Test Date: 05/07/2020

B
CODE

Aakash
Medical|IIT-JEE|Foundations
(Divisions of Aakash Educational Services Limited)

Regd. Office: Aakash Tower, 8, Pusa Road, New Delhi-110005, Ph.011-47623456

Mock Test_CoE_XII

for JEE (Advanced) – 2020

Test – IA (Paper – II)_Actual Pattern-2015

ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(2)	21.	(4)	41.	(9)
2.	(5)	22.	(9)	42.	(4)
3.	(4)	23.	(7)	43.	(2)
4.	(8)	24.	(6)	44.	(5)
5.	(6)	25.	(6)	45.	(6)
6.	(3)	26.	(4)	46.	(4)
7.	(8)	27.	(4)	47.	(1)
8.	(5)	28.	(5)	48.	(0)
9.	(A, B, C, D)	29.	(A, C, D)	49.	(B, C, D)
10.	(A, B, C, D)	30.	(A, B, C, D)	50.	(A, B, C, D)
11.	(A, C, D)	31.	(B, C)	51.	(A, B, C)
12.	(B, C, D)	32.	(A, B, C, D)	52.	(A, B)
13.	(B, C, D)	33.	(A, B, D)	53.	(A, C, D)
14.	(A, B, D)	34.	(B, C)	54.	(A, B, C)
15.	(A, C, D)	35.	(A, B, D)	55.	(A, B)
16.	(A, B, C, D)	36.	(A, C)	56.	(A, B)
17.	(A, B, D)	37.	(B, D)	57.	(A)
18.	(A, B, C)	38.	(C)	58.	(C)
19.	(B, D)	39.	(B, C)	59.	(D)
20.	(A, C)	40.	(A, D)	60.	(B)

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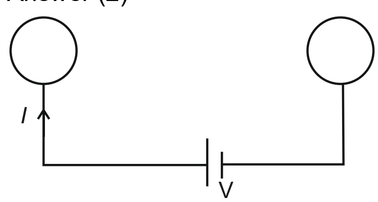
Test – 1A (Paper – II)_Actual Pattern-2015

ANSWERS & SOLUTIONS

PART –I : Physics

1. Answer (2)



For one sphere,

$j \times 4\pi r^2 = I$

$\Rightarrow \frac{1}{\rho} E \times 4\pi r^2 = I$

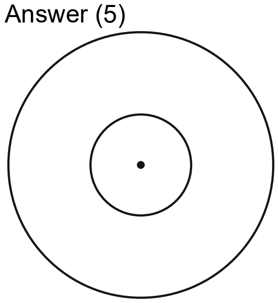
$\Rightarrow E = \frac{\rho I}{4\pi r^2}$

$\therefore V = \int_a^{\infty} E\,dr \times 2$

$\Rightarrow \frac{V}{I} = \frac{\rho}{2\pi a}$

$\Rightarrow n = 2$

2. Answer (5)



$I_C = \frac{5}{2} MR^2$

$Mg \times 2R = \left(\frac{5}{2}MR^2 + M.4R^2\right) \times \left(\frac{a_C}{2R}\right)$

$\Rightarrow 2g = \frac{13}{4} a_C$

$\Rightarrow a_C = \frac{8g}{13}$

$\therefore 2T = M(g - a_C) = \frac{5Mg}{13}$

$T = \frac{5Mg}{26}$

3. Answer (4)

$Q_{initial}$ (on shell's outer surface) $= 3Q$

$Q_{final} = q$ (let)

Then $\frac{kq}{r} + \frac{k \times (3Q)}{3r} = 0$

$\Rightarrow q = -Q$

$\therefore \Delta Q = 4Q$

4. Answer (8)

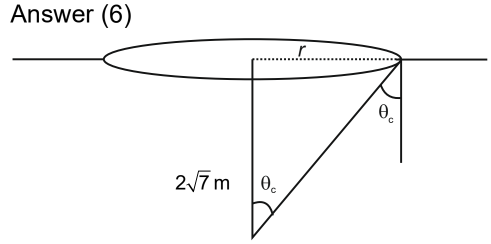
$\frac{dQ}{dt} = \frac{KA(0+4)}{y}$

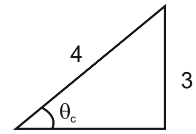
And, $dQ = mL = (\rho A dy) \times L$

$\Rightarrow \frac{dy}{dt} = \frac{k \times 4}{\rho L y}$

$\Rightarrow \Delta t = \frac{\rho L \times 1^2}{2k \times 4} = \frac{\rho L}{8k}$

