



Aakash

Medical | IIT-JEE | Foundations
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Regd. Office :Aakash Tower, 8, Pusa Road, New Delhi-110005, Ph.011-47623456

Time : 3 hrs

Mock Test_CoE_XII

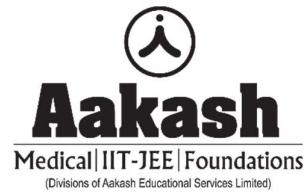
MM : 264

for JEE (Advanced) - 2020

Test - 2A (Paper - I) Actual Pattern-2015

ANSWERS

PHYSICS	CHEMISTRY	MATHEMATICS
1. (3)	21. (7)	41. (6)
2. (8)	22. (3)	42. (0)
3. (4)	23. (3)	43. (2)
4. (2)	24. (4)	44. (2)
5. (6)	25. (4)	45. (6)
6. (3)	26. (2)	46. (6)
7. (8)	27. (1)	47. (2)
8. (4)	28. (2)	48. (5)
9. (B, D)	29. (A, C, D)	49. (B, D)
10. (B, C)	30. (B, C, D)	50. (A, C, D)
11. (A, B)	31. (A, D)	51. (A, C, D)
12. (D)	32. (A, C)	52. (A, B, D)
13. (A, C)	33. (A, D)	53. (C, D)
14. (A, B, C)	34. (A, B, C, D)	54. (A, B, C)
15. (A)	35. (A, B, C)	55. (A, C, D)
16. (A, B, C)	36. (A, B, C)	56. (A, D)
17. (A, B, C)	37. (A, B, C)	57. (A, B, C, D)
18. (A, C)	38. (A, D)	58. (A, B, C)
19. A → (P, S) B → (P, S) C → (R, T) D → (Q, T)	39. A → (P, Q, S) B → (P, T) C → (P, Q, R, S) D → (P, Q, S)	59. A → (Q) B → (P) C → (Q) D → (P)
20. A → (P, S) B → (Q) C → (T) D → (R)	40. A → (R) B → (S, T) C → (P) D → (Q)	. 60. A → (Q) B → (P) C → (S) D → (S)



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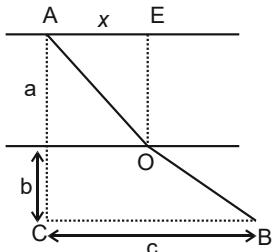
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Test - 2A (Paper - I)_Actual Pattern-2015

HINTS & SOLUTIONS

1. Answer (3)



$$T = t_{A \rightarrow 0} + t_{0 \rightarrow B}$$

$$T = \frac{\sqrt{a^2 + x^2}}{V_1} + \frac{\sqrt{b^2 + (c-x)^2}}{V_2}$$

$$\text{For minimum } T, \frac{dT}{dx} = 0$$

$$\Rightarrow \frac{x / \sqrt{a^2 + x^2}}{(c-x)} = \frac{V_1}{V_2}$$

$$\Rightarrow \frac{\sqrt{b^2 + (c-x)^2}}{\sqrt{b^2 + (c-x)^2}}$$

$$\Rightarrow \frac{\sin \alpha_1}{\sin \alpha_2} = \frac{6}{2} = 3$$

2. Answer (8)

$$V_m (\text{just before collision}) = 2\sqrt{2gh}$$

$$\Rightarrow V_{3m} (\text{just after collision}) = \left(\frac{V_m}{5} \right)$$

$$a_{3m} = \frac{g}{5} (\text{Downward})$$

$$\Rightarrow H_{\max} = \frac{(V_{3m})^2}{2a_{3m}} = \frac{4h_0}{5} = 8 \text{ cm}$$

3. Answer (4)

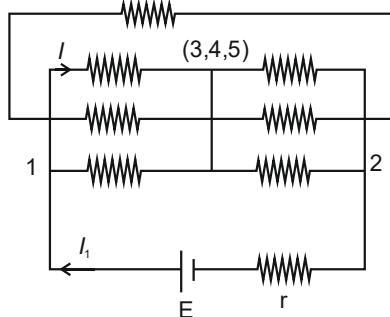
$\vec{L}_i = \vec{L}_f$ (About point on horizontal surface)

$$\Rightarrow \frac{mR^2}{2} \omega_0 = 2 \left(\frac{3}{2} mR^2 \right) \omega$$

$$\omega = \frac{\omega_0}{6} \Rightarrow V_{cm} = \frac{R\omega_0}{6} \Rightarrow J = \frac{mR\omega_0}{6}$$

$$\Rightarrow J = \frac{(2)(1)12}{6} = 4 \text{ kg.m/s}$$

4. Answer (2)



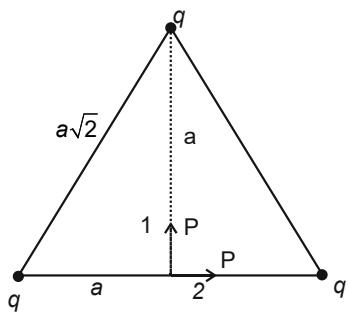
Point 3,4 and 5 will be at same potential

$$\Rightarrow R_{eq}^{1,2} = \frac{2r}{5} + r = \frac{7r}{5}$$

$$\Rightarrow I_1 = \frac{5E}{7r} \Rightarrow I = \left(\frac{3}{5} I_1 \right) \frac{1}{3} = \frac{I_1}{5}$$

$$\Rightarrow I = \left(\frac{E}{7r} \right) = \frac{14}{7 \times 1} = 2A$$

5. Answer (6)



$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$$\vec{F}_1 = 0$$

$$\begin{aligned} |\vec{F}_2| &= 2 \times \frac{2kPq}{a^3} - \frac{kq}{a^3} q \\ &= \frac{3kPq}{a^3} = \frac{3pq}{4\pi \in_0 a^3} = \frac{6Pq}{8\pi \in_0 a^3} \end{aligned}$$

