Aakash
Medical|IIT-JEE|Foundations
(Divisions of Aakash Educational Services Limited)
Regd. Office :Aakash Tower, 8, Pusa Road, New Delhi-110005, Ph.011-47623456

Time : 3 hrs | MM : 264

Mock Test_CoE_XII

for JEE (Advanced) - 2020

Test – 1A (Paper – 1)_Actual Pattern-2015

ANSWERS

PHYSICS

1. (6)
2. (3)
3. (9)
4. (8)
5. (8)
6. (4)
7. (4)
8. (8)
9. (B, C)
10. (B, D)
11. (B, C, D)
12. (A, D)
13. (A, D)
14. (A, B, C)
15. (B, D)
16. (C, D)
17. (A, B, C)
18. (B, D)
19. A $\rightarrow$ (R)
    B $\rightarrow$ (P, S)
    C $\rightarrow$ (T)
    D $\rightarrow$ (Q)
20. A $\rightarrow$ (R, S)
    B $\rightarrow$ (Q)
    C $\rightarrow$ (P)
    D $\rightarrow$ (T)

CHEMISTRY

21. (8)
22. (9)
23. (2)
24. (5)
25. (9)
26. (8)
27. (8)
28. (5)
29. (B, D)
30. (A, C)
31. (A)
32. (A, B)
33. (A, B, C)
34. (A, D)
35. (B, D)
36. (B, C, D)
37. (A, C)
38. (A, B, C)
39. A $\rightarrow$ (P)
    B $\rightarrow$ (R, S)
    C $\rightarrow$ (S)
    D $\rightarrow$ (Q, T)
40. A $\rightarrow$ (S, T)
    B $\rightarrow$ (S, T)
    C $\rightarrow$ (Q, R)
    D $\rightarrow$ (Q, S)

MATHEMATICS

41. (6)
42. (3)
43. (4)
44. (2)
45. (5)
46. (7)
47. (4)
48. (3)
49. (A, C, D)
50. (A, C)
51. (A, C)
52. (B, C)
53. (A, B, C, D)
54. (A, B, D)
55. (A, B, C)
56. (A, D)
57. (A, B, C)
58. (A, D)
59. A $\rightarrow$ (R)
    B $\rightarrow$ (S)
    C $\rightarrow$ (T)
    D $\rightarrow$ (Q)
60. A $\rightarrow$ (Q)
    B $\rightarrow$ (T)
    C $\rightarrow$ (R)
    D $\rightarrow$ (P)

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Mock Test_CoE_XII

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Test – IA (Paper – I)_Actual Pattern-2015

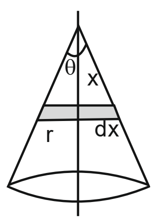
HINTS & SOLUTIONS

1. Answer (6)

From symmetry, we can obtain

$$E_{\text{vertex}} = E_0 \times \left(\frac{60}{360}\right) = \frac{E_0}{6}$$

2. Answer (3)



$$\tan\theta = \frac{R}{h}$$

$$\therefore \int dI = \int \frac{(dm)r^2}{2}$$

$$\Rightarrow I = \frac{3}{10}mR^2$$

$$= \frac{3}{10} \times 10 \times 1^2$$

= 3 kg m^2

3. Answer (9)

$$\frac{30-q}{6} + \frac{24+q}{3} = 0$$

$\Rightarrow q = 6\mu C$

$\therefore \ \Delta H = U_i - U_f$

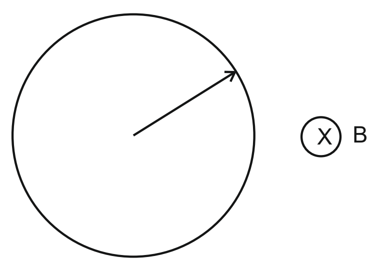
= 9 mJ

4. Answer (8)

$$-\int_0^P dp \times 4\pi r^2 = \int_R^0 \frac{M}{\frac{4}{3}\pi R^3} \times 4\pi r^2\, dr \times \frac{GMr}{R^3}$$

$$\Rightarrow P = \frac{3GM^2}{8\pi R^4}$$

5. Answer (8)



$$E \times 2\pi R = \frac{\pi R^2}{2}\left(\frac{dB}{dt}\right) = \pi R^2 \times 8t$$

$$\Rightarrow E = \frac{R}{2} \times 8t = 4Rt$$

$$\therefore \ E \times Q \times R = \mu \times mg \times R$$

$$\Rightarrow 8R \times Q \times R = \mu mgR$$

$$\Rightarrow \mu = \frac{8QR}{mg}$$

6. Answer (4)

Tension $T = \dfrac{M\omega^2 R}{2\pi}$

$$\therefore U = \frac{1}{2}\cdot\frac{YA}{2\pi R}\times\frac{M^2\omega^4R^4}{(YA)^2}$$

