frac{1}{100} - \frac{1}{120} \right] = 1 ; \frac{K_2}{K_1} = 10$$

$$\frac{R_2}{R_1} = \frac{K_2}{K_1} = 10 ; R_2 = 31 \text{ mol L}^{-1} \text{ hr}^{-1}$$

26. Answer (07)

$$53.55 = 13.6 \times 4 \left[1 - \frac{1}{n^2} \right]; n = 8$$

$\therefore 7^{\text{th}}$ excited state

27. Answer (07)

Pa, Gd, Tm, Fm, Yb, Ce, and Ho are f-Block elements

28. Answer (05)

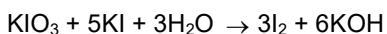
(i), (iv), (v), (vii) and (ix) will show coupling reaction with 2,4,6 trinitrobenzene diazonium cation under appropriate reaction conditions

29. Answer (06)

Except (vi) and (vii) all other molecules will have higher C – H bond energy than C – H bond energy in cyclohexane

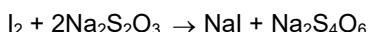
30. Answer (25)

Molarity of $\text{KIO}_3 = 0.05 \text{ M}$



$m \text{ mol of I}_2 = m \text{ mol of KIO}_3 \times 3$

$$= 0.05 \times 50 \times 3 = 7.5$$

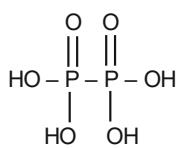


$$n = 2 \quad n = 1$$

$m \text{ equi of Na}_2\text{S}_2\text{O}_3 = m \text{ equi. of I}_2$

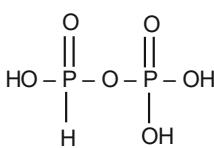
$$0.6 \times V = 7.5 \times 2; V = 25 \text{ ml}$$

31. Answer (09)



Hypophosphoric acid
 $a = 4, c = 1, e = 0$

$$\therefore a + b + c + d + e + f = 9$$



Isohypophosphoric acid
 $b = 3, d = 0, f = 1$

32. Answer (07)



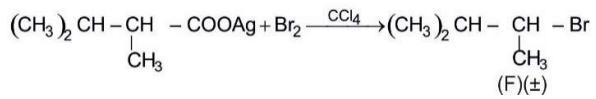
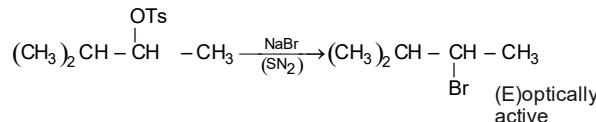
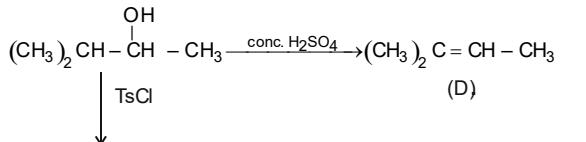
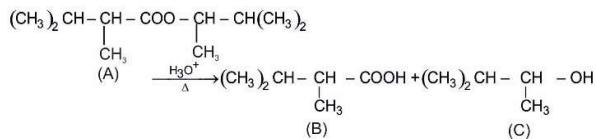
$$x = 1, y = 6, z = 1, m = 6, n = 2$$

$$\frac{y^2 - x^2}{z + m - n} = \frac{36 - 1}{1 + 6 - 2} = 7$$

33. Answer (A)

34. Answer (C)

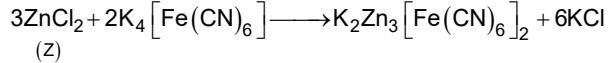
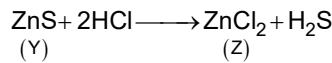
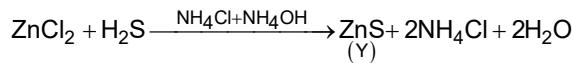
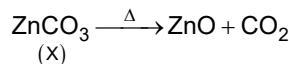
Solution of Q 33 and Q 34