6. Answer (3)

$$T = t_{A \rightarrow B} + t_{B \rightarrow C}$$

$$= \frac{1}{4} 2\pi \sqrt{\frac{l}{g}} + \frac{1}{4} 2\pi \sqrt{\frac{l}{4g}}$$

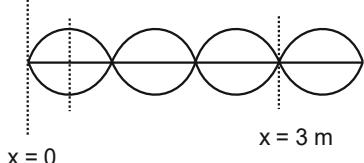
$$= \left(2\pi \sqrt{\frac{l}{g}} \right) \left(\frac{1}{4} + \frac{1}{8} \right)$$

$$= \left(\frac{3}{8} \right) 2\pi \sqrt{\frac{l}{g}}$$

$$\left(\frac{3}{8} \right) \frac{2\pi}{\pi} \times 4 = 3 \text{ sec}$$

7. Answer (8)

$$x = \frac{1}{2} \text{ m}$$



$$\lambda = 1 \text{ m}$$

$$\text{Energy of 1 loop} = \frac{1}{2} \mu \lambda \omega^2 A^2$$

$$= \frac{5}{8} \mu \text{J}$$

8. Answer (4)

$$(\pi x^2) \frac{dy}{dt} = VA$$

$$(\pi x^2) V_0 = (\sqrt{2gy})(A_d)$$

$$\Rightarrow x^4 = cy$$

$$\Rightarrow x = cy^{1/4}$$

$$\Rightarrow \left(k = \frac{1}{4} \right)$$

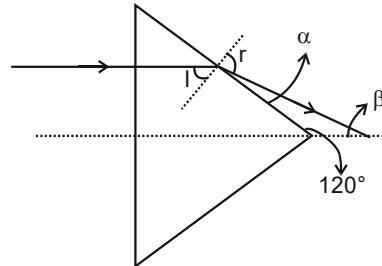
9. Answer (B, D)

$$I = I_1 + I_2 = Ee^{-t} + E(1 - e^{-2t})$$

$$\text{For } I \text{ to be maximum } \frac{dI}{dt} = 0$$

$$\Rightarrow -Ee^{-t} + 2Ee^{-2t} = 0$$

10. Answer (B, C)



$$\frac{\sin i}{\sin r} = \frac{1}{1.44} \Rightarrow \sin r = 0.72$$

$$\alpha = \left(\frac{\pi}{2} - r \right)$$

$$\beta + \alpha + \frac{2\pi}{3} = \pi$$

$$\Rightarrow \beta = \left(\pi - \frac{2\pi}{3} - \alpha \right) = \left(\frac{\pi}{3} - \frac{\pi}{2} + r \right)$$

$$\beta = \left(r - \frac{\pi}{6} \right) = \sin^{-1}(0.72) - \frac{\pi}{6}$$

11. Answer (A, B)

$$E = a(1 + \cos \omega t) \cos \omega_0 t = a \cos \omega_0 t + a \cos \omega t \cos \omega_0 t$$

$$\Rightarrow E = a \cos \omega_0 t + \frac{1}{2} a \cos(\omega + \omega_0)t + \frac{1}{2} a \cos(\omega - \omega_0)t$$

$$K_{\max} = \frac{h}{2\pi} (\omega + \omega_0) - \phi \approx 0.37 \text{ eV}$$

$$\omega_I = |\omega - \omega_0|$$

12. Answer (D)

$$N_A = N_A^\circ e^{-\lambda_A t}$$

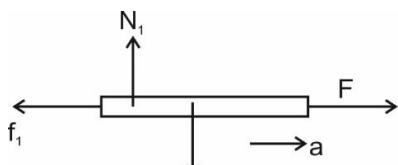
$$N_B = C_1 e^{-\lambda_B t} + \frac{\lambda_A N_A^\circ e^{-\lambda_A t}}{(\lambda_B - \lambda_A)}$$

$$C_1 = 10 N_0$$

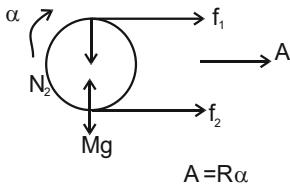
$$\text{Given } \frac{\lambda_A N_A}{\lambda_B N_B} = \frac{9}{22} \Rightarrow \frac{N_A}{N_A} = \frac{3}{11}$$

$$\text{Hence } t = 90 \text{ days}$$

13. Answer (A, C)



14.



$$F - f_1 = ma \quad \dots(i)$$

$$f_1 + f_2 = MA \quad \dots(ii)$$

$$f_1 R - f_2 R = I\alpha \quad \dots(iii)$$

15. Answer (A, B, C)

$$T_B = 8T_0 \Rightarrow T_A = T_0, T_C = 16T_0, T_D = 8T_0$$

$$W_{ij} = Q_{ij} - \Delta U_{ij}$$

Process AD

$$W_{AD} = \frac{1}{2}(P_0 + 2P_0)3V_0 = \frac{9P_0 V_0}{2}$$

$$Q_{AD} = \Delta V_{AD} + W_{AD}$$

$$= \frac{21}{2}P_0 V_0 + \frac{9}{2}P_0 V_0$$

$$Q_{AD} = 15P_0 C_0$$

$$\text{Similarly } \Delta Q_{AB} = 13P_0 V_0, \Delta Q_{BC} = 20P_0 V_0$$

16. Answer (A)

$$R \frac{dq}{dt} + \frac{q}{C} = \alpha t \Rightarrow RC dq + q dt = \alpha C t dt$$

Multiplying both side by $e^{t/RC}$ we get

$$RC e^{t/RC} dq + q e^{t/RC} dt = \alpha C t e^{t/RC} dt$$

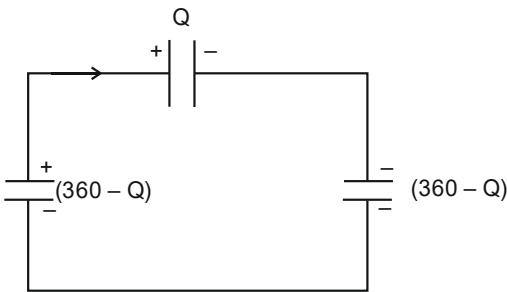
$$\Rightarrow d \left[q e^{\frac{t}{RC}} \right] = \frac{\alpha t}{R} e^{\frac{t}{RC}} dt$$

$$\Rightarrow q = \alpha C [t - RC] + \alpha RC^2 e^{-\frac{t}{RC}}$$

At $t = RC$

$$q = 0 + \frac{\alpha RC^2}{edt}, dq = I_1 V_R = IR$$

17. Answer (A, B, C)



Apply KVL

$$\left(180 - \frac{Q}{2\mu F} \right) + \left(100 - \frac{Q}{3\mu F} \right) = \frac{Q}{2\mu F}$$