$$\frac{M^2\omega^4R^3}{4\pi YA}$$

7. Answer (4)

$$F = \int_{\theta=0}^{\pi/2}\sigma\times 2\pi R\sin\theta\times Rd\theta\times\left(\frac{\sigma_0\cos\theta}{2\epsilon_0}\right)^2\times\cos\theta$$

$$= \frac{\pi R^2\sigma_0^2}{4\epsilon_0}$$

8. Answer (8)

$$\frac{PV}{R} = T_0 + 16V^2 \text{ and } \frac{dP}{dV} = 0$$

$$\Rightarrow V = \sqrt{\frac{T_0}{16}} = \frac{\sqrt{T_0}}{4}$$

$$\therefore P_{\min} = 8R\sqrt{T_0}$$

9. Answer (B, C)

For highly viscous $h_1 = \frac{2S}{\rho gr}$

For zero viscosity $h_2 = \frac{4S}{\rho gr}$

$$\therefore h_1 < h < h_2$$

10. Answer (B, D)

$$F = -\frac{dU}{dx} = -\alpha \quad \text{for } x > 0$$

$$\therefore \text{Acceleration} = \frac{F}{m} = \frac{\alpha}{m}$$

$$\therefore \text{Time of stop } t_1 = \frac{v_0}{(\alpha/m)}$$

$$\therefore T = 4t_1 = 4\times\frac{mv_0}{\alpha}$$

11. Answer (B, C, D)

$$m_1 = \frac{-h_1}{h_0},\ m_2 = \frac{h_2}{h_0} \text{ and } m_1m_2 = 1$$

$$\Rightarrow \frac{m_2}{m_1} = \frac{h_2}{h_1} = 9$$

$$\therefore m_1 = \frac{1}{3},\ m_2 = 3$$

$$\therefore x_1 = 90\text{ cm},\ x_2 = 30\text{ cm}$$

$$\therefore \Delta x = 60\text{ cm}$$

$$\frac{1}{f} = \frac{1}{30} + \frac{1}{90}$$

$$= \frac{3+1}{90} \Rightarrow f = \frac{90}{4} = 22.5\text{ cm}$$

12. Answer (A, D)

Force on cord is independent of shape

F_{mag} = I l_0 B and, shape is circular as force is along normal

13. Answer (A, D)

$$B = \mu_0 nI = \mu_0\frac{N}{l}\times I$$

$$= \frac{\mu_0}{l}\times\frac{\sigma\times 2\pi Rl}{2\pi}\times\omega = \mu_0\sigma Rkt$$

$\Rightarrow$ B is uniform and increasing with time

And $E\times 2\pi r = \pi r^2\times\frac{dB}{dt} = \pi r^2\times\mu_0\sigma Rk$

$$\Rightarrow E = \frac{\mu_0\sigma Rkr}{2}$$

$\Rightarrow$ Non-uniform but constant

14. Answer (A, B, C)

$T_B = T_C = 2\times T_A = 600$ k

$$\Delta Q_{AB} = nC_p\Delta T = 1\times\left(\frac{5R}{2}\times 300\right) = 750R$$

$$W_{BC} + W_{DA} = 1R\times 2T_0\ln 2 - 1R\times T_0\ln 4 = 0$$

$W_{CD} = 0$, $W_{AB} = nR\Delta T = 1\times R\times 300 = 300R$

$$\therefore W_{total} = 300R,\ \therefore \Delta Q_{total} = 300R$$

15. Answer (B, D)

$2\mu t = (2n+1)\frac{\lambda}{2}$ for constructive

$\Rightarrow 2 \times 5 \times 10^{-7} \times 1.5 = (2n + 1)\ \frac{\lambda}{2}$

$\Rightarrow \lambda$ = 600 nm, 430 nm in visible range

16. Answer (C, D)

$$\omega = 2\pi \times \frac{50}{\pi} = 100$$

$$X_C = \frac{1}{\omega_C} = \frac{1}{100 \times 20 \times 10^{-6}} = 500\,\Omega$$
$$X_L = \omega L = 100\Omega$$

$$\therefore Z = \sqrt{R^2 + (X_C - X_L)^2} = 500\Omega$$

$$\therefore i_{rms} = \frac{50}{500} = 0.1 \text{ A}$$

$$V_C = 0.1 \times 500 = 50 \text{ V}$$

Power factor = $\frac{R}{Z} = 0.6$

17. Answer (A, B, C)

$$\frac{n \times (n-1)}{2} = 10 \Rightarrow n = 5 \text{ (orbit)}$$

So, $\Delta E = E_0 Z^2 \left(\frac{1}{n_1^2} - \frac{1}{5^2}\right) = \frac{144E_0}{225}$