35. Answer (B)

36. Answer (C)

Solution of Q 35 and Q36



37. Answer (A, B)

$$\sum_{r=1}^n \sin^{-1} \left[\frac{2r+1}{r(r+1)\sqrt{r^2 + 2r + \sqrt{r^2 + 1}}} \right]$$

$$\sum_{r=1}^n \sin^{-1} \left[\frac{(2r+1)(\sqrt{r^2 + 2r} - \sqrt{r^2 + 1})}{r(r+1)(r^2 + 2r - r^2 - 1)} \right]$$

$$\sum_{r=1}^n \sin^{-1} \left[\frac{(2r+1)(\sqrt{r^2 + 2r} - \sqrt{r^2 + 1})}{r(r+1)(2r-1)} \right]$$

$$\sum_{r=1}^n \sin^{-1} \left[\frac{1}{r} \left(\frac{\sqrt{1-1}}{(1+r)^2} - \frac{1}{(r+1)} \sqrt{\frac{1-1}{r^2}} \right) \right]$$

$$\sum_{r=1}^n \sin^{-1} \left(\frac{1}{r} \right) - \sin^{-1} \left(\frac{1}{r+1} \right)$$

$$\sin^{-1}(1) - \sin^{-1} \left(\frac{1}{n+1} \right)$$

$$= \frac{\pi}{2} - \sin^{-1} \left(\frac{1}{n+1} \right) = \cos^{-1} \left(\frac{1}{n+1} \right)$$

38. Answer (A, D)

$$S_1 = 1$$

$$S_2 = 1 + \frac{1}{2}$$

$$S_3 = 1 + \frac{1}{2} + \frac{1}{3}$$

.

.

$$S_{n-1} = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n-1}$$

$$S_1 + S_2 + S_3 + \dots + S_{21} = (n-1) + \left(\frac{n-2}{2} \right) + \left(\frac{n-3}{3} \right) + \dots + \frac{n-(n-1)}{n-1}$$

$$= n \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n-1} \right) - (1+1+1+1+\dots+1)$$

$$= nS_{(21)} - (n-1)$$

$$= nS_n - n$$

39. Answer (A, D)

$$(z+1)^{90} = -(z-1)^{90}$$

$$\Rightarrow \left(\frac{z+i}{z-i} \right)^{90} = -1$$

$$\Rightarrow \left(\frac{z+i}{z-i} \right)^{90} = \cos(2k+1)\pi + i \sin(2k+1)\pi$$

$$\Rightarrow \left(\frac{z+i}{z-i} \right) = \cos(2k+1)\frac{\pi}{90} + i \sin(2k+1)\frac{\pi}{90}$$

(Where k = 0, 1, 2,89)

Using componendo and dividendo

$$\Rightarrow \frac{2z}{2i} = \frac{\cos(2k+1)\frac{\pi}{90} + i \sin(2k+1)\frac{\pi}{90} + 1}{\cos(2k+1)\frac{\pi}{90} + i \sin(2k+1)\frac{\pi}{90} - 1}$$

$$= \frac{z}{i} = \frac{2\cos^2(2k+1)\frac{\pi}{180} + i2\sin(2k+1)\frac{\pi}{180}\cos(2k+1)\frac{\pi}{180}}{-2\sin^2(2k+1)\frac{\pi}{180} + i2\sin(2k+1)\frac{\pi}{180}\cdot\cos(2k+1)\frac{\pi}{180}}$$

$$= \frac{z}{i} = \frac{\cot(2k+1)^\circ (\cos(2k+1)^\circ + i \sin(2k+1)^\circ)}{(-\sin(2k+1)^\circ + i \cos(2k+1)^\circ)}$$

$$\Rightarrow z = \cot(2k+1)^\circ, k = 0, 1, 2, \dots, 89$$

Roots are $\cot 1^\circ, \cot 3^\circ, \cot 5^\circ, \dots, \cot 175^\circ$

Sum of roots = 0

Product of roots = -1

40. Answer (A, C, D)

$$I = \int_0^{16} \frac{x^{1/4}}{1+x^{1/2}} dx$$

Put $x^{1/4} = \tan \theta$

$$\Rightarrow \frac{1}{4}x^{\frac{3}{4}}dx = \sec^2 \theta d\theta$$

$$\Rightarrow dx = 4\sec^2 \theta \cdot \tan^3 \theta d\theta$$

$$I = 4 \int_0^{\tan^{-1} 2} \frac{\tan^4 \theta \sec^2 \theta}{1 + \tan^2 \theta} d\theta$$

$$I = 4 \int_0^{\tan^{-1} 2} \tan^4 \theta d\theta$$

$$I = 4 \int_0^{\tan^{-1} 2} \tan^2 \theta (\sec^2 \theta - 1) d\theta$$

$$= 4 \left[\frac{\tan^3 \theta}{3} - \tan \theta + \theta \right]_0^{\tan^{-1} 2}$$