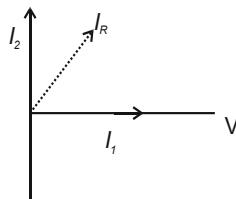
$$\Rightarrow Q = 210\mu F$$

$$\Rightarrow Q_A = 90, Q_B = 150, Q_C = 21$$

18. Answer (A, B, C)

$$I_1 = \frac{V}{R} = 100 \sin(50\pi t)$$

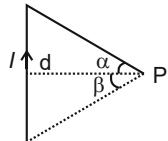
$$I_2 = 100 \sin\left(50\pi t + \frac{\pi}{2}\right)$$



$$I_R = 100\sqrt{2}$$

18. Answer (A, C)

$$|\vec{B}_p| = \frac{\mu_0 I}{4\pi d} (\sin \alpha + \sin \beta)$$



$$\text{also } \hat{B}_x^p = \hat{k}$$

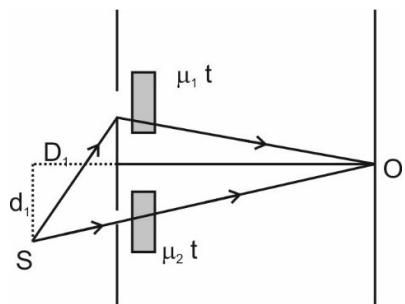
$$\hat{B}_y^p = -\hat{k}$$

19. Answer A – P, S

B – P, S

C – R, T

D – Q, T



At point O

$$\Delta x = \frac{(d_1)(d)}{D_1} - (\mu_1 + \mu_2)t$$

$$\text{If } \frac{(d_1)d}{D_1} > (\mu_2 - \mu_1)t$$

\Rightarrow Central Maxima will be above point O and

$$\beta = \frac{\lambda D}{d}$$

20. Answer A – P, S

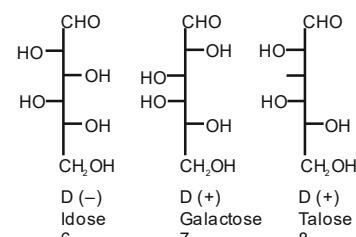
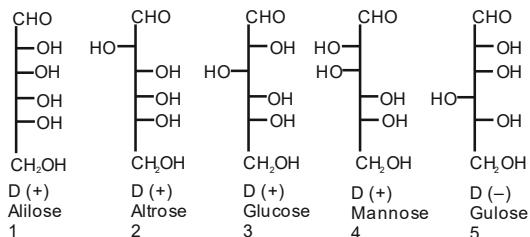
B – Q

C – T

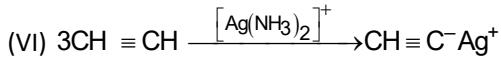
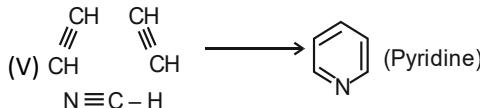
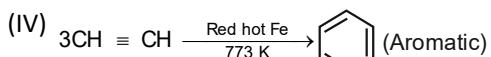
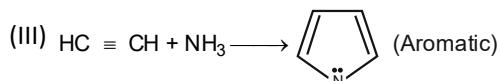
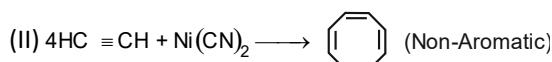
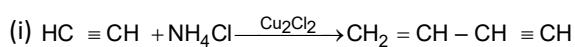
D – R

21. Answer (7)

All statements are correct



22. Answer (3)



23. Answer (3)

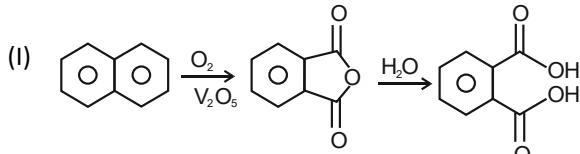
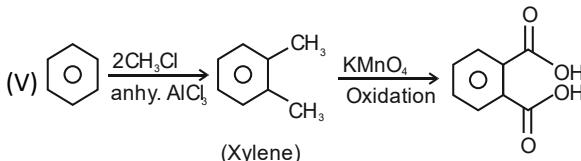
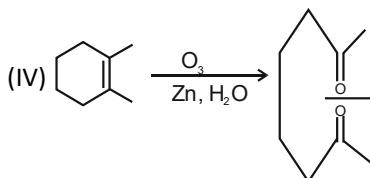
$$\bar{v}_{\text{Be}^{+3}} = R_H \cdot Z^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) \leq R_H (4^2) \left(\frac{1}{1} - \frac{1}{4} \right) \Rightarrow 12 R$$

$$\bar{v}_{\text{He}^+} = R_H \cdot (2)^2 \left(\frac{1}{(1)^2} - \frac{1}{(\alpha)^2} \right) \leq R_H \times 4 \Rightarrow 4 R$$

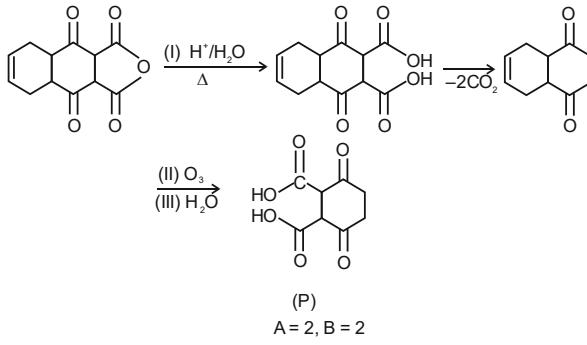
$$\frac{\bar{v}_{\text{Be}^{+3}}}{\bar{v}_{\text{He}^+}} = \frac{12R}{4R} \Rightarrow 3$$

24. Answer (4)

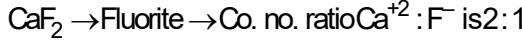
I, II, III and V will give phthalic acid.

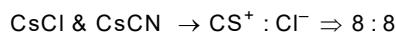
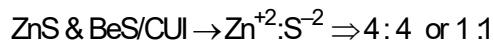
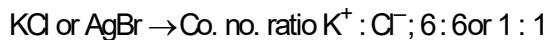
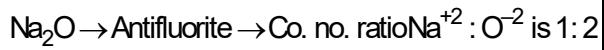


25. Answer (4)



26. Answer (2)





27. Answer (1)

$$r_1 = 2r = K[2P]^x [Q]^y$$

$$2K[P]^x [Q]^y = K[2P]^x [Q]^y \therefore x = 1$$

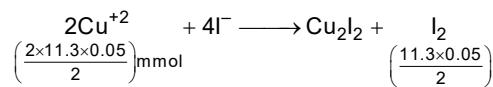
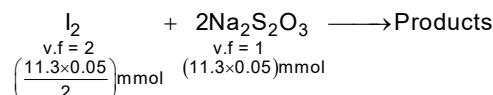
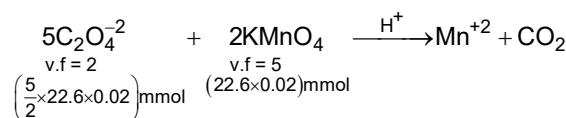
$$r_2 = 5.6r = K[P]^x [2Q]^y$$

$$5.6K[P]^x [Q]^y = K[P]^x [2Q]^y \therefore y = \frac{5}{2}$$

$$\text{Overall order of R}^n = (x + y) \Rightarrow 1 + \frac{5}{2} = 3.5$$

$$\text{Then } 50 - (2 \times 3.5)^2 = 1$$

28. Answer (2)



$$\text{Cu}^{+2} = 11.3 \times 0.05 \text{ mmol} \text{ & } \text{C}_2\text{O}_4^{2-} = \frac{5}{2} \times 22.6 \times 0.02$$

$$\text{Req. ratio } \frac{(\text{C}_2\text{O}_4)^{-2}}{\text{Cu}^{+2}} = \frac{\frac{5}{2} \times 22.6 \times 0.02}{11.3 \times 0.05} = 2$$

29. Answer (A, C, D)

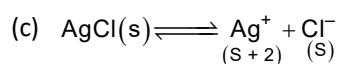
After the water is placed into the freezer it will lose heat to the freezer. So q_{system} is negative and after several hours water turned to ice. So the temperature of the water will be -20°C .