$\Rightarrow$ Z = 3, n_1 = 3

18. Answer (B, D)

$$\frac{k(100 - T_1) \times 4\pi \times 30 \times 60}{(60 - 30)}$$

$$= \frac{k \times (T_1 - 0) \times 4\pi \times 60 \times 30}{(90 - 60)}$$

$$\Rightarrow \frac{(100 - T_1)30}{1} = \frac{T_1 \times 90}{1}$$

$\Rightarrow 100 - T_1 = 3T_1$

$\Rightarrow T_1 = 25°C$

And $\frac{k \times (100 - 50) \times 4\pi \times 30 \times r}{(r - 30)}$

$$= \frac{k \times (50 - 0) \times 4\pi \times r \times 90}{(90 - r)}$$

$$\Rightarrow \frac{30 \times r}{r - 30} = \frac{r \times 90}{90 - r}$$

$$\Rightarrow 90 - r = 3r - 90$$

$\Rightarrow$ 4r = 180 $\Rightarrow$ r = 45 cm

19. Answer A – R

B – P, S

C – T

D – Q

For (A) $a_C = \frac{5}{7} g \sin\theta$

$$f_S = \frac{2}{7} mg \sin\theta$$

$$\mu_{min} = \frac{2}{7} \tan\theta$$

For B- $I = \frac{5mR^2}{2}$

$$\therefore I_i = \frac{5mR^2}{2} + m\left(4R^2\right) = \frac{13mR^2}{2}$$

$$\therefore \frac{13mR^2}{2} \times \frac{a_c}{2R} = mg \sin\theta \times 2R$$

$$\Rightarrow a_c = \frac{8g \sin\theta}{13}$$

$$\therefore f_s = \frac{5mg \sin\theta}{13}$$
$$\therefore \mu_{min} = \frac{5}{13} \tan\theta$$

For (C) $I = \frac{62mR^2}{35}$

$$\therefore I_i = \frac{62mR^2}{35} + 3\left(4R^2\right) = \frac{202}{35} mR^2$$

$$\therefore \frac{202}{35} mR^2 \times \frac{a_c}{2R} = (mg \sin\theta) \times (2R)$$

$$\Rightarrow a_c = \frac{70g \sin\theta}{101}$$

$$\therefore f_s = \frac{31mg \sin\theta}{101}, \mu_{min} = \frac{31}{101} \tan\theta$$

20. Answer A – R, S
B – Q
C – P
D – T

For (A) I (0) = I_0

$$\phi(p) = \frac{2\pi}{\lambda}\cdot\frac{\left(\frac{d}{2}\right).d}{D} = \frac{2\pi}{\lambda}\cdot\frac{d^2}{2D}$$

$$= \frac{\pi}{500\times10^{-9}}\times\frac{10^{-6}}{1.5} = \frac{4\pi}{3}$$

$$\therefore I(P) = I_0\times\cos^2\left(\frac{2\pi}{3}\right) = \frac{I_0}{4}$$

For (B) $I(0) = I_0\cos^2\left(\frac{5\pi}{12}\right)$

$$I(P) = I_0\cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$$

For (C) $\phi(0) = \frac{2\pi}{\lambda}\cdot\frac{\left(\frac{d}{8}\right).d}{D}$

$$= \frac{\pi\times10^{-6}}{500\times10^{-9}\times4\times1.5} = \frac{\pi}{3}$$

$$\therefore\ I(O) = I_0\cos^2\left(\frac{\pi}{6}\right) = \frac{3I_0}{4}$$

$$\phi(P) = \frac{4\pi}{3}+\frac{\pi}{3} = \frac{5\pi}{3}$$

$$\therefore I(P) = \frac{3I_0}{4}$$

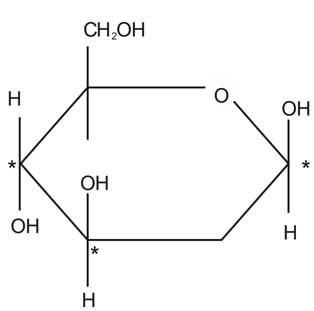
For(D) $\phi(0) = \frac{2\pi}{3}+\frac{\pi}{3} = \pi$

$$\therefore I(0) = 0$$

$$\phi(P) = \frac{2\pi}{3}+\frac{\pi}{3}+\frac{4\pi}{3} = \frac{7\pi}{3}$$

$$\therefore I(p) = I_0\cos^2\left(\frac{7\pi}{6}\right) = I_0\times\frac{3}{4}$$

21. Answer (8)



$2^3 = 8$

3 stereocentres as configuration about C – 5 is fixed (D)

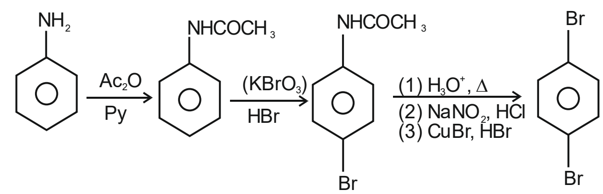
22. Answer (9)