5. Answer (6)





$\sin\theta_C = \frac{1}{\mu} = \frac{3}{4}$

$\therefore r = 2\sqrt{7}\tan\theta_C$

$= 2\sqrt{7} \times \frac{3}{\sqrt{16-9}}$

= 6 m

6. Answer (3)

$$k_{eq} = \frac{\left(\frac{k}{2}\right)\times(k)}{\left(\frac{k}{2}+k\right)} = \left(\frac{k}{3}\right)$$

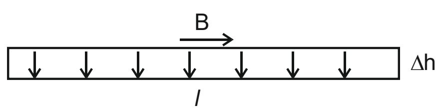
$$\therefore T = 2\pi\sqrt{\frac{3m}{k}}$$

k_{eq} is deduced using the constraint relationships

7. Answer (8)

B on elementary layer = $\frac{\mu_0 nI}{2}$

$$\therefore \Delta F = (nI)(\Delta h)\times\left(\frac{\mu_0 nI}{2}\right)$$



$$\therefore \frac{\Delta F}{\Delta A} = \frac{\Delta F}{l\Delta h} = \frac{\mu_0 n^2 I^2}{2}$$

$\therefore \lambda = 8$

8. Answer (5)

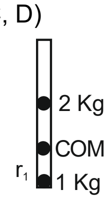
$$v_0 = \frac{1}{4l}\times v = \frac{1}{4\times 0.45}\times(360)$$

$= 200\,\text{Hz}$

$\therefore \nu = 200$ Hz, 600 Hz, 1000 Hz ,1400 Hz, 1800 Hz

$\therefore$ N = 5

9. Answer (A, B, C, D)



$$V_{cm} = \frac{1\times 6}{1+2} = 2 \text{ m/s}$$

$$r_1 = \frac{2\times 1.5}{(1+2)} = 1 \text{ m}$$

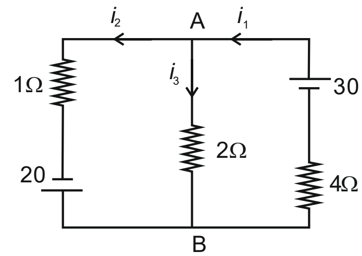
$\therefore 6 \times 1 \times 1 = I_{COM} \times \omega = 3 \times \omega \Rightarrow \omega = 2$ rad/s

$\therefore V_P = 2 + 2 \times 1 = 4$ m/s

$$\Delta_{Loss} = \frac{1}{2}\times 1\times 6^2 - \frac{1}{2}\times 3\times 2^2$$

= 12 J

10. Answer (A, B, C, D)



$$i_{(3\Omega)} = \frac{20}{3}A$$

$$\in_{AB} = \frac{20\times 4 - 30\times 1}{5}$$

= 10 V

$$\therefore i_{(2\Omega)} = \frac{10}{2+\frac{4\times 1}{4+1}} = \frac{25}{7}A$$

$$\therefore i_{(4\Omega)} = \frac{\frac{25}{7}\times 2 + 30}{4} = \frac{65}{7}A$$

$$\therefore i_{(1\Omega)} = \frac{90}{7}A$$

11. Answer (A, C, D)

$$\frac{1}{f_e} = (1.5-1)\left(\frac{1}{20}+\frac{1}{30}\right) = \frac{1}{24}$$

$$\therefore \frac{1}{f_e} = \frac{2}{24} + \frac{1}{15} = \frac{9}{60}$$

$\Rightarrow f_e = \frac{60}{9}$ cm $\Rightarrow$ Image is real

$$\therefore V = -\frac{\left(10\times\frac{60}{9}\right)}{\left(10-\frac{60}{9}\right)} = -20 \text{ cm}$$

$$P = \frac{100}{\left(\frac{60}{9}\right)} = 15 \text{ D}$$

12. Answer (B, C, D)

$$\Delta U = nC_V\Delta T = 1\times\frac{5R}{2}\times T_0$$

TV^2 = Constant $\Rightarrow$ PV^3 = Constant

$$\therefore C = \frac{5R}{2} - \frac{R}{3-1} = 2R$$

$$\therefore \Delta Q = 1\times 2R\times T_0 = 2RT_0$$

$$\therefore \Delta W = \Delta Q - \Delta U = 2RT_0 - \frac{5RT_0}{2}$$

$= -\frac{RT_0}{2}$

13. Answer (B, C, D)

$\in = \frac{1}{2}B\omega l^2$ and O is at higher potential

$\therefore V_0 - V_A = \frac{1}{2}B\omega L^2$

$\therefore V_0 - V_B = \frac{1}{2}B\omega \times \left(\sqrt{2}L\right)^2 = B\omega L^2$