1 mole of liquid water has 6 kJ more enthalpy than the ice.

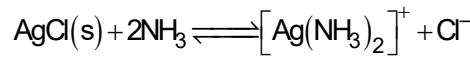
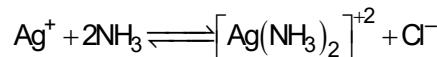
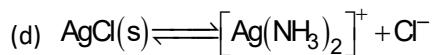
30. Answer (B, C, D)

$$(a) S = \sqrt{K_{\text{SP}}} = 10^{-5} \text{ mol/L}$$

$$(b) S = 10^{-5} \text{ mol/L}$$



$$10^{-10} = (S + 2)(S) \Rightarrow S = \frac{10^{-10}}{2} = 5 \times 10^{-11}$$



$$K = \frac{[\text{Ag}(\text{NH}_3)_2]^+ [\text{Cl}^-]}{[\text{NH}_3]^2} = K_{\text{SP}} \times K_f$$

$$= \frac{S^2}{(2 - 2S)^2} = 10^{+8} \times 10^{-10} \Rightarrow 10^{-2}$$

$$\therefore S = 0.166 \text{ M}$$

31. Answer (A, D)

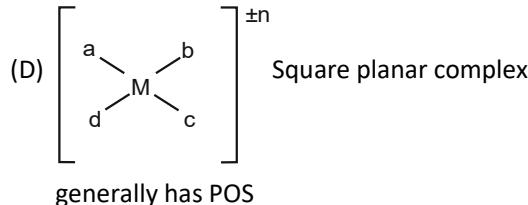
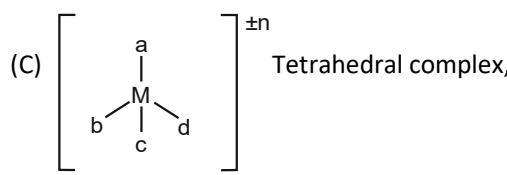
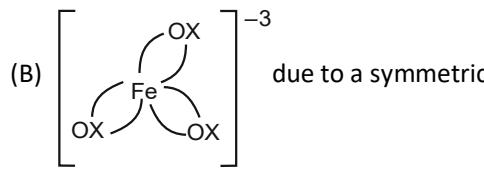
Tyndal effect doesn't shown by true solution due to small size of particles.

Sorption involve adsorption and absorption
Coagulation power \propto Charge

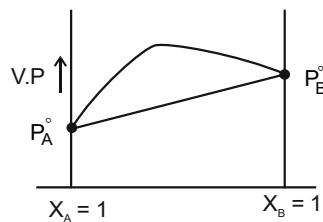
Smaller the gold no greater would be the protecting power

32. Answer (A, C)

(A) $[\text{PtCl}_2(\text{NH}_3)_4]^{+2} \Rightarrow$ both Cis & trans forms are optical inactive

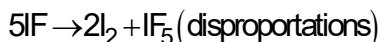


33. Answer (A, D)

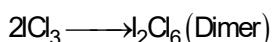


This is positive deviation from Raoult's law hence attraction force is less in mixture B is more Volatile than A

34. Answer (A, B, C, D)

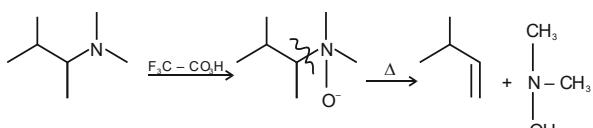


ClF_3 gas is Hypergolic Used in magic shows

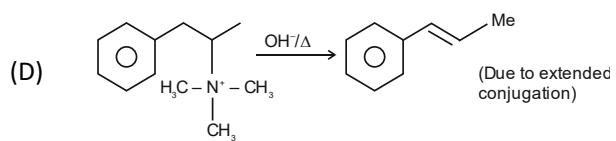


35. Answer (A, B, C)

(A) Cope reaction



- (B) In case of bulky base, Hoffmann alkene is formed
 (C) In case of RF (Poor leaving gp.) whether the base is bulky or non-bulky always gives Hoffmann Alkens



36. Answer (A, B, C)

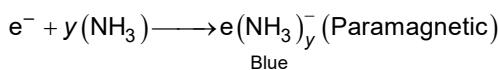
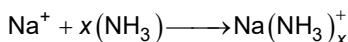
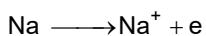
Cupellation process used when lead is present in trace amount and the difference in the M.Pt is high thus A is correct

Parke's process is based on Solubility & Pb quantity is high Mot\ten Zn have higher solubility, So (B) is also correct

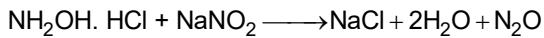
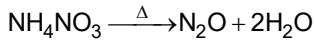
$\text{PbS} + 2\text{PbO} \rightarrow 3\text{Pb} + \text{SO}_2$. Thus (C) is correct

In 4th Lavigation process is used thus this statement is wrong

37. Answer (A, B, C)



38. Answer (A, D)



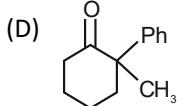
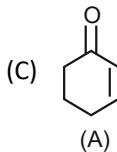
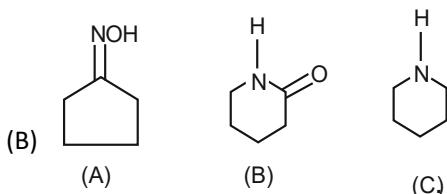
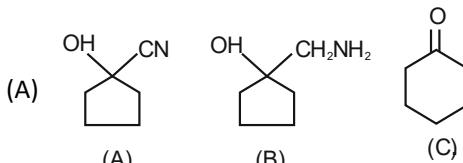
While NH_4NO_2 on heating gives N_2