$Kt_2 = \ln(10)$

$Kt_1 = \ln(8)$

$$\frac{t_2}{t_1} = \frac{\log10}{\log8} = \frac{1}{0.9} = \frac{10}{9}$$

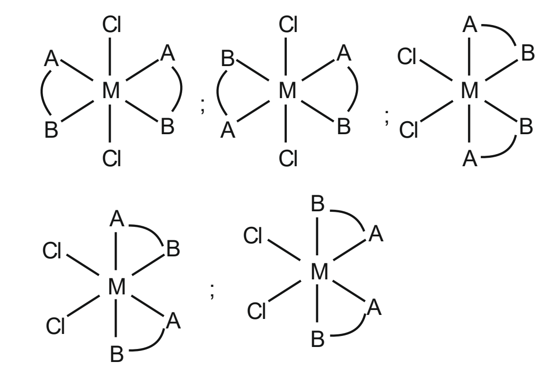
23. Answer (2)



$KBrO_3 + HBr \rightarrow Br_2$ (comproportionation)

24. Answer (5)

gly is a AB type of ligand



25. Answer (9)

For 0.2 M solution

K = G × G*

$$\Rightarrow G^* = \frac{K}{G} = \frac{1.4}{0.02} = 70\ m^{-1}$$

Now, for 0.5 m solution

K = G × G* = $70\times\frac{1}{280} = 0.25\ S\ m^{-1}$

For given system of units

$$\wedge_m = \frac{K}{C\times1000} = \frac{0.25}{0.5\times1000} = 5\times10^{-4}\ S\ m^2\ mol^{-1}$$

x = 5, y = 4

26. Answer (8)

n = 4 l = 0 m = 0

l = 1 m = –1, 0, 1

l = 2 m = –2, –1, 0, 1, 2

l = 3 m = – 3, –2, –1, 0, 1, 2, 3

Total of 4 orbitals *i.e.*, 8 electrons

27. Answer (8)

All except B_2O_3 are amphoteric.

28. Answer (5)

0.558 = i (1.86) (0.1)

i = 3

$\therefore$ Complex is $[CO(NH_3)_5Cl]Cl_2$

$\therefore$ x = 5

29. Answer (B, D)

$$KClO_3 \xrightarrow[MnO_2]{\Delta} O_2 \xrightarrow{P_4} P_4O_{10} \xrightarrow{HNO_3} \underset{\text{or } HPO_3}{H_3PO_4} + N_2O_5$$

30. Answer (A, C)

Diffusion Coefficient is proportional to both the mean free path as well as the mean speed of the gas.

31. Answer (A)

Due to NGP of –COOH group, the product will be having retention of tetrahedral configuration

32. Answer (A, B)

Reaction follows S_N2 mechanism. If R-X is in excess, then primary amine is major. If NH_3 is in excess then quaternary salt is formed as major product

33. Answer (A, B, C)

N_2 : BO = 3, LUMO = π^*, HOMO = σ

O_2^- : BO = 1.5, LUMO = σ^*, HOMO = π^*

C_2 : BO = 2, LUMO = σ, HOMO = π

Be_2^+ : BO = 0.5, LUMO = π, HOMO = σ^*

34. Answer (A, D)

Cr^{2+} acts as reducing agent and converts to Cr^{3+}. Mn^{3+} acts as oxidising agent and converts to Mn^{2+}

35. Answer (B, D)

Frenkel defect is stoichiometric defect

36. Answer (B, C, D)

Aldehydes and α hydroxyl ketones give positive tollen's test.

Benzaldehyde does not give positive Fehling's or Benedict's test

37. Answer (A, C)

Crossover products are not obtained in pinnacol - pinnacolone rearrangement

38. Answer (A, B, C)

$LiAlH_4$ will reduce both aldehyde as well as carboxylic acid to form the respective alcohol

39. Answer A – P

B – R, S

C – S

D – Q, T

Siderite $\rightarrow FeCO_3$

Chromite $\rightarrow FeCr_2O_4$

Cryolite $\rightarrow Na_3AlF_6$

Argentite $\rightarrow Ag_2S$

40. Answer A – S, T

B – S, T

C – Q, R

D – , Q, S

(A) $E^\circ_{cell} < 0, E_{cell} < 0$

(B) $E^\circ_{cell} < 0, E_{cell} < 0$

(C) $E^\circ_{cell} = 0, E_{cell} > 0$

$E^\circ_{cell} = 0, E_{cell} < 0$

41. Answer (6)

$$b^2 = \frac{r^2}{e^2} \Rightarrow b = \frac{r}{e}$$

$$\text{Area } = \pi ab = \frac{\pi b^2}{\sqrt{1-e^2}} = \frac{\pi r^2}{e^2\sqrt{1-e^2}}$$