$\therefore V_A - V_B = \frac{1}{2}B\omega L^2$

14. Answer (A, B, D)

L. C $= \frac{0.5}{100}$ mm $= 0.005$ mm

Zero error = 2 × 0.005 mm (positive)

$\therefore$ Diameter = 1.5 + 70 × 0.005

15. Answer (A, C, D)

From graph

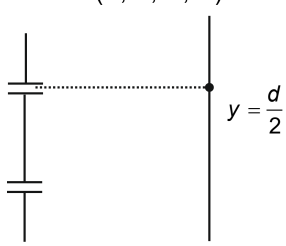
$T_{1/2}(B) < T_{1/2}(A)$

$\Rightarrow \lambda_B > \lambda_A$

$\because T_{av} = \frac{1}{\lambda} \Rightarrow T_{av}(B) < T_{av}(A)$

And $N_0(A) > N_0(B)$

16. Answer (A, B, C, D)



$\Delta\phi = \frac{2\pi}{\lambda}\left(\frac{d}{2} \times \frac{d}{D}\right)$ can be zero intensity

$I = \frac{I_m}{2}$ at $y = \frac{\beta}{4}, \frac{3\beta}{4}$.....

$\therefore (\Delta y)_{\min} = \frac{\beta}{2} = \frac{\lambda D}{2d}$

Now,

$\frac{I_m}{4} = I_m \cos^2\left(\frac{\delta}{2}\right) \Rightarrow \cos\left(\frac{\delta}{2}\right) = \frac{1}{2} \Rightarrow \frac{\delta}{2} = \frac{\pi}{3}$

$\therefore y_1 = \frac{\beta}{3}, \therefore y_{\min} = 2 \times \left(\frac{\beta}{2} - \frac{\beta}{3}\right) = \frac{\beta}{3} = \frac{\lambda D}{3d}$

17. Answer (A, B, D)

$\omega_0 = \frac{V_0}{R}$

$\left(\frac{2}{5}MR^2\right) \times \frac{V_0}{R} = \left(\frac{7}{5}MR^2\right) \times \frac{V_A}{R}$

$\Rightarrow V_A = \frac{2V_0}{7}$

And, $MV_0R = \frac{7}{5}MR^2 \times \frac{V_B}{R}$

$\Rightarrow V_B = \frac{5V_0}{7}$

Work done friction is negative on both

18. Answer (A, B, C)

$\left(\frac{2}{3}MR^2\right) \times \left(\frac{V_0}{R}\right) = \frac{5}{3}MR^2 \times \frac{V_A}{R}$

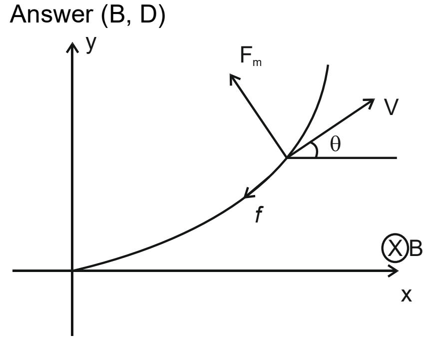
$\Rightarrow V_A = \frac{2V_0}{5}$

And, $MV_0R = \frac{5}{3}MR^2 \times \frac{V_B}{R}$

$\Rightarrow \frac{3V_0}{5} = V_B$

Work done by friction is negative on both

19. Answer (B, D)



$\frac{V\,dV}{dS} = -\frac{bV}{m}$

$\Rightarrow \int_{V_0}^{0} -dV = \frac{b}{m} \times S \Rightarrow \frac{mV_0}{b} = 10$ ..(i)

$\therefore -\frac{dV_x}{dt} = \frac{quB\sin\theta}{m} + \frac{bv}{m}\cos\theta$ …(ii)

And $\frac{dv_y}{dt} = \frac{qvB\cos\theta}{m} - \frac{bv}{m}\sin\theta$ …(iii)

Solving these, $y = \frac{qB}{m}x$

And, $x^2 + y^2 = 6^2 = 36$ …..(iv)