$$= 4 \left[\frac{8}{3} - 2 + \tan^{-1} 2 \right]$$

$$\begin{aligned}
 &= \frac{8}{3} + 4 \tan^{-1} 2 \quad (\text{option A}) \\
 &= \frac{8}{3} + 2 \cdot (2 \tan^{-1} 2) \\
 &= \frac{8}{3} + 2 \left[\pi + \tan^{-1} \frac{2.2}{1-(2)^2} \right] \left[2 \tan^{-1} x = \pi + \tan^{-1} \left(\frac{2x}{1-x^2} \right), x > 1 \right] \\
 &= \frac{8}{3} + 2 \left[\pi - \tan^{-1} \left(\frac{4}{3} \right) \right] \\
 &= 2\pi + \frac{8}{3} - 2 \tan^{-1} \left(\frac{4}{3} \right) \quad (\text{option (C)}) \\
 &= 2\pi + \frac{8}{3} - \left(\pi + \tan^{-1} \frac{\frac{8}{3}}{1 - \frac{16}{3}} \right) \\
 &= 2\pi + \frac{8}{3} - \pi + \tan^{-1} \left(\frac{24}{7} \right) \\
 &= \pi + \frac{8}{3} + \tan^{-1} \left(\frac{24}{7} \right) \quad (\text{option (D)})
 \end{aligned}$$

41. Answer (A, C)

$$\begin{aligned}
 &(2+x)(2+2x)(2+3x)\dots(2+kx) \\
 &= a_0 + a_1 x + a_2 x^2 + \dots + a_k x^k \quad \dots(i)
 \end{aligned}$$

Comparing the co-efficient of x^k in L.H.S and R.H.S

$$\Rightarrow 1.2.3\dots.k = a_k$$

$$\Rightarrow a_k = |k| \quad \dots(ii)$$

Put $x = 1$ in equation (i)

$$\Rightarrow 3.4.5\dots(2+k)$$

$$= a_0 + a_1 + a_2 + \dots + a_k$$

$$\Rightarrow a_0 + a_1 + \dots + a_{k-1} + a_k = \frac{(1.2.3\dots(k+2))}{2}$$

$$\Rightarrow (a_0 + a_1 + a_2 + \dots + a_{k-1}) + |k| = \frac{|k+2|}{2} \text{ from (ii)}$$

$$\Rightarrow a_0 + a_1 + \dots + a_{k-1} = \frac{|k+2|}{2} - |k|$$

$$= \frac{(k+2)(k+1)|k|}{2} - |k|$$

$$= |k| \left[\frac{k^2 + k + 2k + 2 - 2}{2} \right]$$

$$= |k| \frac{(k^2 + 3k)}{2}$$

$$\Rightarrow a_0 + a_1 + \dots + a_{k-1} = \frac{k(k+3)|k|}{2}$$

Note : $k(k+3)$ will be an even number

Now $2020 = 101 \times 20$ (101 is prime)

$2021 = 47 \times 43$ (47 is prime)

$2.22 = 337 \times 6$ (337 is prime)

\Rightarrow If $a_0 + a_1 + \dots + a_{k-1}$ is divisible by

$2020 = 101 \times 20 \Rightarrow k = 98 \Rightarrow a = 98$

$2021 = 47 \times 43 \Rightarrow k = 44 \Rightarrow b = 44$

$2022 = 337 \times 6 \Rightarrow k = 334 \Rightarrow c = 334$

$$\text{Now } \frac{c+b}{a-b} = \frac{334+44}{98-44} = \frac{378}{54} = 7$$

$$\frac{b+c-a}{c-6b} = \frac{44+334-98}{334-6 \times 44}$$

$$= \frac{280}{334-264} = \frac{280}{70} = 4$$

Option 'A' and 'C' are correct

42. Answer (A, B, C, D)

Equation of tangent to $y^2 = x$

$$ty = x + \frac{t^2}{4}$$

Also tangent to circle

\Rightarrow Perpendicular distance from (2,0) is 2

$$\frac{\left| 2 + \frac{t^2}{4} \right|}{\sqrt{1+t^2}} = 2$$

$$\Rightarrow (t^2 + 8)^2 = 64(1 + t^2)$$

$$\Rightarrow t^4 - 48t^2 = 0$$

$$\Rightarrow t = 0, \pm 4\sqrt{3}$$

Equation of tangent

$$x = 0, \pm 4\sqrt{3} \quad y = x + 12$$

Equation of tangent to parabola $y^2 = -x$

$$ty = -x + \frac{t^2}{4}$$

Perpendicular from $(2, 0) = 2$

$$\frac{|-2 + \frac{t^2}{4}|}{\sqrt{1+t^2}} = 2$$

$$(t^2 - 8)^2 = 64(1+t^2)$$

$$t^4 - 80t^2 = 0$$

$$t = 0, t = \pm 4\sqrt{5}$$

Equation of tangent

$$x = 0, \pm 4\sqrt{5} \quad y = -x + 20$$

43. Answer (18)

$$S = \lim_{n \rightarrow \infty} \sum_{r=2}^n \frac{rC_2}{3^r} \\ = \lim_{n \rightarrow \infty} \left(\frac{1}{9} + \frac{3}{27} + \frac{6}{81} + \dots \right)$$