39. Answer A – P, Q, S

B – P, T

C – P, Q, R, S

D – P, Q, S



40. Answer A – R

B – S, T

C – P

D – Q

$$y = \frac{1}{V^2} \text{ or } \sqrt{y} = \frac{1}{V}; P = x \text{ and } P = \frac{\text{constant}}{V}$$

$$(A) x = (K)\sqrt{y} \Rightarrow y = k'x^2$$

- (B) $V = KT$; $y = V$ & $\frac{1}{T} = x \therefore y = \frac{k}{x}$ inversely proportional (Rectangular Hyperbola)
- (C) $P = KT$; $PT = KT^2$ or $y = kx$
- (D) $V = \frac{C}{P} \Rightarrow y = C\sqrt{x}$; $y^2 = Cx$

41. Answer (6)

$$P = \lim_{x \rightarrow \infty} \frac{1 + [x] + [x^2] + \dots + [x^{2020}]}{1 + x + x^2 + \dots + x^{2020}}$$

$$= 1 - \lim_{x \rightarrow \infty} \frac{\{x\} + \{x^2\} + \dots + \{x^{2020}\}}{1 + x + x^2 + \dots + x^{2020}} = 1$$

42. Answer (0)

$$S_n(\theta) = \operatorname{Re} \left(\sum_{r=1}^n \frac{Z^{r-1}}{(\cos \theta)^{r-1}} \right)$$

(where $Z = \cos \theta + i \sin \theta$)

$$\operatorname{Re} \left(\frac{1 - \left(\frac{Z}{\cos \theta} \right)^n}{1 - \frac{Z}{\cos \theta}} \right) = \operatorname{Re} \left(\frac{1 - \frac{\cos n\theta + i \sin n\theta}{\cos^n \theta}}{-i \tan \theta} \right)$$

$$= \frac{\sin n\theta}{\sin n\theta \cos^{n-1} \theta}$$

$$\text{So, } S_{3\lambda} \left(\frac{\pi}{3} \right) = 0$$

43. Answer (2)

$$2 \cos^4 x + (5 - \sqrt{5}) = \cos^2 x (\sqrt{5} + 1 + 2 \sin x) + (\sqrt{5} - 1) \sin x$$

$$\Rightarrow 2 \cos^4 x - 2 \cos^2 x - \cos^2 x (2 \sin x + \sqrt{5} - 1) + (2 + (3 - \sqrt{5})) - (\sqrt{5} - 1) \sin x = 0$$

$$\Rightarrow 2 \cos^4 x + 2 \sin^2 x - \cos^2 x (2 \sin x + \sqrt{5} - 1) + (3 - \sqrt{5}) - (\sqrt{5} - 1) \sin x = 0$$

$$\Rightarrow \cos^4 x + \sin^2 x + \frac{3 - \sqrt{5}}{2} - \cos^2 x \left(\sin x + \frac{\sqrt{5} - 1}{2} \right) - \frac{(\sqrt{5} - 1)}{2} \sin x = 0$$

$$\Rightarrow (\cos^2 x)^2 + (\sin x)^2 + \left(\frac{\sqrt{5} - 1}{2} \right)^2 - (\sin x)(\cos^2 x) - \frac{(\sqrt{5} - 1)}{2} \cos^2 x - \left(\frac{\sqrt{5} - 1}{2} \right) \sin x = 0$$

$$\Rightarrow (\cos^2 x - \sin x)^2 + \left(\sin x - \frac{\sqrt{5} - 1}{2} \right)^2 + \left(\cos^2 x - \frac{\sqrt{5} - 1}{2} \right)^2 = 0$$

$$\Rightarrow \cos^2 x = \sin x = \frac{\sqrt{5} - 1}{2}$$

$$\Rightarrow \cos^2 x = \sin x$$

$$\Rightarrow \sin^2 x + \sin x = 1 \quad \dots(i)$$

$$\sin^6 x + 4 \sin^5 x + 6 \sin^4 x + 4 \sin^3 x + \sin^2 x + \sin x + 1 = \sin^2 x (\sin^2 x + \sin x)^4 + \sin x + 1 \text{ from (i)}$$

$$= \sin^2 x + \sin x + 1$$

$$= 2$$

44. Answer (2)

Common focus

$$\Rightarrow a^2 = 4 - 3$$

$$\Rightarrow a = 1 (\because a > 0)$$

Point of intersection of curves

$$3x^2 + 16x - 12 = 0$$

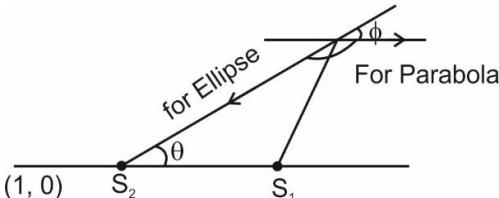
$$\Rightarrow 3x^2 + 15x - 2x - 10 = 0$$

$$\Rightarrow 3x(x + 6) - 2(x + 6) = 0$$

$$\Rightarrow x = \frac{2}{3} \cdot x = -6$$

$$\Rightarrow y = \pm 4 \sqrt{\frac{2}{3}}$$

Now, (Let S be common Source)

 S_2 is another focus of ellipse

$$\text{Now } \tan \phi = \frac{4 \sqrt{\frac{2}{3}}}{\frac{2}{3} + 1} = \frac{4 \sqrt{5}}{5}$$

$$\text{Now } \tan(\pi - \theta) = \frac{4\sqrt{6}}{5}$$

$$= \tan \theta = -\frac{4\sqrt{6}}{5}$$

$$\Rightarrow P = 5$$

45. Answer (6)

Let m be the slope of common Tangent

$$\Rightarrow m^2 + 2 = 2m^2 + p^2 = 3m^2 + 2p$$

$$\Rightarrow 2 - p^2 = p^2 - 2p$$

$$\Rightarrow 2p^2 - 2p - 2 = 0$$

$$\Rightarrow p^2 - p - 1 = 0$$

$$\Rightarrow p = \frac{\sqrt{5} + 1}{2}$$

Now

$$2p \sin 18^\circ$$

$$= \frac{(\sqrt{5} + 1)(\sqrt{5} - 1)}{4} = 1$$

46. Answer (6)

Let E be the event of obtaining one head and two tails in three tosser and A, B, C, D denote the events of selecting A, B, C and D respectively

$$P\left(\frac{E}{A}\right) = {}^3C_2 \times \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)^2 = \frac{3}{8}$$

$$P\left(\frac{E}{B}\right) = {}^3C_2 \times \left(\frac{1}{3}\right) \left(\frac{2}{3}\right)^2 = \frac{12}{27} = \frac{4}{9}$$

$$P\left(\frac{E}{C}\right) = {}^3C_2 \times \left(\frac{2}{3}\right) \left(\frac{1}{3}\right)^2 = \frac{6}{27} = \frac{2}{9}$$