Area is minimum when $e = \sqrt{\frac{2}{3}}$

42. Answer (3)

$\vec{t} + (\vec{t}\times\vec{r}) = \vec{s}$ ...(1)

taking the dot product, with $\vec{s}$

$\Rightarrow \vec{t}.\vec{s} + [\vec{t}\ \vec{r}\ \vec{s}] = 1$

$\Rightarrow [\vec{t}\ \vec{r}\ \vec{s}] = 1 - \vec{t}.\vec{s}$...(2)

taking dot product with $\vec{t}$ on (1) we get

$|\vec{t}|^2 = \vec{t}.\vec{s}$ ...(3)

by squaring the equation (1)

$|\vec{t}|^2 + |\vec{t}\times\vec{r}|^2 + 2\vec{t}.(\vec{t}\times\vec{r}) = |\vec{s}|^2$

$\Rightarrow |\vec{t}|^2 + |\vec{t}^2|^2 \sin^2\theta = 1$

$\Rightarrow |\vec{t}|^2 = \frac{1}{1+\sin^2\theta} \ge \frac{1}{2}$

$\Rightarrow \vec{t}.\vec{s} \ge \frac{1}{2}(\text{since }(3))$

$\Rightarrow 1 - [\vec{t}\ \vec{r}\ \vec{s}] \ge \frac{1}{2}(\text{since }(3))$

$\Rightarrow [\vec{t}\ \vec{r}\ \vec{s}] \le \frac{1}{2} \Rightarrow \text{Max. valume} = \frac{1}{2}$

$\Rightarrow \frac{1}{2} = \frac{3}{2K} \Rightarrow K = 3$

43. Answer (4)

$$\frac{a}{r_1} = \frac{4R\sin A/2\cos A/2}{4R\sin A/2\cos B/2\cos C/2}$$

$$= \tan\frac{B}{2} + \tan\frac{C}{2}$$

$$\left(\text{By using} \cos A/2 = \sin\frac{(B+C)}{2}\right)$$

now $\sum\frac{a}{r_1} = 2\Sigma\tan\frac{A}{2}$

$= 2\sum\frac{r_1}{s}$

$= 2\left(\frac{r_1+r_2+r_3}{s}\right)$

$= 4\left(\frac{r_1+r_2+r_3}{2s}\right)$

$\Rightarrow \left(\frac{a}{r_1}+\frac{a}{r_2}+\frac{a}{r_3}\right)\left(\frac{a+b+c}{r_1+r_2+r_3}\right) = 4$

44. Answer (2)

$x = \frac{1}{t}$

$$\lim_{x\to\infty}\left(\frac{f\left(3+\frac{3}{x}\right)}{f(3)}\right)^x = \lim_{x\to\infty}\left(\frac{f(3+3t)}{f(3)}\right)^{1/t}$$

$= e^{\lim_{t\to 0}\left(\frac{f(3+3t)-f(3)}{t\,f(3)}\right)}$

by L-hospital's rule

$= e^{\lim_{t\to 0}\frac{3f'(3+3t)}{f(3)}}$

$= e$

$\Rightarrow L = e \Rightarrow [L] = 2$

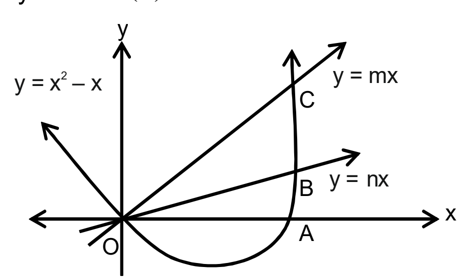
$\Rightarrow \frac{[L]}{4} = \frac{1}{2} = 0.50$

45. Answer (5)

Let $y = x^2 - x$ ...(1)

$y = mx$ ...(2)

$y = nx$ ...(3)



by solving (1) and (2)

x=0, x=m+1

similarly by (2) and (3) x=0, x=n+1

required area $= ar(OABCO) - ar(OABO)$

$$\frac{37}{6} = \int_0^{m+1}(mx - x^2 + x)dx - \int_0^{n+1}(nx - x^2 + x)dx$$

$$\frac{37}{6} = \frac{(m+1)^3}{6} - \frac{(n+1)^3}{6}$$

$\Rightarrow a^3 - b^3 = 37 \quad (\text{Let } m+1=a, n+1=b)$

$\Rightarrow (a-b)(a^2+ab+b^2) = 37$

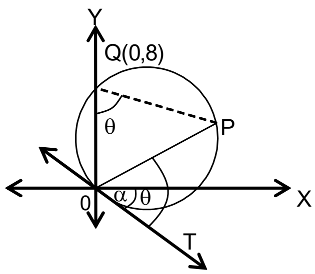
$\Rightarrow$ a-b =1 and $a^2+b^2+ab=36$