$$S = \frac{1}{9} + \frac{3}{27} + \frac{6}{81} + \dots \infty$$

$$\frac{S}{3} = \frac{1}{27} + \frac{3}{81} + \dots \infty$$

$$\frac{2S}{3} = \frac{1}{9} + \frac{2}{27} + \frac{3}{81} \dots \infty$$

$$\frac{2S}{9} = \frac{1}{27} + \frac{2}{81} + \dots \infty$$

$$\frac{2S}{3} - \frac{2S}{9} = \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots \infty$$

$$\frac{4S}{9} = \frac{\frac{1}{9}}{1 - \frac{1}{3}}$$

$$\Rightarrow \frac{4S}{9} = \frac{1}{6}$$

$$\Rightarrow S = \frac{3}{8}$$

$$\Rightarrow S = 0.38$$

44. Answer (23)

$$f(x) = f(-x) \quad \forall x \in R$$

$$\Rightarrow ([a]-7)([a]-4)x^3 - (4\{a\}-1)(2\{a\}-1)x + \sin x \cdot \text{sgn}(x) =$$

$$([a]-7)([a]-4)x^3 + (4\{a\}-1)(2\{a\}-1)x - \sin x \cdot \text{sgn}(-x)$$

$$\Rightarrow 2([a]-7)([a]-4)x^2 - 2(4\{a\}-1)(2\{a\}-1)x \\ + \sin x (\text{sgn}(x) + \text{sgn}(-x)) = 0$$

$$\Rightarrow ([a]-7)([a]-4)x^3 - (4\{a\}-1)(2\{a\}-1)x = 0$$

$$[\text{sgn}(x) + \text{sgn}(-x) = 0]$$

$$\Rightarrow ([a]-7)([a]-4) = 0 \quad \text{and} \quad (4\{a\}-1)(2\{a\}-1) = 0$$

Case (1) $[a] - 7 = 0 \quad \& \quad 4\{a\} - 1 = 0$

$$\Rightarrow a = 7 + \frac{1}{4} = \frac{29}{4}$$

Case (2) $[a] - 7 = 0 \quad \& \quad 4\{a\} - 1 = 0$

$$\Rightarrow a = 7 + \frac{1}{2} = \frac{15}{2}$$

Case (3) $[a] - 4 = 0 \quad \& \quad 4\{a\} - 1 = 0$

$$\Rightarrow a = 4 + \frac{1}{4} = \frac{17}{4}$$

Case (4) $[a] - 4 = 0 \quad \& \quad 2\{a\} - 1 = 0$

$$\Rightarrow a = 4 + \frac{1}{2} = \frac{9}{2}$$

$$\text{Sum of values of } a = \frac{29}{4} + \frac{15}{2} + \frac{17}{4} + \frac{9}{2}$$

$$= 23.50$$

45. Answer (01)

$$\cos^2 A = \tan A$$

$$\cos^3 A = \sin A$$

$$\sin 2A + \tan A = \sin^2 A + \cos^2 A = 1$$

46. Answer (08)

$$\prod_{n=1}^2 \frac{1}{1 - \tan^2 \frac{1}{2^n}} = \frac{1}{\cos \frac{1}{2^{n-1}}} \times \cos^2 \frac{1}{2^n} = \tan 1 = \tan k$$

$$[k^2] = 1$$

47. Answer (09)

$$f(x) = \left(3 - \sqrt{4 - x^2}\right)^2 + \left(1 + \sqrt{4 - x^2}\right)^3$$

Let us consider

$$1 + \sqrt{4 - x^2} = a$$

48. Answer (08)

$$8 \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{r^{1/a} \left[n^a \cdot n^{-1/a} + r^a \cdot e^{-1/a} \right]}{n^a \cdot n}$$

$$8 \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n} \left[\left(\frac{r}{n}\right)^{\frac{1}{a}} + \left(\frac{r}{n}\right)^a \right]$$

$$= 8 \int_0^1 \left(x^{\frac{1}{a}} + x^a \right) dx$$

$$= 8 \left[\frac{x^{\frac{1}{a}+1}}{\frac{1}{a}+1} + \frac{x^{a+1}}{a+1} \right]_0^1$$