Solving these, $B_0 = \frac{4b}{3q}$

20. Answer (A, C)

$B = 2B_0 = 2 \times \frac{4b}{3q}$

$y = \frac{qB}{m}x$ and $\frac{mv_0}{b} = 10$

$$\therefore x = \frac{\frac{mv_0}{B}}{1 + \frac{q^2B^2}{b^2}} = \frac{10}{1 + \frac{64}{9}} = \frac{90}{73}$$

$$\therefore y = \frac{8}{3}x$$

$$\therefore d = \sqrt{x^2 + y^2} = \frac{30}{\sqrt{73}}m$$

PART –II : CHEMISTRY

21. Answer (4)

Mean free path (λ)

$$\lambda = KT/\left(2^{1/2}\sigma P\right)$$

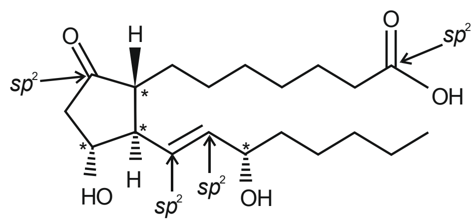
$$\sigma = \pi d^2 \Rightarrow \pi \times \left(4 \times 10^{-10}\right)^2$$

$\Rightarrow 5 \times 10^{-19}\ m^2$

$$\lambda = \frac{\left(1.4 \times 10^{-23}\ JK^{-1}\right) \times 298\ K}{2^{1/2} \times \left(5 \times 10^{-19}\ m^2\right) \times \left(1 \times 10^{-9}\ torr\right) \times \left(\frac{1 atm}{760\ torr}\right) \times \left(\frac{1.013 \times 10^5\ Pa}{1\ atm}\right)}$$

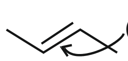
$= 4.5 \times 10^4$ m

22. Answer (9)

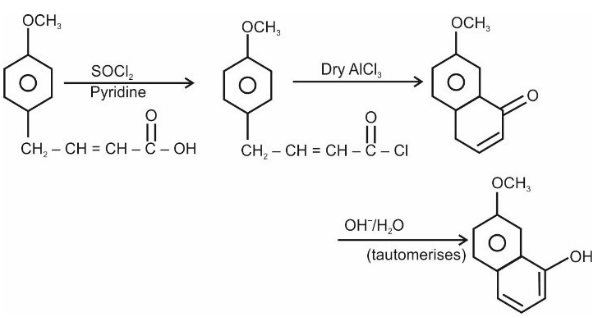


Asymmetric centres are 4

sp^2(C) = 4

*  (E) Configuratior

23. Answer (7)



24. Answer (6)

$$\Delta E_{3\to 2} = E_3 - E_2 = -\left(\frac{A}{3^2} - \frac{A}{2^2}\right)$$

$$= 2.179 \times 10^{-18}\ J\left(\frac{1}{4} - \frac{1}{9}\right) \Rightarrow 0.3026 \times 10^{-18}\ J$$

From planks equation, E = hν

$$\nu = \frac{0.3026 \times 10^{-18}\ J}{6.626 \times 10^{-34}\ JS} \Rightarrow 0.457 \times 10^{15}\ HZ$$

$$\bar{\nu} = \frac{\nu}{c} = \frac{0.457 \times 10^{15} S^{-1}}{3 \times 10^8\ m/s} \Rightarrow 1.52 \times 10^6\ metre^{-1}$$

$\therefore$ Value of P = 6

25. Answer (6)

Acidified $K_2Cr_2O_7$, $CuSO_4$, H_2O_2, Cl_2, O_3, $FeCl_3$ Oxidise aq. Iodide to iodine. Alkaline $KMnO_4$ Oxidise aqueous iodide to IO_3^-

* $Na_2S_2O_3$ is a strong reducing agent which on reaction with I_2 produces I^-

$Na_2S_2O_3 + I_2 \longrightarrow 2I^- + Na_2S_4O_6$

26. Answer (4)

Volume of O_2 = $375 \times \frac{24}{100} \Rightarrow 90$ ml

Volume of gas Hydrocarbon = 15 ml

$$CxHy + \left(x + \frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O(l)$$

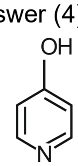
$15\left(x + \frac{y}{x}\right)O_2 = 90 \ \therefore x + \frac{y}{4} = 6$ ....(i)

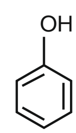
Excess air = 375 – 90 = 285

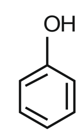
Volume of CO_2 + 285 = 345 $\therefore$ Vol. of CO_2 = 60

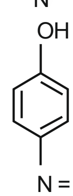
x = 4 and $x + \frac{y}{4} = 6 \ \therefore y = 8$

27. Answer (4)

(A) 

(B) 

(C) 

(D) 

28. Answer (5)

$$K_0 = \frac{A_0}{2 \times t_{1/2}} \text{ and } K_1 = \frac{0.693}{t_{1/2}}$$

We get $K_0 = \frac{1.386}{2 \times 20}$ ....(i)