$\Rightarrow (a,b)=(4,3) \Rightarrow (m,n)=(3,2)$

46. Answer (7)

equation of tangent to $x^2+y^2-6x-8y=0$ at (0,0) is 3x+4y=0

$\Rightarrow$ Slope of $OT = \frac{-3}{4}$



Let $\angle TOX = \alpha \Rightarrow \tan\alpha = \frac{3}{4}$

$\angle OQP = \angle TOX = \theta \Rightarrow \angle XOP = \theta - \alpha$

given $\tan^{-1}\left(\frac{5}{4}\right) = \theta \Rightarrow \tan\theta = \frac{5}{4}$

slope of $OP = \tan(\theta-\alpha)$

$= \frac{\tan\theta - \tan\alpha}{1+\tan\theta\tan\alpha} = \frac{8}{31}$

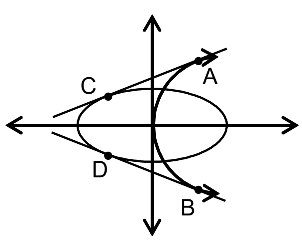
equation of OP is 8x-31y=0

47. Answer (4)

equation of tangent to $y^2=4ax$ at $(at^2, 2at)$ is $yt=x+at^2$

equation of tangent to $y^2=4x$ is $y = \frac{x}{t} + t$

this is also tangent to ellipse



$\Rightarrow c^2 = a^2m^2 + b^2$

$\Rightarrow t^4 - 6t^2 - 16 = 0$

$\Rightarrow t = \pm 2\sqrt{2}$

$\Rightarrow A = (8, 4\sqrt{2})$ and $B = (8, -4\sqrt{2})$

equation of tangent at A is $x - 2\sqrt{2y} = -8$ ...(1)

let equation of tangent to ellipse at C be $3x_1x+8y_1y=48$ ...(2)

(1) and (2) represents same line

$\Rightarrow C = \left(-2, \frac{3}{\sqrt{2}}\right)$

similarly we get $D = \left(-2, \frac{-3}{\sqrt{2}}\right)$

now equation circle with AB as diameter is $x^2+y^2-16x+32=0$ and $S_1 = 32\pi$

equation of circle with CD as diameter is

$x^2 + y^2 + 4x - \frac{1}{2} = 0$ and $S_2 = \frac{9\pi}{2}$

48. Answer (3)

$xy = 4 \Rightarrow P = \left(2t, \frac{2}{t}\right)$

if the normal at $P(t_1)$ intersects the hyperbola again at t_2 then $t_1^3\, t_2 = -1$

$\Rightarrow Q = \left(\frac{-2}{t^3}, -2t^3\right)$

Slope of $OP = \frac{1}{t^2}$

Slope of $OQ=t^6$

Now $\tan\alpha = \frac{\frac{1}{t^2} - t^6}{1+t^4} = \frac{1-t^4}{t^2}$

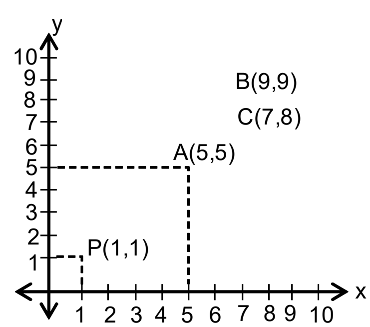
Slope of normal chord $PQ=t^2$

$\Rightarrow \tan\beta = \frac{t^2 - \frac{1}{t^2}}{1+1} = \frac{t^4-1}{2+2}$

Now $\frac{\tan\alpha}{\tan\beta} = -2 \Rightarrow \frac{\sin\alpha\cos\beta}{\sin\beta\cos\alpha} = -2$

$\Rightarrow \frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)} = 3$

49. Answer (A, C, D)



Let P=(1,1)

A=(5,5), B=(9,9), C=(7,8)

Let $L_1=(x+2, y) \to$ horizontally 2 units

$L_2=(x+4, y) \to$ horizontally 4 units

$U_1=(x, y+2) \to$ Vertically 2 units

$U_2=(x,y+4) \to$ vertically 4 units

to reach from P to A 4 units horizontally and 4 units vertically to be travel

$L_1 L_1 U_1 U_1 = \frac{4!}{2!2!} = 6$ ways

$L_1L_1U_2 = \frac{3!}{2!} = 3$ ways

$L_2U_1U_1 = \frac{3!}{2!} = 3$

$L_2U_2 = 2! = 2$

total no.of ways=14

no.of ways to travel from P to B = (no.of ways from P to A). (no.of ways from A to B)