$$= 8 \left[\frac{a}{a+1} + \frac{1}{a+1} \right] = 8$$

49. Answer (80)

$$1! + 2! + 3! + 4! = 33$$

for $x \geq 4$ unit place of $\sum_{r=1}^x r!$ is 3

But square of any number cannot have unit place 3

$\Rightarrow x < 4$ is only possibility

$$\text{for } x = 1 \quad k^2 = 1 \Rightarrow k = 1$$

$$\text{for } x = 3 \quad k^2 = 9 \Rightarrow k = 3$$

$$\Rightarrow \alpha = 1, \beta = 3$$

$$\text{Now } (a + \sqrt{b})^{x^2 - (1+3+5)} + (a - \sqrt{b})^{x^2 - 3 \times 3} = 2a$$

$$(a + \sqrt{b})^{x^2 - 9} + (a - \sqrt{b})^{x^2 - 9} = 2a$$

As $a^2 - b = 1$, then only possibility of above equation is when

$$x^2 - 9 = 1 \quad \text{or} \quad x^2 - 9 = -1$$

$$x^2 = 10 \quad \text{or} \quad x^2 = 8$$

$$\text{Hence } x = \pm \sqrt{10}, \pm \sqrt{8}$$

$$|a_1 + a_2 + a_3 + a_4 + a_1 a_2 a_3 a_4|$$

$$= |+\sqrt{10} - \sqrt{10} + \sqrt{8} - \sqrt{8} + (\sqrt{10})(\sqrt{-10})(\sqrt{8})(\sqrt{-8})|$$

$$= 80$$

50. Answer (46)

Consider the pattern

$$a = \sqrt[3]{20x + \sqrt[3]{20x + \sqrt[3]{20x + \dots \sqrt[3]{20x + \sqrt[3]{20x + 13}}}}$$

infinitely many terms

$$a = \sqrt[3]{20x + \sqrt[3]{20x + \sqrt[3]{20x + \sqrt[3]{20x + 13}}}}$$

No change (infinitely many terms)

$$\text{Hence } \sqrt[3]{20x + 13} = 13$$

$$x = \frac{13^3 - 13}{20} = \frac{546}{5}$$

51. Answer (B)

$$x_1 x_2 + x_1 x_3 + x_2 x_4 + x_3 x_4 = x_1 x_2 + x_3 x_4$$

$$= \frac{x_1}{x_4} + \left(-\frac{x_4}{x_1} \right) \text{ where } x_1 \text{ and } x_4 \text{ are roots of}$$

$$x^2 - x - 2 = \sqrt{5}$$

$$\frac{(x_1 - x_4)}{x_1 x_4} = -\frac{b\sqrt{D}}{ac} = +\frac{\sqrt{9 + 4\sqrt{5}}}{-(2 + \sqrt{5})} = -1$$

52. Answer (D)

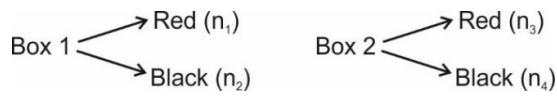
$$x_2^3 + x_4^3 = x_4^3 + \frac{1}{x_4^3} \quad (\text{where } x_4 \text{ is the larger root of}$$

$$x^2 - x - 2 - \sqrt{5} = 0, \quad \text{which is } \frac{3 + \sqrt{5}}{2}$$

$$= \left(\frac{3 + \sqrt{5}}{2} \right)^3 + \left(\frac{3 - \sqrt{5}}{2} \right)^3$$

$$= \frac{2(27 + 45)}{8} = 18$$

53. Answer (A)



$$P(\text{red ball}) = \frac{1}{2} \left(\frac{n_1}{n_1 + n_2} \right) + \frac{1}{2} \left(\frac{n_3}{n_3 + n_4} \right)$$

$$P\left(\frac{R}{B}\right) = \frac{\frac{1}{2} \left(\frac{n_3}{n_3 + n_4} \right)}{\frac{1}{2} \left(\frac{n_1}{n_1 + n_2} + \frac{n_3}{n_3 + n_4} \right)} = \frac{n_3}{\frac{n_1}{n_1 + n_2} + \frac{n_3}{n_3 + n_4}} = \frac{1}{3}$$

By option verification only (A) is correct

54. Answer (C)

Given

$$\frac{n_1}{n_1 + n_2} \cdot \frac{n_1 - 1}{n_1 + n_2 - 1} + \frac{n_2}{n_1 + n_2} \cdot \frac{n_1}{n_1 + n_2 - 1} = \frac{1}{3}$$

By solving $2n_1 = n_2$

□ □ □