$$P\left(\frac{E}{D}\right) = {}^3C_2 \times \left(\frac{3}{4}\right) \left(\frac{1}{4}\right)^2 = \frac{9}{64}$$

$$\text{Now } P\left(\frac{D}{E}\right) = \frac{P(D).P\left(\frac{E}{D}\right)}{\sum P\left(\frac{E}{D}\right).P(D)} = \frac{P\left(\frac{E}{D}\right)}{\sum P\left(\frac{E}{D}\right)}$$

$$\therefore P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$$

$$\Rightarrow P\left(\frac{D}{E}\right) = \frac{\frac{9}{64}}{\frac{3}{8} + \frac{4}{9} + \frac{2}{9} + \frac{9}{64}}$$

$$= \frac{\frac{9}{64}}{\frac{227}{64 \times 3}}$$

$$P = \frac{27}{227}$$

$$454P = 454 \times \frac{27}{227} = 54$$

47. Answer (2)

$$S_n = \frac{1}{(x+1)^2} \begin{vmatrix} 1+2x^2+x^2 & x+2x^3+x & 2x+1 \\ 2+4x^2+x & 2x+4x^3+1 & 2+2x+1 \\ x+2x+x^2 & x^2+2x^2+1 & x+1+x \end{vmatrix}$$

$$= \frac{1}{(x+1)^2} \begin{vmatrix} 1 & x & x \\ 2 & 2x & 1 \\ x & 1 & x \end{vmatrix} \begin{vmatrix} 1 & x & 1 \\ 2x & 2x^2 & 1 \\ x & 1 & 1 \end{vmatrix}$$

$$S_n = \frac{1}{(x+1)^2} \begin{vmatrix} 1 & x & 1 \\ 2 & 2x^2 & 1 \\ x & 1 & x \end{vmatrix}$$

$$= \frac{1}{(x+1)^2} \begin{vmatrix} 1 & x & x \\ 0 & 2x-1 & 2x^2-x \\ 0 & -x & 1-2x^2 \end{vmatrix}^2$$

$$= \frac{(2x-1)^2}{(x+1)^2} (x^2-1)^2$$

$$\text{Now } E_p = \sqrt{5p+1} - \sqrt{5p}$$

$$= 2((p+1)^2 - p^2) - 3((p+1) - p)$$

$$= 2(2p+1) - 3$$

$$= 4p - 1 \quad \text{A.P}$$

$$\text{Now, } \sum_{P=1}^{2020} E_p = \frac{2020}{2} (3 + 8079)$$

$$= 2020 \times 4041$$

$$\Rightarrow \left[\sum \frac{\epsilon_p}{(2020)^2} \right] = 2$$

48. Answer (5)

$$b-d = b^2$$

$$1 \ 1$$

$$2 \ 4$$

$$3 \ 9$$

Note : $\because b < d \Rightarrow b \neq d$

Also $d < e \Rightarrow$ max value of d can be 8

$\Rightarrow b \neq 3$

Hence only one possibility

i.e. $b = 2$ and $d = 4$

$\Rightarrow c = 3$

Hence $a \frac{2}{b} \frac{3}{c} \frac{4}{d} e$

$\Rightarrow a = 1$

\Rightarrow Total numbers will be total values taken by e and i.e. 5

49. Answer (B, D)

$$\because f'(y) > 0 \forall y \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \Rightarrow f\left(\frac{2\pi}{5}\right) < f\left(\frac{3\pi}{7}\right)$$

$$\therefore f(y) = \int_{\frac{\pi}{4}}^y \left((\tan x)^x - (\cot x)^x \right) \left(\frac{x}{\tan x} + \frac{\ln(\tan x)}{\sec^2 x} \right) \sec^2 x dx$$

$$= \int_1^{\tan y} \left(t^{\tan^{-1}t} - t^{-\tan^{-1}t} \right) \left(\frac{\tan^{-1}t}{t} + \frac{\ln t}{1+t^2} \right) dt$$

$$= \int_1^{\tan y} t^{\tan^{-1}t} \left(\frac{\tan^{-1}t}{t} + \frac{\ln t}{1+t^2} \right) dt - \int_1^{\tan y} t^{-\tan^{-1}t} \left(\frac{\tan^{-1}t}{t} + \frac{\ln t}{1+t^2} \right) dt \\ = \left[t^{\tan^{-1}t} + t^{-\tan^{-1}t} \right]_1^{\tan y} = (\tan y)^y + (\cot y)^y - 2$$

50. Answer (A, C, D)

Let $2^x = a, 3^x = b$

Given equation converts to

$$2a^2 + 2b^2 + a^2b^2 - 2ab - 4a - 4b + 5 = 0$$

$$\Rightarrow (a^2a + b^2b - 2ab) + (a^2b^2 - 2ab + 1) + (a^2 + b^2 - 4a - 4b + 2ab + 1) = 0$$

$$\Rightarrow (a - b)^2 + (ab - 1)^2 + (a + b - 2)^2 = 0$$

$$\Rightarrow 2^x = 3^x \Rightarrow x = 0$$

$$6^x = 1 \Rightarrow x = 0$$

$$2^x + 3^x - 2 = 0 \Rightarrow x = 0$$

$$\Rightarrow p = 0$$

$$\Rightarrow A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\text{Now } |A - \lambda I|$$

$$\Rightarrow \begin{bmatrix} 1-\lambda & 0 & 1 \\ 0 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{bmatrix} = 0$$

$$\Rightarrow (1-\lambda)(\lambda^2 - 1) + \lambda = 0$$

$$\Rightarrow \lambda^3 - \lambda^3 - 2\lambda + 1 = 0$$

$$\lambda \rightarrow A$$

$$\Rightarrow A^3 - A^2 - 2A + I_3 = 0$$

$$\Rightarrow \alpha = -1, \beta = -2, \gamma = 1$$

Also

$$\beta + \gamma = -1 = \alpha$$

$$\text{Also } |\alpha| + |\gamma| = 1 + 1 = 2 = |\beta|$$

51. Answer (A, C, D)

$$\vec{a} \times (\vec{b} \times \vec{c}) = (-x^2 - 4x + 1)\vec{b} + \cos y \vec{c}$$

$$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = (-x^2 - 4x + 1)\vec{b} + \cos y \vec{c}$$