And $K_1 = \frac{0.693}{40}$ ....(ii)

$$\frac{K_1}{K_0} = \frac{0.693}{40} \times \frac{2 \times 20}{1.386} = 0.5 \text{ mol}^{-1} \text{ dm}^3$$

$$\frac{x}{10} = 0.5 \therefore x = 5$$

29. Answer (A, C, D)

(A) $H_2C_2O_4$ (acid) reacts with KOH (Base)

$$H_2\overset{+3}{C_2}O_4 + KMnO_4 \rightarrow CO_2 + Mn^{+2}$$

(B) KOH(S.B) titrates $LiHC_2O_4$ (salt) with n-factor = 1

n-factor = 1 $\therefore$ eq.wt $= \frac{M.W}{1}$

$\therefore$ Statement 'B' incorrect

(C) $Fe^{2+} + MnO_4^- \xrightarrow{H^+} Fe^{3+} + Mn^{2+}$

n = 1 n = 5

$Fe^{2+} + Cr_2O_7^{2-} \xrightarrow{H^+} Fe^{3+} + Cr^{3+}$

n = 1 n = 6

To oxidise 1 mol (or/equiv.) of Fe^{2+} ion,

$$\frac{\text{No. of moles of } Cr_2O_7^{2-}}{\text{No. of moles of } MnO_4^-} = \frac{5}{6}$$

(D) No. of equivalents of HCl = No. of equivalents of $K_2C_2O_4$

$\therefore$ 1 × 0.12×8 ×10^{-3} L = 2 × M × 10 × 10^{-3}

$\therefore$ M = 4.8 × 10^{-2}M

meq $KMnO_4$

= 5 × 0.12 × 9.6 × 10^{-3} L = 9.6 × 10^{-4}

meq $K_2C_2O_4$ = [2 × 4.8 × 10^{-2} × 10 × 10^{-3}]

= 9.6 × 10^{-4}

30. Answer (A, B, C, D)

(A) Polar Aprotic Solvent favour S_N2

(B) Polar Protic Solvent favour S_N1 elimination is not possible

(C) E_2, methoxide ion is a strong base

(D) Polar Protic solvent favour S_N1

31. Answer (B, C)

$h\nu = W_0 + K.E$ (K.E of e^- does not depend on intensity of light)

32. Answer (A, B, C, D)

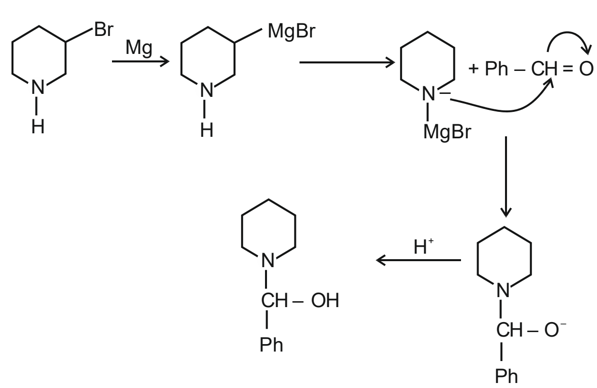
(A) Reversible adiabatic process is iso-entropic process

(B) When $(\Delta G_{system})_{T,P} < 0$ then process may be Endo/Exothermic

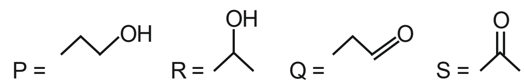
(C) dG = VdP – SdT considering only PV work

spontaneous Reaction

33. Answer (A, B, D)



34. Answer (B, C)



35. Answer (A, B, D)

(A) $\rightarrow$ In liq. State (NO) dimerises into $(NO)_2$ and odd e^- disappears giving diamagnetic property

$$\underset{(\text{Paramagnetic})}{2NO} \longrightarrow \underset{(\text{Diamagnetic})}{O = N - N = O}$$

(B) If central atom belongs to same group and peripheral atoms are same then bond angle decreases down the group

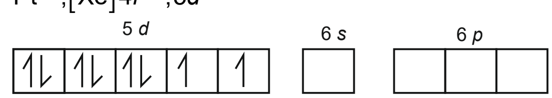
(C) Oxidising power of halogen: $F_2 > Cl_2 > Br_2 > I_2$

(D) Fe is more electropositive than Hydrogen, displace H^+ ions from acid solution as:

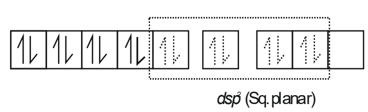
$Fe + 2HCl \longrightarrow FeCl_2 + H_2 (\uparrow)$