=14×14=196

50. Answer (A, C)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

total number of matrices = 2^9

$S_1S_2S_3t_1t_2t_3$ is an odd integer if

(i) all the elements are 1

Number of matrices =1

(ii) four zeroes and five ones (where zeros must occupy all the positions of any minor)

number of matrices=9

(iii) Six zeros and three ones

number of matrices ${}^3C_1\ {}^2C_1\ 1 = 6$

required probability $= \frac{16}{2^9} = \frac{1}{32}$

$\Rightarrow$ m=1 and n=32

number of divisors of mn=(5+1)=6

51. Answer (A, C)

$$I_m = \int_0^{m\pi} |\sin x|\, e^{\cos 4x} dx$$

by $\int_0^{kT} f(x)dx = k\int_0^{T} f(x)dx$

$$I_m = m\int_0^{\pi} |\sin x|\, e^{\cos 4x} dx$$

$$\Rightarrow I_m = 2m\int_0^{\pi/2} \sin x\ e^{\cos 4x} dx \ ...(1)$$

now $S_n = \int_0^{n\pi} x\,|\cos x|\,e^{\cos 4x} dx$ ....(2)

apply $\int_0^{b} f(a+b-x)dx = \int_a^{b} f(x)dx$

$$S_m = \int_0^{n\pi} (n\pi - x)\,|\cos x|\,e^{\cos 4x}\ dx \ ...(3)$$

by (2) + (3) $\Rightarrow 2S_n = n\pi\int_0^{n\pi} |\cos x|\, e^{\cos 4x} dx$

$$2S_n = n^2\pi \int_0^{\pi} |\cos x|\, e^{\cos 4x} dx$$

$$= 2n^2\pi\int_0^{\frac{\pi}{2}} \cos x\ e^{\cos 4x} dx$$

$$S_n = n^2\pi\int_0^{\frac{\pi}{2}} \sin x\ e^{\cos 4x} dx \ ...(4)$$

Now $(4) \div (1) \Rightarrow \frac{S_n}{I_m} = \frac{n^2\pi}{2m}$

52. Answer (B, C)

Let $\frac{2+\sin^2 x}{1+\sin^2 x} = t \Rightarrow t \in \left[\frac{3}{2}, 2\right]$

$t^2 - (a-3)t + (a-4) = 0$

$\Rightarrow t = 1$ or $a - 4$

Clearly $a - 4 \in \left[\frac{3}{2}, 2\right]$

$a \in \left[\frac{11}{2}, 6\right]$

53. Answer (A, B, C, D)

Expand C_3 and write as sum of 2 determinants. The value of determinate will be zero

54. Answer (A, B, D)

$$I = \int_{-1}^{2} \frac{\sin x\, f(x^2)}{2 + f(x+3)} dx$$

$$= \int_{-1}^{0} \frac{\sin x(0)}{2+0}dx + \int_{0}^{1} \frac{\sin x(0)}{2+0}dx + \int_{1}^{\sqrt{2}} \frac{\sin x(1)}{2+0}dx + \int_{\sqrt{2}}^{\sqrt{3}} \frac{\sin x(0)}{2+0}dx + \int_{\sqrt{3}}^{2} \frac{\sin x(0)}{2+0}$$

$$I = \frac{1}{2}\int_1^{\sqrt{2}} \sin x dx = -\frac{1}{2}\left[\cos\sqrt{2} - \cos 1\right]$$

$$I = \sin\left(\frac{\sqrt{2}+1}{2}\right)\sin\left(\frac{\sqrt{2}-1}{2}\right)$$

$$\Rightarrow k = 1, \alpha = \frac{\sqrt{2}+1}{2}, \beta = \frac{\sqrt{2}-1}{2}$$

55. Answer (A, B, C)

$f(x) = \sin x + ax + b = 0$

$f'(x) = \cos x + a$

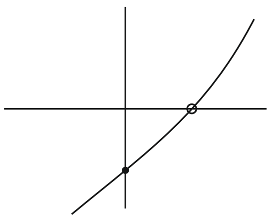
If a > 1 f′(x) > 0 $\forall x \in R \Rightarrow f(x)$ increasing

If a < – 1 $f'(x) < 0$ $\forall x \in R \Rightarrow f(x)$ decreasing

Option 1 . a > 1, b < 0

$f(0) = b < 0$

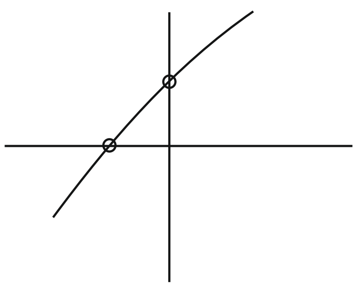
True



Option2: a = 1 b > 0

$f(0) = b > 0$

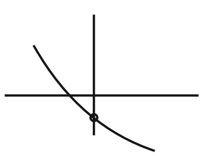
True



Option 3: a < – 1, b < 0

$f(0)$ = b > 0

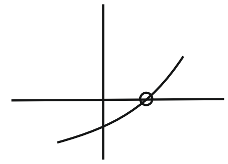
True



Option 4: a > 1, b < 0

$f(0)$ = b < 0

False



56. Answer (A, D)

AM > GM

$$\Rightarrow \sum k \sum \frac{1}{k} > (2001)^2$$

$$\Rightarrow \sum \frac{1}{k} > 1 \Rightarrow S > 1$$

Also

$$S < \frac{500}{1000} + \frac{500}{1500} + \frac{500}{2000} + \frac{500}{2500} + \frac{1}{3001}$$

$$S < \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{3000} = \frac{3851}{3000} < \frac{4}{3}$$