$$\text{and } \vec{a} \cdot \vec{b} = -\cos y$$

$$\text{Also } \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = 6$$

$$-\cos y - x^2 - 4x + 1 = 6$$

$$-x^2 - 4x - 5 = \cos y$$

$$-(x^2 + 4x + 4) - 1 = \cos y$$

$$-(x+2)^2 - 1 = \cos y$$

$$\text{Range of } -(x+2)^2 - 1 = (-\infty, -1)$$

$$\text{Range of } \cos y = [-1, 1]$$

Hence solution is possible only when

$$\text{L.H.S} = \text{R.H.S} = -1$$

$$\Rightarrow x = -2 \text{ and } \cos y = -1$$

$$x = 2 \Rightarrow y = (2n+1)\pi, n \in \mathbb{Z}$$

$$(x, y) \equiv (-2, (2n+1)\pi); n \in \mathbb{Z}$$

Now

$$\text{Option (A) put } x = -2$$

$$-4 + y - \pi + 4 = 0$$

$$\Rightarrow y = \pi$$

$$(-2, \pi) \text{ satisfies}$$

$$(\text{E}) -2 - y + 2\pi + 2 = 0$$

$$\text{If } y = 2\pi$$

$$(-2, 2\pi) \text{ does not satisfy}$$

$$(\text{F}) x = -2 \text{ satisfy}$$

$$(\text{G}) -14 - y + 15\pi + 14 = 0$$

$$(-2, 15\pi) \text{ satisfies}$$

52. Answer (A, B, D)

$$x^3 - 2020x^2 + 2021 = (x+1)(x^2 - 2021x + 2021)$$

$$\alpha + \beta = 2021 \text{ and } \alpha \cdot \beta = 2021$$

$\because \alpha^n + \beta^n$ can be represented as a polynomial of $\alpha + \beta$ and $\alpha \cdot \beta$.

So, $a_n \in \mathbb{N}$

Also

$$\alpha^3 + 2021 = 2020\alpha^2$$

$$\Rightarrow \alpha^{n+3} + 2021\alpha^n = 2020\alpha^{n+2}$$

$$\Rightarrow a_{n+3} + 2021 a_n = 2020 a_{n+2}$$

$$\Rightarrow \frac{a_{n+3} + a_n}{a_{n+2} - a_n} = 2020$$

53. Answer (C, D)

$$\begin{aligned} \ln L &= \sum_{r=0}^n \frac{r+2n}{\sqrt{m}(r+n)} \ln \left(\sqrt{\frac{r}{n}} + \tan^{-1} \sqrt{\frac{r}{n}} \right) \\ &= \int_0^1 \frac{2+x}{1+x} \frac{\ln(\sqrt{x} + \tan^{-1} \sqrt{x})}{\sqrt{x}} dx \end{aligned}$$

$$\text{Let } \sqrt{x} + \tan^{-1} \sqrt{x} = t \Rightarrow \left(\frac{2+x}{1+x} \right) \frac{dx}{\sqrt{x}} = 2dt$$

$$\begin{aligned} \ln L &= 2 \int_0^{\frac{\pi}{4}} \ln t dt = 2t \ln \left(\frac{t}{e} \right) \Big|_0^{\frac{\pi}{4}} = \left(\frac{\pi+4}{2} \right) \ln \left(\frac{\pi+4}{4e} \right) \\ &L \left(\frac{\pi+4}{4e} \right)^{\left(\frac{\pi+4}{2} \right)} \end{aligned}$$

54. Answer (A, B, C)

$$y^2 = 16x$$

$$a = 4$$

Let two points $P(4t_1^2, 8t_1)$ and $Q(4t_2^2, 8t_2)$ on parabola PQ is normal chord

$$t_2 = -t_1 - \frac{2}{t_1}$$

$$\text{Normal chord} = PQ = \sqrt{a^2(t_1^2 - t_2^2)^2 + 4a^2(t_1 - t_2)^2}$$

$$PQ^2 = \left[16 \left[(t_1^2 - t_2^2)^2 + 4(t_1 - t_2)^2 \right] \right]$$

$$= 16(t_1 - t_2)^2 \left[(t_1 + t_2)^2 + 4 \right]$$

$$= 16 \left(t_1 + t_1 + \frac{2}{t_1} \right)^2 \left[\left(\frac{-2}{t_1} \right)^2 + 4 \right]$$

$$= 16 \left(2t_1 + \frac{2}{t_1} \right)^2 \left(\frac{4}{t_1^2} + 4 \right)$$

$$= 256 \frac{(t_1^2 + 1)^2}{t_1^2} \frac{(1 + t_1^2)}{t_1^2}$$

$$= 256 \frac{(t_1^2 + 1)^3}{t_1^4}$$

$$= 256 \left(\frac{1+t_1^2}{t_1^{4/3}} \right)^3$$

$$= 256 \left(t_1^{2/3} + t^{-4/3} \right)^3$$

$$PQ^2 = 256 \left(\frac{1}{2} t_1^{2/3} + \frac{1}{2} t_1^{2/3} + t_1^{4/3} \right)^3$$

$$PQ^2 \geq 256 \left(3\sqrt[3]{\frac{1}{4}} \right)^3$$

Using

$AM \geq GM$

$$PQ^2 \geq 256 \times \frac{27}{4}$$

Minimum value of $PQ = 24\sqrt{3}$

55. Answer (A, C, D)

(1) $z_1, z_3, z_5, z_7 \dots$ are always lies on the ray

emanating from origin at an angle $-\frac{2\pi}{3}$

Hence $z_1, z_3, z_5, z_7 \dots$ are on a line (a) $z_2, z_4, z_6 \dots$ are lies on Real axis at $(1, 0), (2, 0), (3, 0)$ respectively

$$\therefore \arg(z_3 - z_1) - \arg(z_4 - z_2) = -\frac{2\pi}{3}$$

56. Answer (A, D)

Sides are in A.P and $a < \min\{b, c\}$

\Rightarrow Order of A.P can be b, c, a or c, b, a

Case I: If $2c = a + b$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{b^2 + c^2 + (2c-b)^2}{2bc} = \frac{4b-3c}{2b}$$

Case II: If $2b = a + c$

$$\cos A = \frac{(b^2 + c^2) - (2b-c)^2}{2bc} = \frac{4c-3b}{2c}$$

57. Answer (A, B, C, D)

The given equation can be written as

$$y^2 + (2x - 20)y + 9x^2 - 92x + 244 = 0$$

$$D \geq 0$$

$$(2x - 30)^2 - 4(9x^2 - 92x + 244) \geq 0$$

$$\Rightarrow -8x^2 + 72x - 144 \geq 0$$

$$\Rightarrow x^2 - 9x + 18 \leq 0$$

$$\Rightarrow x \in [3, 6]$$

The given equation can be re-written as

$$= 9\sqrt{2} \int_{-\pi/2}^{\pi/2} \cos^2 \theta d\theta$$

$$= 9\sqrt{2} \times 2 \int_0^{\pi/2} \cos^2 \theta d\theta$$

$$= 18\sqrt{2} \times \frac{1}{2} \times \frac{\pi}{2} - \frac{9\sqrt{2}}{2} = \frac{9\pi}{\sqrt{2}} \text{ units}^2$$