36. Answer (A, C)

$Pt^{+2}, [Xe]4f^{14}, 5d^8$



$[Pt(NH_3)Cl_2]$



37. Answer (B, D)

$$M(S) + M^+(aq, 1M) \longrightarrow M^+(aq, 0.05 M) + M(S)$$

$$E_{cell} = 0 - \frac{2.303RT}{F}\log\frac{0.05}{1} > 0$$

Hence $|E_{cell}| = E_{cell}$ = 0.070 V and $\Delta G < 0$ for

38. Answer (C)

$$E_{cell} = E° - \frac{0.0538}{1}\log 0.0025$$

$\Rightarrow 0.139$ V $\simeq 140$ mV

39. Answer (B, C)

A is diamagnetic, square planar complex because of strong ligand CN^-

B is Paramagnetic, tetrahedral because of weak ligand Cl^-

40. Answer (A, D)

P $\rightarrow dsp^2$

Q $\rightarrow sp^3$

PART –III : Mathematics

41. Answer (9)

Let no. of men be n

Total number of games men played with women

$= \left({}^3C_1 \times {}^3C_1\right)3 = 3(3n)$

Now $= 3\left({}^3C_2\right) - 3(3n) = 27$

$$\frac{n(n-1)}{2} - 3n = 9$$

42. Answer (4)

Given ellipse $\frac{x^2}{4} + \frac{y^2}{8} = 1$

$\tan^4 x + \cot^4 x + 3 = 5\sin^2 y$

Consider $\frac{\tan^4 x + \cot^4 x}{2} = \sqrt{\tan^4 x + \frac{1}{\tan^4 x}}$

$\tan^4 x + \cot^4 x \geq 2$

$\tan^4 x + \cot^4 x + 3 \geq 5$

$5\sin^2 y \geq 5$

$\sin^2 y \geq 1$

$\Rightarrow \sin y = \pm 1$

$y = (2n+1)\frac{\pi}{2}$ and $x = (2n+1)\frac{\pi}{4}$

So points satisfying given condition are $\left(\pm\frac{\pi}{4}, \pm\frac{\pi}{2}\right)$

Total points 4

43. Answer (2)

Given lines in carter ion form

$$L_1 = \frac{x-0}{1} = \frac{y+1}{1} = \frac{z-0}{1} = \lambda$$

$$L_2 = \frac{x+1}{2} = \frac{y-0}{1} = \frac{z-0}{1} = \mu$$

$\therefore$ any point on L_1 be P $(\lambda, \lambda - 1, \lambda)$

L_2 be Q$(2\mu - 1, \mu, \mu)$

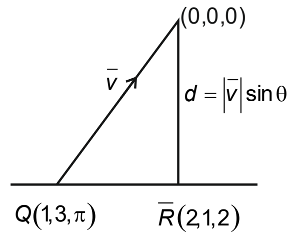
Dr's of line joining PQ = Dr's at given line

$(2m - 1 - \lambda, \mu - \lambda + 1, \mu - \lambda) = (2, 1, 2)$

On compiling and simplifying

$\mu = 1$ $\lambda = 3$

$\therefore$ P(3, 2, 3) Q(1,1,1)



$$\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-1}{2}$$

$$d = \frac{|\bar{v} \times \bar{c}|}{|\bar{c}|} = \frac{v_2}{3}$$

$$d^2 = \frac{2}{9} = 0.22$$

44. Answer (5)

For $\log_3(ax^2 + 4x + a)$ to be valid

$ax^2 + 4x + a > 0$ $\forall x \in R$

$a > 0$ and $4 - a^2 < 0 \Rightarrow a > 2$

Given inequality $\log_3\left(\frac{ax^2 + 4x + a}{x^2 + 1}\right) \geq 1$

$ax^2 + 4x + a \geq 3\left(x^2 + 1\right)$

$(a-3)x^2 + 4x + a - 3 \geq 0$ $\forall x \in R$

For above expression to be true

$a > 3$ and $\Delta \leq 0$

$16 - 4(a-3)^2 \leq 0$

$a > 3$ and $a - 3 \geq 2 \Rightarrow a \geq 25$

so $a \geq 5$

45. Answer (6)

$n(E_2) = a_{c_2} = 36$

$n(E_1 \cap E_2) = 3 \times 3 + 3 \times 3 = 18$

$$\therefore P\left(\frac{E_1}{E_2}\right) = \frac{18}{36}$$

46. Answer (4)

Let $x = n + f$

$$\Rightarrow \frac{1}{n} + \frac{1}{2n + 2f} = f + \frac{1}{3}$$

Case 1: $0 \leq f < \frac{1}{2}$

$\Rightarrow \frac{1}{n}+\frac{1}{2n}=f+\frac{1}{3}$

$\Rightarrow \frac{1}{n}+\frac{1}{2n}=f+\frac{1}{3}$

But $n = 1 \Rightarrow f > \frac{1}{2}$

$\therefore n = 2,3,4$

Case ii : $\frac{1}{2} \le f < 1$

$\Rightarrow \frac{1}{n}+\frac{1}{2n+1}=f+\frac{1}{3} \ge \frac{1}{2}+\frac{1}{3}$