57. Answer (A, B, C)

$N^2 = 2^4 \times 3^6 \times 7^8$

(A) Total number of factors = 5 × 7 × 9 = 315

Number of factors less than N = 156

Number of factors of N = 58

$\therefore$ Answer = 156 – 58 = 98

(B) If the factor of the form 3^α, 7^β, where $\alpha + \beta$ is even is of the form 4K + 1

Answer = 9

(C) $\frac{N}{54} = 2 \times 7^4$ Number of factors = 10

58. Answer (A, D)

59. Answer A – R

B – S

C – T

D – Q

(A) x + y + z = 0

$\therefore$ The number of integral solutions of x +y + z = 0 is

= coefficient of x^0 in$\left(x^{-3} + x^{-2} + x^{-1} + x^0 + x^3 + x^1\right)^1 = 25$

(B) Let the position vectors of P, Q and S be $\bar{o},\bar{q},\bar{s}$

The equation of

$$AB : \frac{\bar{q}+4(\bar{q}+S)}{5}, \bar{B}=\frac{\bar{q}+S+4\bar{S}}{5}$$

$$\therefore \overline{PG}=\frac{21}{5}\overline{PR}$$

(C) Let the position vectors of A, B, C and D are $\bar{a},\bar{b},\bar{c}$ & $\bar{o}$

$$\Rightarrow \left|\overline{AB}\times\overline{CD}+\overline{BC}\times\overline{AD}+\overline{CA}\times\overline{BD}\right|$$

$$= 2\left|\overline{AC}\times\overline{AB}\right| = 4\frac{1}{2}\left|\overline{AC}\times\overline{AB}\right| = 4\Delta$$

(D) x + y + z = 3

$$\frac{10x}{3-x}+\frac{10y}{3-y}+\frac{10z}{3-z}=10\left(-3+3\left(\frac{1}{3-x}+\frac{1}{3-y}+\frac{1}{3-z}\right)\right)$$

$$\geq 10(-3+3(3/2))$$

$$\geq 15$$

60. Answer A – Q

B – T

C – R

D – P

$$f(x+y)=f(x)+f(y)-xy$$

By x = y = 0 $\Rightarrow f(0)=0$

Now $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$=\lim_{h\to 0}\frac{f(x)+f(h)-hx-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(h)}{h}-x$$

$$=\lim_{h\to 0}\frac{f(h)-f(0)}{h}-x \text{ (since f(0)=0)}$$

$$\Rightarrow f'(x)=f'(0)-x$$

$$\Rightarrow f(x)=x-\frac{x^2}{2}+c \Rightarrow f(x)=x-\frac{x^2}{2} \text{ (since c=0)}$$

now $x^2y\frac{d^2y}{dx^2}+\left(x+\frac{dy}{dx}-y\right)^2=0$

$$\Rightarrow x^2\left[y\frac{d^2y}{dx^2}+\frac{dy}{dx}\right]^2-\left[2xy\frac{dy}{dx}-y^2\right]=0$$

$$\Rightarrow \frac{d}{dx}\left(y\frac{dy}{dx}\right)-\frac{d}{dx}\left(\frac{y^2}{x}\right)=0$$

by integrating

$$\Rightarrow y\frac{dy}{dx}-\frac{y^2}{x}=c_1$$

Let $y^2=v \Rightarrow 2yy'=\frac{dv}{dx}$

$$\Rightarrow \frac{dv}{dx}-\frac{2}{x}v=2c_1 \text{ is L.D.E}$$

$$\Rightarrow \frac{v}{x^2}=\frac{-2c_1}{x}+c_2 \Rightarrow y^2=-2c_1x+c_2x^2$$

as g(x) passes through (1, 0) and (2, 1). then

$$\Rightarrow c_1=\frac{1}{4}, c_2=\frac{1}{2}$$

$$\Rightarrow g(x)=\frac{x^2-x}{2}$$

(A) the graph of $y=|f(|x|)|$ will have 3 sharp edges hence no.of non-differentiable points is 3.

(B) $g(x)=\sqrt{\frac{x^2-x}{2}} \Rightarrow$ g(0) = 0 has one local minima

(C) by solving g(x) = f(x) we get two solutions

(D) Required area

$$A=\int_0^2\left(x-\frac{x^2}{2}\right)dx=\left[\frac{x^2}{2}-\frac{x^3}{6}\right]_0^2=\frac{2}{3}$$

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