58. Answer (A, B, C)

Let

$$\vec{a} = p\hat{i} + q\hat{j} + r\hat{k}$$

$$\because L_1 \perp_r L_2 \Rightarrow \vec{a}(\hat{i} - \hat{j} + \hat{k}) = 0$$

$$\Rightarrow p - q + r = 0 \quad \dots(i)$$

$$\text{Also } [\vec{a} \ \vec{b} \ \vec{c}] = 0$$

$$\Rightarrow \begin{vmatrix} p & q & r \\ 1 & 2 & 1 \\ -1 & 4 & 1 \end{vmatrix} = 0 \Rightarrow p + q = 3r \quad \dots(ii)$$

$$\text{Also } \vec{a} \cdot \vec{b} = 5 \Rightarrow 2p + q + r = 5 \quad \dots(iii)$$

From (i), (ii), (iii)

$$\Rightarrow p = 1, q = 2, r = 1 \quad \dots(iv)$$

$$\Rightarrow \vec{a} = \hat{i} + 2\hat{j} + \hat{k} \quad \dots(v)$$

$$\text{Now } |\vec{a}| = \sqrt{1+4+1} = \sqrt{6}$$

$$\vec{a} \cdot \vec{c} = (\hat{i} + 2\hat{j} + \hat{k})(-\hat{i} + 4\hat{j} + \hat{k})$$

$$= -1 + 8 + 1 = 8$$

$$\text{Also } |\vec{a} \times (\vec{b} \times \vec{c})| = |(\hat{i} + 2\hat{j} + \hat{k}) \times (-3\hat{i} - 3\hat{j} + 9\hat{k})|$$

$$= 3 \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 1 & 1 & -3 \end{vmatrix} = 3\sqrt{66}$$

59. Answer A - Q

B - P

C - Q

D - P

$$L = \lim_{x \rightarrow 0} \frac{\left(\sin^{-1} x - x\right)}{x^3} + \frac{(x - \sin x)}{x^3}$$

$$= \frac{\frac{2 \times 1}{6}}{2 \times -\frac{1}{3}} = -\frac{1}{2}$$

$$\text{Now } 2|L| = 2 \times \frac{1}{2} = 1$$

$$(Q) f(x) = \operatorname{sgn}(\{x\}^{2020} (1 - \{x\}^2))$$

$$\text{Now } 0, \{x\} < 1$$

$\Rightarrow \{x\}^{2020}$ will always be positive ..(i)

Also $1 - \{x\}^2 > 0 \quad [\because 0 \leq \{x\} < 1]$..(ii)

From (i) & (ii)

$\{x\}^{2020} (1 - \{x\}^2)$ will always between

$$\Rightarrow f(x) = 1$$

\Rightarrow No points of discontinuity

$$(R) \sqrt{1+2x} + \sqrt{2+x} = \sqrt{1-2x} + \sqrt{2-x}$$

Note $x = 0$ is a solution

Now

$$f(x) = \sqrt{1+2x} + \sqrt{2+x}$$

$$f'(x) = \frac{2}{2\sqrt{1+2x}} + \frac{1}{2\sqrt{2+x}} > 0$$

$$\text{Since } g(x) = \sqrt{1-2x} + \sqrt{2-x}$$

$$g'(x) < 0$$

$\therefore f(x)$ is always increasing and $g(x)$ is always decreasing

\Rightarrow only one solution $x = 0$

$$(S) \text{ R.H.S will always be an Integer}$$

\Rightarrow L.H.S must also always be an integer

Now For L.H.S to be integer (i.e. only 0)

But its given $x \neq$ Integer

\Rightarrow No solution

60. Answer A – Q

B – P

C – S

D – S

(A) According to Cauchy Schwarz Inequality

$$(l_1 m_2 + m_1 n_2 + n_1 l_2)^2 \leq (l_1^2 + m_1^2 + n_1^2)(m_2^2 + n_2^2 + l_2^2)$$

$$\text{Now } El_1^2 = 1, El_2^2 = 1$$

\Rightarrow Maximum value of $l_1 m_2 + m_1 n_2 + n_1 l_2$ is 1

Q. $\bar{p}, \bar{q}, \bar{r}$ are coplanar

$$\Rightarrow \begin{bmatrix} \lambda & 1 & 2\lambda \\ 1 & \lambda & 1 \\ 1 & 1 & 2\lambda \end{bmatrix} = 0$$

$$\Rightarrow \lambda(2\lambda^2 - 1) - (2\lambda - 1) + 2\lambda(1 - \lambda) = 0$$

$$2\lambda^3 - \lambda - 2\lambda + 1 + 2\lambda - 2\lambda^2 = 0$$

$$2\lambda^3 - 2\lambda^2 - \lambda + 1 = 0$$

$$2 = 1 \text{ is one solution} \quad (\text{i})$$

$$2\lambda^3 - 2\lambda^2 - \lambda + 1 = (\lambda - 1)(2\lambda^2 - 1)$$

$$\text{Other values of } \lambda = 1 - \frac{1}{\sqrt{2}}$$

$$\Rightarrow |\lambda| = \frac{1}{\sqrt{2}}$$

$$\Rightarrow [\lambda] = 0$$

\Rightarrow Number of values of $[\lambda] = 2$

(R) According to Cauchy Schwanz inequality

$$\frac{x_1^2}{y_1} + \frac{y_2^2}{y_2} \geq \frac{(x_1 + x_2)^2}{y_1 + y_2}$$

$$\Rightarrow \frac{\sin^4 \theta}{(9/5)} + \frac{\cos^4 \theta}{(16/5)} \geq \frac{(\sin^2 \theta + \cos^2 \theta)^2}{\frac{9}{5} + \frac{16}{5}} \geq \frac{1}{5}$$

$$\Rightarrow P = 5$$

(S) For no sides common

$${}^n C_3 - n - n(n-4) = 0$$

$$\Rightarrow \frac{n(n-1)(n-2)}{6} - n - n(n-4) = 6$$

$$\Rightarrow n \left[\frac{n^2 - 3n + 2}{6} - 1 - n + 4 \right] = 0$$

$$\Rightarrow n \left[\frac{n^2 - 3n + 2}{6} + 3 - n \right] = 0$$

$$\Rightarrow n[n^2 - 3n + 2 + 18 - 6n] = 0$$

$$\Rightarrow n[n^2 - 9n + 20] = 0$$

$\Rightarrow x = 0$ not possible

$n = 4$

$n = 5$

\Rightarrow Maximum value of n is 5