$\Rightarrow 10n^2-13n-16 \le 0$

$\Rightarrow n = 1$ and $f = 1$

Not possible

47. Answer (1)

Two distinct tangents exist when one point of contact is point of inflection let it be P

$\therefore$ P = (2, 2b –a – 16)

tangent at P : y = (b – 12)x – a + 8 …(i)

Let Q – (2 + h, 3h – 1)

On lies an 3x – y = 7 ….(ii)

(i) & (ii) represents same

$\Rightarrow$ a = 15, b = 15

48. Answer (0)

Let P(x) = $a_0 x^4$ + …..+ a_4

P'(1) = 0, P'(2) = 0

Also $\lim_{x\to 0}\left(\frac{P(x)}{x^2}\right)=1 \Rightarrow a_x=0, a_3=0$ and

$a_2=1, a_1=-1, a_0=\frac{1}{4}$

$\therefore P(x)=\frac{1}{4}x^4-x^3+x^2$

$\Rightarrow P(2)=0$

49. Answer (B, C, D)

r^{th} term $T_r=\frac{8r}{4r^4+1}$

$=\frac{8r}{(2r^2+2r+1)(2r^2-2r+1)}$

$=2\left\{\frac{2r^2+2r-1-(2r^2-2r+1)}{(2r^2+3r+1)(2r^2-3r+1)}\right\}$

$=2\left\{\frac{1}{2r^2-2r+1}-\frac{1}{2r^2+2r+1}\right\}$

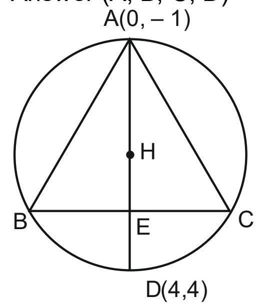
$=2\left\{\frac{1}{v_r}-\frac{1}{v_{r+1}}\right\}$

$\sum_{r=1}^{\infty}T_r=2\left[\left(\frac{1}{1}-\frac{1}{5}\right)-\left(\frac{1}{5}-\frac{1}{13}\right)+....\right]$

= 2

$1<\sum_{r=1}^{n}T_r<2$

50. Answer (A, B, C, D)



Slope of BC = $\frac{-1}{\text{Slope of H.D}}=\,=\frac{-1}{2}$

2y – 10 = –x + 3

x + 2y = 13

If ΔABC is equilateral then

Orthocentre = Centroid

If A(n, k) then H divides A, E in 2 : 1 ratio

$\left(\frac{6+h}{3},\frac{10+k}{3}\right)=(2,3)$

h = 0 k = –1

A(0, –1)

E = Midpoint of HD (3,5)

In radius $=r=\frac{R}{2}=\frac{AH}{2}=\sqrt{5}$

Area of triangle $=\frac{L^2}{\sqrt{3}}=15\sqrt{3}$

51. Answer (A, B, C)

$\because 1\le|\sin x|+|\cos x|\le\sqrt{2}$

$\Rightarrow [|\sin x|+|\cos x|]=1$

For f(x) to be defined

$\sin^2 x + 2\sin Ax + \frac{11}{4} \ge 2$

$(\sin x+1)^2 \ge \frac{1}{4}$

$\Rightarrow \sin x+1\ge\frac{1}{2}$ or $\sin x+1\le-\frac{1}{2}$

$\sin x\ge-\frac{1}{2}$ or $\sin x\le-\frac{3}{2}$(not possible)

52. Answer (A, B)

$\lim_{x\to 1^+}h(x)=\lim_{x\to 1^+}\left\{\lim_{n\to\infty}\frac{x^m f(x)+g(x)+3}{2x^n+4x+1}\right\}$

$\lim_{n\to\infty}\lim_{x\to 1^+}\left\{\frac{f(x)+\frac{g(x)+3}{x^m}}{2+\frac{4x+1}{x^n}}\right\}=\frac{f(1)}{2}$

$f(1) = 2e^2$

Similarly $\lim_{x\to 1^-}g(x)=\frac{h(1)+3}{5}$

$n(1) = 5e^3 - 3$

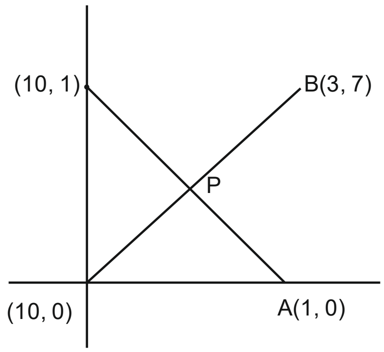
52. Answer (A, C, D)

Let $z = x + iy$

$f(x + iy) = |x + iy| + |x - 1 + iy| + |x + (y - 1)i| + |x - 3 + (y - 4)i|$

Let 0(0, 0) A(1, 0) B(3, 4) C(0, 1)

Now for f(z) to be minimum (x, y) must be point of interaction of OB and AC



$\therefore f(z)_{\min} = OB + AC$

$= 5 + \sqrt{2}$

Which occurs at point of interaction $P(x, y) = \left(\frac{3}{7}, \frac{4}{7}\right)$

53. Answer (A, B, C)

$x = 0, y = 1 \Rightarrow f(1) = 0$

Now $f(x + h)\ \ f(x)\, f(h) = f(xh + 1)$

$$\Rightarrow \frac{f(x+h)-f(x)}{h} + \frac{f(x)[f(h)+1]}{h} = \left[\frac{f(xh+1)-f(1)}{xh}\right]x$$

taking $h \to 0$

$\Rightarrow f'(x) + f(x)f'(0) = xf'(1)$

$\Rightarrow f'(x) + f(x) = x$

$\Rightarrow e^x[f'(x) + f(x)] = e^x x$

$\Rightarrow$ f(x) = x – 1

54. Answer (A, B, C)

$x = 0, y = 1 \Rightarrow f(1) = 0$

Now $f(x + h) + f(x)\, f(h) = f(xh + 1)$

$$\Rightarrow \frac{f(x+h)-f(x)}{h} + \frac{f(x)[f(h)+1]}{h} = \left[\frac{f(xh+1)-f(1)}{xh}\right]x$$

taking $h \to 0$

$\Rightarrow f'(x) + f(x)f'(0) = xf'(1)$

$\Rightarrow f'(x) + f(x) = x$

$\Rightarrow e^x[f'(x) + f(x)] = e^x x$

$\Rightarrow$ f(x) = x – 1

55. Answer (A, B)

As maximum of f(x) = 6

$\therefore$ f(x) = 6 has equal roots

$\Rightarrow \frac{x^2 - 2x + d}{x^2 + 3x + d} = 6$ has equal roots

$\Rightarrow$ d = 4 and $a = \frac{q}{7}$

56. Answer (A, B)

Put x = 0

$\Rightarrow \int_0^1 f(t)e^{-1}\, dt = \frac{1}{a}$

$\Rightarrow \int_0^x f(t)e^x - e^{2x}$

$\Rightarrow f(x) = e^x - e^{2x}$

57. Answer (A)

Let equation of circle s be $(x - r)^2 + (y - r)^2 = r^2$

It passes through (α, β) for $r = r_1$ and $r = r_2$

For orthogonal of two circles

$2(-r_1)(-r_2) + 2(-r_1)(-r_2) = r_1^2 + r_2^2$

$(r_1 + r_2)^2 = 6r_1r_2$

$4(\alpha + \beta)^2 = 6(\alpha^2 + \beta^2)$

$\alpha^2 + \beta^2 = 4\alpha\beta$

58. Answer (C)

Let $S_i = x^2 + y^2 - 2r_i(a + y) + r_i^2 = 0$ where i = 1, 2

Equation of common chord

$2(r_2 - r_1)(x + y) = r_1^2 + r_2^2 = 0$

$2(x + y) = (r_1 + r_2) = 2(\alpha + \beta)$

$x + y = \alpha + \beta$

59. Answer (D)

Substitute $x = \sqrt{i}\sqrt{t}$

We get $a = \sqrt{j}\int_0^\infty \frac{e^{-jt}}{2\sqrt{t}}dt$

$$\int_0^\infty \frac{e^{-jx}}{\sqrt{x}}dx = \frac{2a}{\sqrt{i}}$$

60. Answer (B)

$$\frac{d}{d\alpha}(f(\alpha)) = \sin\alpha x \frac{e^{-x2}}{2}\int_0^\infty - \frac{\alpha}{2}\int_0^\infty e^{-x^2}\cos\alpha x\, dx$$

$$f'(ga) = \frac{-\alpha}{2}f(\alpha) \Rightarrow f(\alpha) = ke^{-\frac{\alpha^2}{4}}$$

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